A NEW LMS/NEWTON ALGORITHM FOR ROBUST ADAPTIVE FILTERING IN IMPULSIVE NOISE

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ABSTRACT

This paper proposes a new LMS/Newton algorithm for robust adaptive filtering in impulse noise. The new algorithm is obtained by applying the non-linear filtering technique and the robust statistic approach to the conventional fast LMS/Newton method. A robust method for estimating the required threshold parameters for impulse suppression is also given. Simulation results show that aside from retaining the advantages of the LMS/Newton algorithm such as low complexity and numerical stability, the new algorithm is more robust and effective in suppressing the adverse effects of the impulses.

I. INTRODUCTION

Adaptive filters have been widely used in communications, control, and many other systems in which the statistical characteristics of the signals to be filtered are either unknown a priori or, in some cases, slowly time varying. Two commonly used families of adaptive filtering algorithms are based on the least mean squares (LMS) and the recursive least squares (RLS) algorithms. The RLS-based adaptive algorithms are well known for their fast convergence speed, as compared with the LMS–based algorithms. The convergence speed of the latter is usually very sensitive to the eigenvalue spread of the correlation matrix of the input signal. The LMS-based algorithms, however, have a very low arithmetic complexity of $O(N)$ (where N is the number of taps in the adaptive filter), as compared with $O(N^2)$ for the conventional RLS algorithm. Different approaches have been proposed to improve the convergence property of the LMS algorithm. Interested readers are referred to [1] – [4] for various aspects of adaptive filters.

One very efficient class of algorithms is the fast Newton algorithm [5] – [7]. The basic idea is to approximate the Kalman gain vector in the update of the weight vector of the adaptive algorithm. In the fast Newton transversal filters (FNTF) [5] and LMS/Newton [6] algorithms, the input signal of the adaptive filter is modeled as a low-order auto-regressive (AR) process so that the Kalman gain vector can be efficiently determined. By so doing, the FNTF [5] has a complexity of $2N+5M$ arithmetic operations, which is very close to that of the normalized LMS (NLMS) algorithm. The LMS/Newton algorithm [6] is based on a similar concept and it was found to be more stable than the FNTF algorithm with a complexity of $2N+6M$. Since AR signal modeling has been found to provide a sufficiently accurate representation for many different types of signals, such as speech processing, it is expected the FNTF and the LMS/Newton algorithms will find applications related to the processing of speech signals, such as acoustic each cancellation described in [6] where the length of the required adaptive filter is very large.

In this paper, an improved LMS/Newton algorithm for robust adaptive filtering in impulsive noise environment is proposed. Impulsive interference, which results from nature or man-made sources, can significantly affect the performance of linear adaptive filters and it represents an important problem in communications systems. The proposed robust LMS/Newton algorithm is based on the robust statistics concept, where an *M*-estimate distortion measure [8] – [10] is minimized instead of the conventional least squares error. Simulation results show that the proposed algorithm offers improved robustness over conventional LMS/Newton algorithm in impulsive noise environment. The paper is organized as follows: the conventional Fast LMS/Newton algorithm is described in section II. The new Robust Fast LMS/Newton algorithm is presented in Section III. Experimental and comparison results are given in section IV. Finally, conclusions are drawn in Section V.

II. THE LMS/NEWTON ALGORITHM

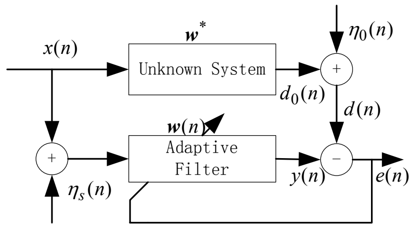


Fig. 1. System Identification Structure

Without loss of generality, consider the system identification problem in Fig. 1. The system input, $x(n)$, is passed through the unknown system to obtain the desired output $d_0(n)$. Simultaneously, it might be corrupted by additive interference $\eta_s(n)$ and is fed to the adaptive filter to generate the estimate $y(n)$. $d_0(n)$ is assumed to be corrupted by background noise, modeling error and interference which can be integrally represented by $\eta_0(n)$. The resultant reference signal is denoted by $d(n)$. The parameter vector of the unknown system and that of the adaptive filter are denoted respectively by $\boldsymbol{w}^*$ and $\boldsymbol{w}(n)$. In the Newton algorithm, the weight update equation is given by

$$e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{w}(n) \tag{1}$$

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) + \mu \cdot e(n)\hat{\boldsymbol{R}}^{-1}(n)\boldsymbol{x}(n), \tag{2}$$

