Precoding for Space-Time Block Codes in (Non-)Kronecker Correlated MIMO Channels

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***Abstract*— A memoryless linear precoder is designed for space-time block codes (STBC) for quasi static non-frequency selective correlated Rayleigh fading multiple-input multiple-output (MIMO) channels. The precoder is designed to minimize a symbol error-based metric as function of the joint transmit-receiver channel correlation coefficients which are supposed to be fed back to the transmitter. The correlation may or may not follow the *Kronecker structure*. We demonstrate in particular the impact of the precoder on receive correlated channels when the Kronecker model does not hold. A numerical optimization method is proposed that can be used for invertible correlation matrices. Monte Carlo simulations show that the proposed precoder outperforms a system not having a precoder for highly correlated channels.**

I. Introduction

In the area of efficient communications over non-reciprocal MIMO channels, recent research has demonstrated the value of feeding back to the transmitter information about channel state observed at the receiver. Clearly, the type of feedback may vary largely, depending on its nature, e.g., required rate, instantaneous, or statistical channel state information (CSI), leading to various transmitter design schemes, e.g., [1], [2], [3]. Among those, there has been a growing interest in transmitter schemes that can exploit low-rate long-term statistical CSI in the form of antenna correlation coefficients. So far, emphasis has been on designing precoders for space-time block coded (STBC) [2] signals or spatially multiplexed streams that are adjusted based on the knowledge of the transmit correlation only while the receiving antennas are uncorrelated [4], [5], [6], [7]. These techniques are well suited to downlink situation where an elevated access point (situated above the surrounding clutter) transmits to a subscriber placed in a rich scattering environment. However, to the best of our knowledge, the corresponding uplink case has not been addressed before nor the case where both transmit and receive antennas exhibit correlation, except to some extent in [3] where the instantaneous channel state side information feedback case is treated. Although simple models exist for the joint transmit receiver correlation based on the well known Kronecker structure [2], the accuracy of these models has recently been questioned in the literature based on measurement campaigns [8]. Therefore, there is interest in investigating the precoding of STBC signals for MIMO channels that *do not* necessarily follow the Kronecker structure.

In this paper, we address the problem of linear precoding of STBC signals launched over a jointly transmit-receive correlated MIMO channel. Our contributions are three fold:

1) We propose a technique reminiscent of [4] in that we minimize certain bounds on the pair-wise error probability (PEP) of the STBC signal, where the choice of the STBC is given in advance. The precoder is obtained via an iterative algorithm which uses the knowledge of the full transmit-receive correlation, regardless of whether the Kronecker structure is valid or not.
2) We show that in the case that correlation *happens to be* Kronecker based, *and* the transmit antennas are uncorrelated, then the receive antenna correlation does not have any impact on the precoder design for STBC signals. In contrast, we point out that if the transmit antennas are correlated, then the receive correlation *does* play a role in the precoder design.
3) Finally, we exhibit intuitive, closed-form solutions for the precoder in the special case where only the receive antennas are correlated yet the Kronecker structure does not hold. We give a practical example for this situation.

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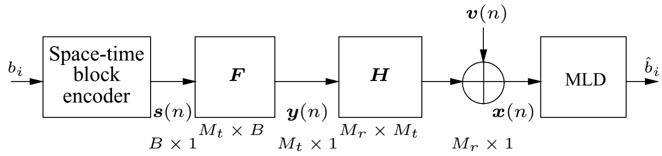

Fig. 1. Block model of the linear precoded STBC MIMO system.

We measure the bit error rate (BER) versus signal to noise ratio (SNR) system performance by Monte Carlo simulations over correlated non-frequency selective Rayleigh fading channels and provide additional perspectives.

II. System Description

A. STBC Signal Model

Figure 1 shows the block MIMO system model with M_t and M_r transmitter and receiver antennas, respectively. The original bits sent from the transmitter are denoted b_i and the decoded bits $\hat{b}_i$. We consider the problem of linearly precoding signals originating from a space-time block encoder with codeword matrix of size $B \times N$ where B and N are the space and time dimension, respectively, and where the codewords are mapped from the input bits in some unspecified way.