where $\hat{\boldsymbol{R}}^{-1}(n)$ is the inverse of the estimated covariance matrix and μ is the stepsize. Both the FNTF and the LMS/Newton algorithms explore the structure of $\hat{\boldsymbol{R}}^{-1}(n)$ when $x(n)$ is an *M*-th order AR process. Further, if *M* is much smaller than *N*, then the computational complexity is similar to the LMS algorithm, while offering significant performance improvement. More precisely, if the input can be sufficiently modeled as an M-th order AR process, then $\hat{\boldsymbol{R}}^{-1}(n)$ can be factored into the following form:

$$\hat{\boldsymbol{R}}^{-1}(n) = \boldsymbol{L}_M^{-T}(n)\boldsymbol{D}^{-1}(n)\boldsymbol{L}_M^{-1}(n) \tag{3}$$

where

$$\boldsymbol{L}_M^{-1}(n) = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ a_{1,1}(n) & 1 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{M,M}(n) & a_{M,M-1}(n) & \cdots & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & a_{M,M}(n-1) & \cdots & a_{M,1}(n-1) & 1 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & a_{M,M}(n-N+M+1) & a_{M,M-1}(n-N+M+1) & \cdots & 1 \end{bmatrix}$$

is an $(N \times N)$ lower triangular matrix where $a_{p,i}(n)$ is the i-th coefficient of the p-th order backward predictor for $x(n)$, and $\boldsymbol{D}(n)$ is a diagonal matrix whose i-th element is the estimated power of the i-th backward prediction error. It can be seen that the $(M+1)$-th through the N-th rows are shifted version of each other. By extending the input and coefficient vector $\boldsymbol{x}(n)$ and $\boldsymbol{w}(n)$:

$$\boldsymbol{x}_E(n) = [x(n+M), \ldots x(n-N-M+1)]^T, \tag{4}$$

$$\boldsymbol{w}_E(n) = [w_{-M}(n), \ldots, w_{N+M-1}(n)]^T. \tag{5}$$

the LMS/Newton algorithm can be written as

$$e(n) = d(n) - \boldsymbol{x}^{\boldsymbol{T}}(n)\boldsymbol{w}(n), \tag{6}$$

$$\boldsymbol{w}_E(n+1) = \boldsymbol{w}_E(n) + 2\mu e(n)\boldsymbol{u}(n),$$

$$\boldsymbol{u}(n) = \boldsymbol{L}_2(n)\tilde{\boldsymbol{D}}^{-1}(n)\boldsymbol{L}_1(n)\boldsymbol{x}_E(n). \tag{7}$$

where $\boldsymbol{L}_1(n)$ and $\boldsymbol{L}_2(n)$ are respectively (N+M)-by-(N+2M) and N-by-(N+M) matrices whose rows are consecutive shifted and delayed coefficients of the M-th order forward and backward predictor $[a_{M,M}(n), a_{M,M-1}(n), \ldots 1]$ and $[1, a_{M,1}(n), \ldots a_{M,M}(n)]$ [6]. Using the shifting property of $\boldsymbol{u}(n)$ and that of $\boldsymbol{b}_E(n) = \boldsymbol{L}_1(n)\boldsymbol{x}_E(n)$, it is possible to reduce the computational complexity to $2N + 6M$ multiplications and additions. The predictor parameters can be efficiently computed using a lattice predictor and the Levinson-Durbin algorithm.

III. THE ROBUST LMS/NEWTON ALGORITHM

Since the LMS/Newton algorithm in (6) and (7) is based on the LMS criterion, its performance will deteriorate considerably when the desired or the input signal is corrupted by impulsive noise. Nonlinear techniques are usually employed to reduce the hostile effects of impulsive noise on LMS-type of algorithms. These include the order statistic least mean square algorithm (OSLMS) [11], the adaptive threshold nonlinear algorithm (ATNA) [12] and the Least mean M-estimate (LMM) algorithms, which are based on the concept of robust statistics. In the LMM algorithm, an least mean M-estimate distortion measure $J_\rho = E[\rho(e(n))]$ is minimized instead of the least mean squares distortion measure $J_{LS} = E[e^2(n)]$, where $\rho(e)$ is a robust M-estimate function such as the Hampel's three parts re-descending function as shown in equation (8) and Fig. 2.