A codeword $\boldsymbol{S}(n)$ is formed by N successive code vectors $\boldsymbol{s}(k)$

$$\boldsymbol{S}(n) = [\boldsymbol{s}(Nn) \;\; \boldsymbol{s}(Nn-1) \;\; \cdots \;\; \boldsymbol{s}(Nn-N+1)], \qquad (1)$$

where it is assumed with little loss of generality[1] that

$$E\left[\boldsymbol{S}(n)\boldsymbol{S}^H(n)\right] = \kappa \boldsymbol{I}_B, \qquad (2)$$

where κ is a positive constant given by the STBC used.

Before each code vector is launched into the channel, it is precoded with a memoryless matrix $\boldsymbol{F}$ of size $M_t \times B$, so the $M_r \times 1$ receive signal model becomes

$$\boldsymbol{x}(n) = \boldsymbol{H}\boldsymbol{F}\boldsymbol{s}(n) + \boldsymbol{v}(n), \qquad (3)$$

where the additive noise on the channel $\boldsymbol{v}(n)$ is complex Gaussian circularly distributed with independent components having variance σ^2.

[1] This assumption is exact for orthogonal block codes.

B. Correlated Channel Models

A quasi-static non-frequency selective correlated Rayleigh fading channel model [2] is assumed. Let $\boldsymbol{R}$ be the general $M_tM_r \times M_tM_r$ positive definite autocorrelation matrix for the channel coefficients. A channel realization of the correlated channel can then be found by

$$\operatorname{vec}(\boldsymbol{H}) = \boldsymbol{R}^{1/2}\operatorname{vec}(\boldsymbol{H}_w), \tag{4}$$

where $\boldsymbol{R}^{1/2}$ is the unique positive definite matrix square root [9] of $\boldsymbol{R}$, $\boldsymbol{H}_w$ has size $M_r \times M_t$ and is complex Gaussian circularly distributed with independent components all having unit variance, and the operator $\operatorname{vec}(\cdot)$ stacks the columns of the matrix it is applied to into a long column vector [9].

Kronecker model: A special case of the model above is as follows [2]

$$\boldsymbol{H} = \boldsymbol{R}_r^{1/2}\boldsymbol{H}_{\boldsymbol{w}}\boldsymbol{R}_t^{1/2}, \tag{5}$$

where the matrices $\boldsymbol{R}_r$ and $\boldsymbol{R}_t$ are the correlations matrices of the receiver and transmitter, respectively, and their sizes are $M_r \times M_r$ and $M_t \times M_t$. The full autocorrelation matrix $\boldsymbol{R}$ of the model (4) is then given by

$$\boldsymbol{R} = E\left[\operatorname{vec}(\boldsymbol{H})\operatorname{vec}^H(\boldsymbol{H})\right] = \boldsymbol{R}_t^T \otimes \boldsymbol{R}_r, \tag{6}$$

where the operator $(\cdot)^T$ denotes transposition and $\otimes$ is the Kronecker product. This channel model was used in [4], to find a linear precoder when $\boldsymbol{R}_r = \boldsymbol{I}_{M_r}$. We address below the precoding problem in the general case of Equation (4). Unlike Equation (6), the general model considers that the receive (or transmit) correlation depends on at which transmit (or receive) antenna the measurements are performed.

III. Precoding of STBC signals

A. Optimal Precoder Problem Formulation

Maximum likelihood decoding is assumed at the receiver. The goal is to find the matrix $\boldsymbol{F}$ such that an upper bound for the pairwise error probability (PEP) is minimized under an appropriate power constraint, for given channel correlation properties.

The receiver is assumed to know the channel matrix $\boldsymbol{H}$ exactly and it performs a maximum likelihood decoding (MLD) of blocks of length N. The transmitter knows $\boldsymbol{R}$.

Suppose codeword $\boldsymbol{S}_k(n)$ is transmitted while $\boldsymbol{S}_l(n)$ is detected. Let $\boldsymbol{E}_{k,l}(n) = \boldsymbol{S}_k(n) - \boldsymbol{S}_l(n)$ be the error matrix of size $B \times N$. The probability of transmitting the block $\boldsymbol{S}_k(n)$ and decoding the block $\boldsymbol{S}_l(n)$ for a given channel is

$$\begin{aligned}&\Pr\{\boldsymbol{S}_k(n) \to \boldsymbol{S}_l(n)|\boldsymbol{H}\}\\ &= Q\left(\sqrt{\frac{\operatorname{Tr}\{\boldsymbol{H}\boldsymbol{F}\boldsymbol{E}_{k,l}(n)\boldsymbol{E}_{k,l}^H(n)\boldsymbol{F}^H\boldsymbol{H}^H\}}{2\sigma^2}}\right).\end{aligned} \tag{7}$$

Equation (7) follows from [2]. Let the operator $\det(\cdot)$ denote the determinant of the matrix it is applied to. If the statistics of the channel $\boldsymbol{H}$ are taken into consideration and the expression for the Q-function given in [10] is used, then the probability $\Pr\{\boldsymbol{S}_k(n) \to \boldsymbol{S}_l(n)\}$ can be found:

$$\begin{aligned}&\Pr\{\boldsymbol{S}_k(n) \to \boldsymbol{S}_l(n)\}\\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{d\theta}{\det(\boldsymbol{R})\det\left(\boldsymbol{R}^{-1} + \frac{\boldsymbol{F}\boldsymbol{E}_{k,l}(n)\boldsymbol{E}_{k,l}^H(n)\boldsymbol{F}^H}{4\sigma^2\sin^2\theta} \otimes \boldsymbol{I}_{M_r}\right)}.\end{aligned} \tag{8}$$

The performance measure that used in this article is an upper bound for the pair-wise error probability given by Equation (8). Analogous to [4], define $\boldsymbol{E} = \underset{\{k\neq l,n\}}{\operatorname{argmin}} \det\left(\boldsymbol{E}_{k,l}(n)\boldsymbol{E}_{k,l}^H(n)\right)$. For orthogonal STBC $\boldsymbol{E}_{k,l}(n)\boldsymbol{E}_{k,l}^H(n) = \beta_{k,l}\boldsymbol{I}_B$. Let $\beta = \min_{k\neq l}\{\beta_{k,l}\}$.

The total block error probability is decided by many terms of the type given in Equation (8) for different values of k and l. The term that is used as the optimization in this article is the following:

$$\det\left(\boldsymbol{R}^{-1} + \frac{\boldsymbol{F}\boldsymbol{E}\boldsymbol{E}^H\boldsymbol{F}^H}{4\sigma^2} \otimes \boldsymbol{I}_{M_r}\right). \tag{9}$$

This criterion is closely related to the criteria used in [3], [4].

Using Equation (2), the power constraint on the transmitted block $\boldsymbol{Y}(n) = \boldsymbol{F}\boldsymbol{S}(n)$ can be formulated as

$$\kappa\operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P, \tag{10}$$

where P is the average power used by the transmitted block $\boldsymbol{Y}(n)$.

We propose that the optimal precoder is given by the following optimization problem:

Problem 1:

$$\begin{aligned}&\max_{\{\boldsymbol{F}\in\mathbb{C}^{M_t\times B}\}} \det\left(\boldsymbol{R}^{-1} + \boldsymbol{F}\boldsymbol{E}\boldsymbol{E}^H\boldsymbol{F}^H \otimes \frac{\boldsymbol{I}_{M_r}}{4\sigma^2}\right)\\ &\text{subject to} \quad \kappa\operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\} = P.\end{aligned}$$

B. Properties of the Optimal Precoder

In this subsection, lemmas characterizing the optimal precoder in special cases are presented.

Lemma 1: If $\boldsymbol{F}$ is an optimal solution of Problem 1 for an orthogonal STBC, then the precoder $\boldsymbol{F}\boldsymbol{U}$, where $\boldsymbol{U} \in \mathbb{C}^{B\times B}$ is unitary, is also optimal.