It can be seen that $\rho(e)$ is an even real-valued function and ξ, Δ_1 and Δ_2 are the threshold parameters used to control the degree of suppression of the outliers. The contribution of the error e to $\rho(e)$ is reduced when its magnitude is increased beyond these thresholds. There, the smaller the values of ξ, Δ_1

$$\rho(e) = \begin{cases} e^2/2, & 0 \le |e| < \xi \\ \xi|e| - \xi^2/2, & \xi \le |e| < \Delta_1 \\ \dfrac{\xi}{2}(\Delta_2 + \Delta_1) - \dfrac{\xi^2}{2} + \dfrac{\xi}{2}\dfrac{(|e| - \Delta_2)^2}{\Delta_1 - \Delta_2}, & \Delta_1 \le |e| < \Delta_2 \\ \dfrac{\xi}{2}(\Delta_2 + \Delta_1) - \dfrac{\xi^2}{2}, & \Delta_2 \le |e| \end{cases} \tag{8}$$

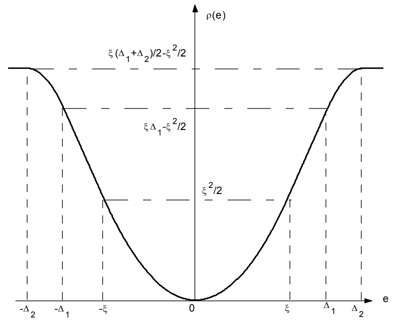


Fig. 2. The Hampel's three parts re-descending M-estimate function.

and Δ_2, the greater the suppression will be of the outliers. These threshold parameters are usually estimated continuously, which will be discussed later. $J_\rho = E[\rho(e(n))]$ is therefore capable of smoothing out momentary fluctuation caused by the impulses. By setting the first-order partial derivatives of J_ρ, with respect to $\boldsymbol{w}(n)$, to zero, it was shown in [9] that optimal weight vector satisfies the M-estimate normal equation:

$$\boldsymbol{R}_\rho \boldsymbol{w}_\rho^* = \boldsymbol{P}_\rho, \tag{9}$$

where $\boldsymbol{R}_\rho = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))\boldsymbol{x}(i)\boldsymbol{x}^T(i)$ and $\boldsymbol{P}_\rho = \sum_{i=1}^{n} \lambda_i^{(n)} q(e(i))d(i)\boldsymbol{x}(i)$ are the M-estimate autocorrelation matrix of $\boldsymbol{x}(n)$ and the M-estimate cross-correlation vector of $d(n)$ and $\boldsymbol{x}(n)$, respectively, and $\rho'(e) = \partial\rho(e)/\partial e = q(e) \cdot e$. Note that (9) is a nonlinear function of $\boldsymbol{w}$ and iterative method is required to solve for the optimal solution. When $\rho(e)$ is chosen as the square function, (9) reduces to the conventional normal equation which is a system of linear equation. The solution is given by

$$\boldsymbol{w}_{LS}^* = \boldsymbol{R}^{-1}\boldsymbol{P}. \tag{10}$$

In fact, the Newton adaptive filtering algorithm (2) is based on the following modification of (10): for a given initial weight vector $\boldsymbol{w}$, the gradient vector is

$$\nabla_{\boldsymbol{w}} = 2\boldsymbol{R} \cdot \boldsymbol{w} - 2\boldsymbol{P}. \tag{11}$$

Multiplying both sides of (11) by $\frac{1}{2}\boldsymbol{R}^{-1}$, one gets

$$\boldsymbol{w}_{LS}^* = \boldsymbol{w} - \tfrac{1}{2}\boldsymbol{R}^{-1}\nabla_{\boldsymbol{w}}. \tag{12}$$

To reduce the effect of noise and modeling errors, a stepsize can be introduced to yield the following update equation for the Newton method:

$$\boldsymbol{w}(n+1) = \boldsymbol{w}(n) - \tfrac{1}{2}\mu\boldsymbol{R}^{-1}(n)\nabla_{\boldsymbol{w}(n)}. \tag{13}$$

In practice, both $\boldsymbol{R}^{-1}$ and $\nabla_{\boldsymbol{w}(n)}$ have to be approximated. In the LMS/Newton method, $\boldsymbol{R}^{-1}$ is estimated by (3), while $\frac{1}{2}\nabla_{\boldsymbol{w}(n)}$ is approximated by its instantaneous gradient $\frac{1}{2}\hat{\nabla}_{\boldsymbol{w}(n)} = \frac{1}{2}\nabla_{\boldsymbol{w}}\{e^2(n)\} = -e(n)\boldsymbol{x}(n)$. This yields the equation update in (2).