Proof: For an orthogonal STBC, we have $\boldsymbol{E}\boldsymbol{E}^H = \beta\boldsymbol{I}$. Let $\boldsymbol{F}$ be an optimal solution of Problem 1 and $\boldsymbol{U} \in \mathbb{C}^{B\times B}$, be an arbitrary unitary matrix. It is then seen by insertion that the objective function and the power constraint are unaltered by the unitary matrix. ■

Lemma 2: Assume that $B = M_t$ and that only receiver correlation is present. Let the total correlation matrix be given by

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{r_0} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{R}_{r_1} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{R}_{r_{M_t-1}} \end{bmatrix}, \tag{11}$$

where $\boldsymbol{R}_{r_i}$ is the receive correlation matrix seen by transmitter number i and the matrix $\boldsymbol{0}_{k\times l}$ has size $k \times l$ containing only zeroes. Then, the optimal $\boldsymbol{F}$ can be chosen diagonal up to a unitary matrix.

Proof: Let the eigenvalue decomposition of $\boldsymbol{R}_{r_i}$ be given by

$$\boldsymbol{R}_{r_i} = \boldsymbol{V}_{r_i}\boldsymbol{\Lambda}_{r_i}\boldsymbol{V}_{r_i}^H, \tag{12}$$

where $\boldsymbol{V}_{r_i} \in \mathbb{C}^{M_r\times M_r}$ is unitary and $\boldsymbol{\Lambda}_{r_i} \in \mathbb{R}^{M_r\times M_r}$ is diagonal with positive diagonal elements λ_{i_k}. It follows that the eigenvalue decomposition of $\boldsymbol{R} = \boldsymbol{V}\boldsymbol{\Lambda}\boldsymbol{V}^H$ is given by the matrices

$$\boldsymbol{V} = \begin{bmatrix} \boldsymbol{V}_{r_0} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{V}_{r_1} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{V}_{r_{M_t-1}} \end{bmatrix}, \tag{13}$$

and

$$\boldsymbol{\Lambda} = \begin{bmatrix} \boldsymbol{\Lambda}_{r_0} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{\Lambda}_{r_1} & \cdots & \boldsymbol{0}_{M_r\times M_r} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{0}_{M_r\times M_r} & \boldsymbol{0}_{M_r\times M_r} & \cdots & \boldsymbol{\Lambda}_{r_{M_t-1}} \end{bmatrix}. \tag{14}$$

The objective function of Problem 1 can now be rewritten as:

$$\det\left(\boldsymbol{\Lambda}^{-1} + \frac{1}{4\sigma^2}\boldsymbol{V}^H\left(\boldsymbol{F}\boldsymbol{F}^H \otimes \boldsymbol{I}_{M_r}\right)\right). \tag{15}$$

Block element number (k,l) of size $M_r \times M_r$ of the second term within the determinant of Equation (15) can be expressed as: $\left(\boldsymbol{F}\boldsymbol{F}^H\right)_{k,l} \boldsymbol{V}_{r_k}^H \boldsymbol{V}_{r_l} / \left(4\sigma^2\right)$. Let the second term within the determinant of Equation (15) be denoted $\boldsymbol{A}$. By using Hadamard's inequality [11] on $\det\left(\boldsymbol{\Lambda}^{-1} + \boldsymbol{A}\right)$, this determinant is maximized when $\boldsymbol{A}$ is diagonal. From the structure of $\boldsymbol{A}$ and due to the fact that the matrices $\boldsymbol{V}_{r_i}$ are unitary, it follows that $\boldsymbol{F}\boldsymbol{F}^H$ is diagonal. Hence, $\boldsymbol{F}$ is diagonal up to a unitary matrix. ■

Lemma 3: Let $B = M_t$, and let the total correlation matrix be given by Equation (11). The diagonal element number i of the optimal diagonal product $\boldsymbol{F}\boldsymbol{F}^H$ is denoted α_i. Let eigenvalue number k of the correlation matrix $\boldsymbol{R}_{r_i}$ be denoted $\lambda_{r_{i_k}}$. The optimization problem that must be solved is the following: Find α_i such that the following product is maximized:

$$\prod_{i=0}^{M_t-1} \prod_{k=0}^{M_r-1} \left(1 + \frac{\alpha_i \lambda_{r_{i_k}}}{4\sigma^2}\right) \tag{16}$$

subject to

$$\kappa \sum_{i=0}^{M_t-1} \alpha_i = P, \quad \alpha_i \geq 0. \tag{17}$$

Proof: Under the assumptions in the lemma, the determinant on Equation (15) can be written as

$$\begin{aligned} &\prod_{i=0}^{M_t-1} \det\left(\boldsymbol{\Lambda}_{r_i}^{-1} + \alpha_i \boldsymbol{I}_{M_r}\right) \\ &= \left(\prod_{i=0}^{M_t-1} \det\left(\boldsymbol{\Lambda}_{r_i}^{-1}\right)\right)\left(\prod_{i=0}^{M_t-1} \det\left(\boldsymbol{I}_{M_r} + \alpha_i \boldsymbol{\Lambda}_{r_i}\right)\right). \end{aligned} \tag{18}$$

The first product in the last line of this equation is a constant and the second factor can be rewritten to Equation (16). The power constraint reformulation follows directly by inserting the diagonal matrix product $\boldsymbol{F}\boldsymbol{F}^H$ into Equation (10). Since α_i is diagonal element number i of a positive semi-definite matrix, it must be non-negative. ■

Example 1: Let the assumptions of Lemma 3 be valid. Let $M_t = M_r = 2$ with $\boldsymbol{R}_{r_0} = \boldsymbol{I}_2$ and $\boldsymbol{R}_{r_1} = \boldsymbol{1}_{2\times 2}$, where the matrix $\boldsymbol{1}_{k\times l}$ has size $k \times l$ containing only ones. In this case, $\lambda_{r_{0_0}} = \lambda_{r_{0_1}} = 1$ and $\lambda_{r_{1_0}} = 2$ and $\lambda_{r_{1_1}} = 0$. If the optimization problem in Lemma 3 is solved, it is found that $\alpha_0 = 2/3$ and $\alpha_1 = 1/3$. This makes intuitive sense, since more power is poured into the channel exhibiting more diversity.

Lemma 4: Let the correlation model of the channel follow the Kronecker model in Equation (6) and assume that an orthogonal STBC is used. If $\boldsymbol{R}_t = \boldsymbol{I}_{M_t}$, then the optimal precoder is independent of the receiver correlation matrix $\boldsymbol{R}_r$.

Proof: See [12]. ■

C. An Analytical Solution for Receiver Correlation

In this subsection, we show that a closed-form solution can be found if only receiver correlations are present. The solution is constructed upon the premise that, on average, the gain coming from all diversity branches should be equal, regardless whether the channels are correlated or uncorrelated. This is the same criterion behind complex orthogonal block codes where symbols are also spread with equal energy across all channels, and guarantee an equal gain coming from all channels. This has shown to be optimal from an SNR point of view [13].

For simplicity, in the derivation we assume $B = N = M_t = 2$, $P/\kappa = 1$, and the Alamouti code [14] being employed. An extension to more than two transmitters may be derived in a similar fashion. Since $M_t = 2$, the correlation matrix $\boldsymbol{R}$, in Equation (11), contains two correlation matrices $\boldsymbol{R}_{r_0}$ and $\boldsymbol{R}_{r_1}$. From Lemma 2, we know that $\boldsymbol{F}$ can be chosen diagonally. Let the diagonal elements of $\boldsymbol{F}$ be f_0 and f_1, satisfying $f_0^2 + f_1^2 = P/\kappa = 1$. Here, it is assumed that $f_i \in \mathbb{R}$. Let the elements in $\boldsymbol{R}_{r_0}$ bee denoted by $\rho_{i,j}$ while those in $\boldsymbol{R}_{r_1}$ by $\varrho_{i,j}$ and $\rho_{i,i} = \varrho_{i,i} = 1$.

Lemma 5: To make certain that all diversity branches provide an equal gain $\sum_{i=0}^{M_r-1}(1 + \sum_{j=0,j\neq i}^{M_r-1} |\rho_{i,j}|^2) f_0^2 = \sum_{i=0}^{M_r-1}(1 + \sum_{j=0,j\neq i}^{M_r-1} |\varrho_{i,j}|^2) f_1^2$ must hold under the energy constraint.