For the *M*-estimate normal equation, we can approximate (9) locally as a quadratic function and it gives rise to a similar update as in (13), except that $\boldsymbol{R}^{-1}$ should now be replaced by some estimate of $\boldsymbol{R}_\rho^{-1}$ and $\frac{1}{2}\hat{\nabla}_{\boldsymbol{w}(n)} = \frac{1}{2}\nabla_{\boldsymbol{w}}\{e^2(n)\}$ should be replaced by the instantaneous gradient of the *M*-estimate distortion function, which is given by [8]:

$$\frac{1}{2}\hat{\nabla}_{w_\rho} = \frac{1}{2}\nabla_{w_\rho}\{\rho(e(n))\} = q(e(n))e(n)\boldsymbol{x}(n). \tag{14}$$

This will improve the robustness of the algorithm against impulse noise in the desired signal. Since $\boldsymbol{R}_\rho^{-1}$ depends on the weight vector $\boldsymbol{w}_\rho^*$ and its role is to improve the robustness against input impulse, we still employ $\boldsymbol{R}^{-1}$ to retain the simplicity of the fast LMS/Newton algorithm. Finally, we get the following robust LMS/Newton algorithm:

$$e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{w}(n), \tag{15}$$

$$\boldsymbol{w}_E(n+1) = \boldsymbol{w}_E(n) + 2\mu \cdot q(e(n))e(n)\boldsymbol{u}(n),$$

$$\boldsymbol{u}(n) = \boldsymbol{L}_2(n)\tilde{\boldsymbol{D}}^{-1}(n)\boldsymbol{L}_1(n)\boldsymbol{x}_E(n). \tag{16}$$

This robust LMS/Newton algorithm will reduce to the LMS/Newton algorithm when $e(n)$ is smaller than ξ. When $e(n)$ is considerably larger than ξ, it will be treated as an outlier and will be suppressed. When $|e(n)| > \Delta_2$, it will be completely ignored.

The threshold parameters ξ, Δ_1 and Δ_2 can be determined by the method proposed in [8]. Basically, the distribution of the estimation error $e(n)$ is assumed to be Gaussian distributed and is corrupted with additive impulse noise. By estimating the variance of "impulse-free" component of $e(n)$, it helps to detect and reject the impulses in $e(n)$. According to [8], the probability of $|e(n)|$ greater than a given threshold T_h is

$$\theta_T(n) = P_r\{|e(n)| > T_h\} = erfc(T_h/(\sqrt{2}\hat{\delta}_e(n))), \tag{17}$$

where $ercf(x) = (2/\sqrt{\pi})\int_x^\infty e^{-x^2}dx$ is the complementary error function and $\hat{\delta}_e(n)$ is the estimated standard deviation of the "impulse-free" error. Let $\theta_\xi = P_r\{|e(n)| > \xi\}$, $\theta_{\Delta_1} = P_r\{|e(n)| > \Delta_1\}$ and $\theta_{\Delta_2} = P_r\{|e(n)| > \Delta_2\}$ be the probabilities that $e(n)$ is greater than ξ, Δ_1 and Δ_2, respectively. By appropriate choosing θ_ξ, θ_{Δ_1} and θ_{Δ_2} (i.e.-different confidence in distinguishing the outlier from an ordinary sample with very high amplitude), the values of ξ, Δ_1 and Δ_2 can be determined. For θ_ξ, θ_{Δ_1} and θ_{Δ_2} equal to 0.05, 0.025, 0.01, respectively, the values of ξ, Δ_1 and Δ_2 are respectively $1.96\hat{\delta}_e(n)$, $2.24\hat{\delta}_e(n)$ and $2.576\hat{\delta}_e(n)$.

The robust estimation of $\hat{\delta}_e(n)$ can be effectively implemented by

$$\hat{\delta}_e^2(n) = \lambda_e\hat{\delta}_e^2(n-1) + c_1(1-\lambda_e)med(A_e(n)), \tag{18}$$

where $c_1 = 1.483(1+5/(N_w-1))$ is a finite sample correction factor, med(.) stands for median operator, $A_e(n) = \{e^2(n),...,e^2(n-N_w+1)\}$, λ_e is the forgetting factor and N_w is the length of the estimation window.