Proof: The total transfer matrix may be written as $\boldsymbol{H}\boldsymbol{F} = [f_0 \boldsymbol{h}_0 \;\; f_1 \boldsymbol{h}_1]$ where the two column vectors of $\boldsymbol{H} = [\boldsymbol{h}_0 \;\; \boldsymbol{h}_1]$ are related to the two column vectors of $\boldsymbol{H}_w = [\boldsymbol{h}_{w_0} \;\; \boldsymbol{h}_{w_1}]$ by $\boldsymbol{h}_0 = \boldsymbol{R}_{r_0}^{1/2} \boldsymbol{h}_{w_0}$ and $\boldsymbol{h}_1 = \boldsymbol{R}_{r_1}^{1/2} \boldsymbol{h}_{w_1}$. To work directly with the coefficients of $\boldsymbol{R}_{r_i}$, we pre-multiply column number i of the total transfer matrix by $\boldsymbol{R}_{r_i}^{1/2}$ to arrive at the two column vectors $\boldsymbol{g}_0 = f_0 \boldsymbol{R}_{r_0} \boldsymbol{h}_{w_0}$ and $\boldsymbol{g}_1 = f_1 \boldsymbol{R}_{r_1} \boldsymbol{h}_{w_1}$.

With Alamouti coding the total gain is observed as the sum of absolute channel coefficients squared: $\gamma = \boldsymbol{g}_0^H \boldsymbol{g}_0 + \boldsymbol{g}_1^H \boldsymbol{g}_1 = f_0^2 \boldsymbol{h}_{w_0}^H \boldsymbol{R}_{r_0}^2 \boldsymbol{h}_{w_0} + f_1^2 \boldsymbol{h}_{w_1}^H \boldsymbol{R}_{r_1}^2 \boldsymbol{h}_{w_1} = \eta + \zeta$ where $\eta = \sum_{i=0}^{M_r-1} |h_{w_{0,i}}|^2 (1 + \sum_{j=0,j\neq i}^{M_r-1} |\rho_{i,j}|^2) f_0^2 + \sum_{i=0}^{M_r-1} |h_{w_{1,i}}|^2 (1 + \sum_{j=0,j\neq i}^{M_r-1} |\varrho_{i,j}|^2) f_1^2$. ζ contains several cross-interference terms of the form $h_{w_{i,k}} h_{w_{i,l}}^*$, such that $E[\zeta] = 0$. In order to make certain that the expected value of all channels experience equal gain, the lemma follows from the expression of η. ■

Example 2: Assuming $M_r = 2$ with $\rho_{i,j} = \rho \in \mathbb{R}$ and $\varrho_{i,j} = \varrho \in \mathbb{R}$, lead to $\gamma = f_0^2((1+\rho^2)|h_{w_{0,0}}|^2 + (1+\rho^2)|h_{w_{0,1}}|^2 + 2\rho(h_{w_{0,0}}^* h_{w_{0,1}} + h_{w_{0,1}}^* h_{w_{0,0}})) + f_1^2((1+\varrho^2)|h_{w_{1,0}}|^2 + (1+\varrho^2)|h_{w_{1,1}}|^2 + 2\varrho(h_{w_{1,0}}^* h_{w_{1,1}} + h_{w_{1,1}}^* h_{w_{1,0}}))$. This gives $E[\gamma] = 2f_0^2(1+\rho^2) + 2f_1^2(1+\varrho^2)$, and the equal-gain requirement imposes $f_0^2(1+\rho^2) = f_1^2(1+\varrho^2)$. The explicit closed-form solution is therefore $f_0^2 = \frac{1+\varrho^2}{2+\rho^2+\varrho^2}$, $f_1^2 = \frac{1+\rho^2}{2+\rho^2+\varrho^2}$.

Example 3: Assume no cross-correlation at all, i.e., $\rho_{i,j} = \varrho_{i,j} = 1 \;\forall\; i = j$, otherwise 0, gives $f_0^2 = f_1^2$, i.e., equal power scaling.

Example 4: Assume full cross-correlation as seen from the second emitter only, i.e, $\rho_{i,j} = 0 \;\forall\; j \neq i$ and $\varrho_{i,j} = 1 \;\forall\; i,j$. This results in $f_0^2 = M_r f_1^2$ or $f_0^2 = \frac{M_r}{M_r+1}$, $f_1^2 = \frac{1}{M_r+1}$.

IV. Optimization Algorithm

Let the matrix $\boldsymbol{K}_{k,l}$ be the commutation matrix [11] of size $kl \times kl$. The constrained maximization Problem 1 can be converted into an unconstrained optimization problem by introducing a Lagrange multiplier μ. This is done by defining the following Lagrange function:

$$\mathcal{L}(\boldsymbol{F}) = \det\left(\boldsymbol{R}^{-1} + \boldsymbol{F}\boldsymbol{E}\boldsymbol{E}^H\boldsymbol{F}^H \otimes \frac{\boldsymbol{I}_{M_r}}{4\sigma^2}\right) - \mu' \operatorname{Tr}\left\{\boldsymbol{F}\boldsymbol{F}^H\right\}. \tag{19}$$

Since the objective function should be maximized, μ' should be positive.

Lemma 6: The precoder that is optimal for Problem 1 must satisfy

$$\begin{aligned} \operatorname{vec}(\boldsymbol{F}) = \mu \boldsymbol{K}_{B,M_t} &\left(\boldsymbol{I}_{M_t} \otimes \frac{\boldsymbol{E}^*\boldsymbol{E}^T\boldsymbol{F}^T}{4\sigma^2}\right) \boldsymbol{L} \\ &\times \operatorname{vec}\left(\left(\boldsymbol{R}^{-1} + \frac{\boldsymbol{F}\boldsymbol{E}\boldsymbol{E}^H\boldsymbol{F}^H}{4\sigma^2} \otimes \boldsymbol{I}_{M_r}\right)^{-T}\right), \end{aligned} \tag{20}$$

where $(\cdot)^*$ means complex conjugation and the matrix $\boldsymbol{L}$ is given by

$$\boldsymbol{L} = \left(\boldsymbol{I}_{M_t^2} \otimes \operatorname{vec}^T(\boldsymbol{I}_{M_r})\right)\left(\boldsymbol{I}_{M_t} \otimes \boldsymbol{K}_{M_t,M_r} \otimes \boldsymbol{I}_{M_r}\right), \tag{21}$$

and μ is a positive scalar chosen such that the power constraint in Equation (10) is satisfied

Proof: The necessary condition for the optimality of Problem 1 is found by setting the derivative of the Lagrangian in Equation (19) equal to zero. By finding the derivative [11], [15] with respect to

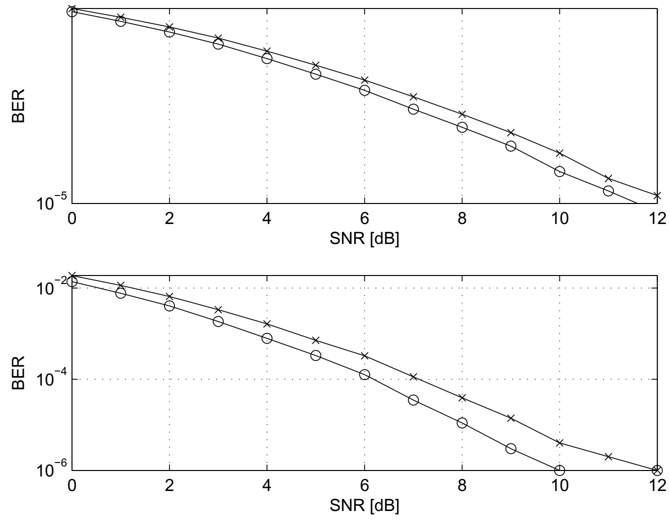

Fig. 2. Scenario 1: BER versus SNR performance of the proposed system $-\circ-$ and a system not employing a precoder $-\times-$. In the upper plot, $M_r = 4$ and in the lower plot $M_r = 6$.

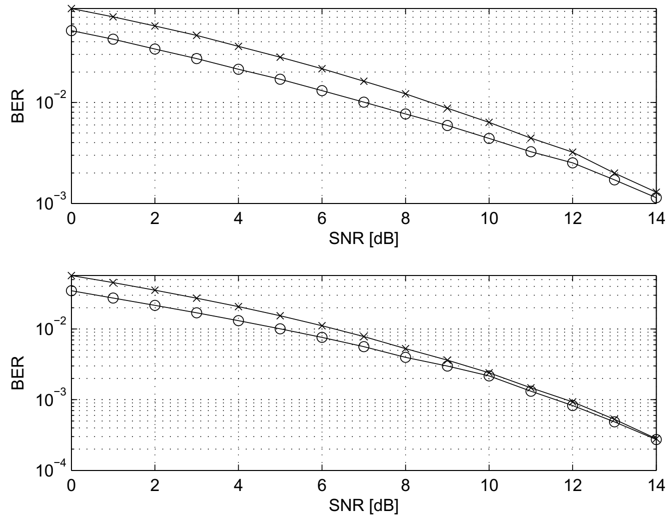

Fig. 3. Scenario 2: BER versus SNR performance of the proposed system $-\circ-$ and a system not employing a precoder $-\times-$. In the upper plot, $M_r = 4$ and in the lower plot $M_r = 6$.

the complex valued matrix $\boldsymbol{F}^*$, of Lagrangian in Equation (19), the result in Equation (20) is found. For more details, see [12]. ∎

Equation (20) can be used in a fixed point iteration.

V. Results and Comparisons

In this section, Monte Carlo simulation results are presented for two different non-Kronecker correlation scenarios using 1 000 000 bits. Comparisons are made against a system not employing any precoding, i.e., $\boldsymbol{F} = \boldsymbol{I}_{M_t}/\sqrt{M_t}$.

The following parameters are used in the examples: $N = B = M_t = 2$, $P/\kappa = 1$, and $M_r \in \{4, 6\}$. The SNR is defined as: $\mathrm{SNR} = 10\log_{10}\frac{P}{\kappa\sigma^2}$. The signal constellation is Gray-coded unit variance 4QAM. The Alamouti code [14] was used as the STBC, leading to $\boldsymbol{E}_{k,l}(n)\boldsymbol{E}_{k,l}^H(n) = \beta_{k,l}\boldsymbol{I}_B$, where $\beta_{k,l} \in \{2, 4, 6, 8\}$. In this case, $\beta = \min_{k \neq l}\{\beta_{k,l}\} = 2$.

Scenario 1: Let the correlation matrix $\boldsymbol{R}$ be given by Equation (11) with $\boldsymbol{R}_{r_0} = \boldsymbol{I}_{M_r}$ and $\boldsymbol{R}_{r_1} = a\mathbf{1}_{M_r \times M_r} + (1-a)\boldsymbol{I}_{M_r}$, where $a = 0.9999$. Since the numerical method developed is valid for invertible correlation matrices $\boldsymbol{R}$, the parameter a is chosen different from one. This corresponds to a downlink situation where the two transmitter antennas are widely separated, possibly originating from different access points, and experiences totally different channel conditions.

Scenario 2: Let the correlation matrix $\boldsymbol{R}$ be given by

$$(\boldsymbol{R})_{k,l} = 0.99^{|k-l|}, \tag{22}$$

where the notation $(\cdot)_{k,l}$ picks out element with row number k and column number l.

Figures 2 and 3 show the BER versus SNR performance for the non-precoded reference system and the proposed precoded system. From the figures, it is seen that proposed precoder outperforms the reference system. For Scenario 1, the performance gain is bigger for $M_r = 6$ than for $M_r = 4$ and for Scenario 2, it is bigger for low values of SNR.

VI. Conclusions

A precoder is proposed that minimizes a certain upper bound for the pair-wise error probability for transmission of STBC over quasi-static correlated Rayleigh MIMO channels. Several features of the optimal solution were derived for special cases, and one iterative numerical optimization technique was proposed for invertible channel correlation matrices.

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