To suppress the impulses in the input signal, we make use of the lattice predictor in (7). The one-step ahead prediction of the input signal $\hat{x}(n+1)$ and the error $e_p(n)$ can be written as

$$\hat{x}(n+1) = -\sum_{i=0}^{M-1}k_{i+1}(n)b_i(n),\ e_p(n) = x(n+1) + \sum_{i=0}^{M-1}k_{i+1}(n)b_i(n) \tag{19}$$

The threshold parameters and the variance of $e_p(n)$, $\hat{\delta}_{e_p}^2(n)$, can be computed by a similar parameter estimation technique described above with $e(n)$, $\hat{\delta}_e^2(n)$, N_w and λ_e replaced, respectively, by $e_p(n)$, $\hat{\delta}_{e_p}^2(n)$, N_p and λ_p. If $e_p(n)$ is greater than the estimated threshold, an impulse is said to occur in the input signal, and it will be replaced by its predicted value, i.e., the input to the adaptive filter is now $\tilde{x}(n+1) = q(e_p(n))x(n+1) + (1-q(e_p(n)))\hat{x}(n+1)$. Where $q(\cdot)$ is the weighting function defined earlier, and $\hat{x}(n+1)$ is the predicted value of $x(n+1)$ in (19). Since the suppressing or filtering of the impulses in $x(n)$ is derived from the lattice predicting process, no additional pre-processor is required.

IV SIMULATION RESULTS

We now evaluate the performance of the proposed algorithm using computer simulation. For the sake of comparison, the order of the unknown system is set to be 30 with the coefficients being randomly generated as in [6]. The AR process is with coefficients [1 -0.65 0.693 -0.22 0.309 -0.177] and the normalized unit power. The signal-to-noise ratio at the system output is given by $SNR = 10\log 10(\delta_{d_0}^2/\delta_g^2)$, where $\delta_{d_0}^2$ is the variance of the output of the unknown system. The interference $\eta_0(n) = \eta_g(n) + \eta_{im}(n) = \eta_g(n) + b(n)\eta_w(n)$ is chosen as a contaminated Gaussian (CG) noise with $\eta_g(n)$ and $\eta_w(n)$ being i.i.d. zero mean Gaussian processes with variance δ_g^2 and δ_w^2, respectively. $b(n)$ is an i.i.d. Bernoulli random process assuming a value of either 1 or 0 with occurrence probabilities $P_r(b(n)=1) = p_r$ and $P_r(b(n)=0) = 1-p_r$. The strength and frequency of the impulsive noise are specified by the ratio $r_{im} = p_r\delta_w^2/\delta_g^2$. The parameters in our experiment are: $SNR = 30dB$, $p_r = 0.005$, and $r_{im} = 100$. For illustration purpose, the interference noise $\eta_0(n)$ is an additive Gaussian noise from time $n=1$ to $n=2000$ and 2801 to 7000. From $n=2001$ to 2800, the CG noise is applied. In order to visualize more clearly the effect of impulses in the desired signal, the locations of impulses are respectively fixed at $n = 2253,2620$ but the amplitudes of impulses are independent variables governed by $\eta_w(n)$. Similarly, for the same reason one impulse is added to the input signal of the filter at $n = 5000$. The threshold parameters are obtained according to (17): $\xi = 1.96\hat{\delta}_e(n)$, $\Delta_1 = 2.24\hat{\delta}_e(n)$ and $\Delta_2 = 2.576\hat{\delta}_e(n)$. The forgetting factor λ_e, λ_p are set equal to 0.99, and the window length N_w, N_p are both chosen to be 14.

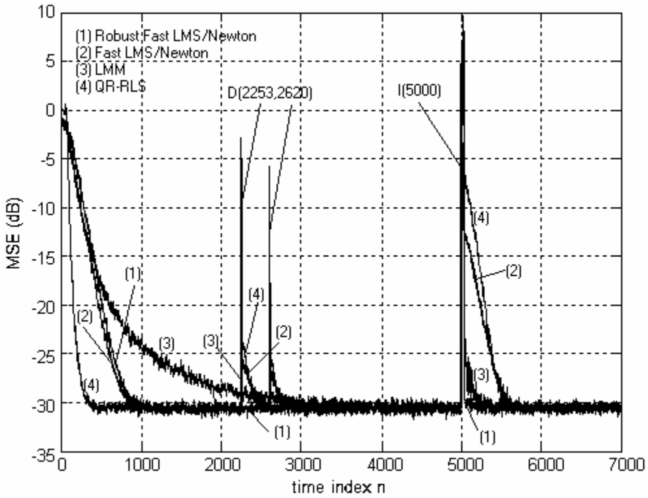
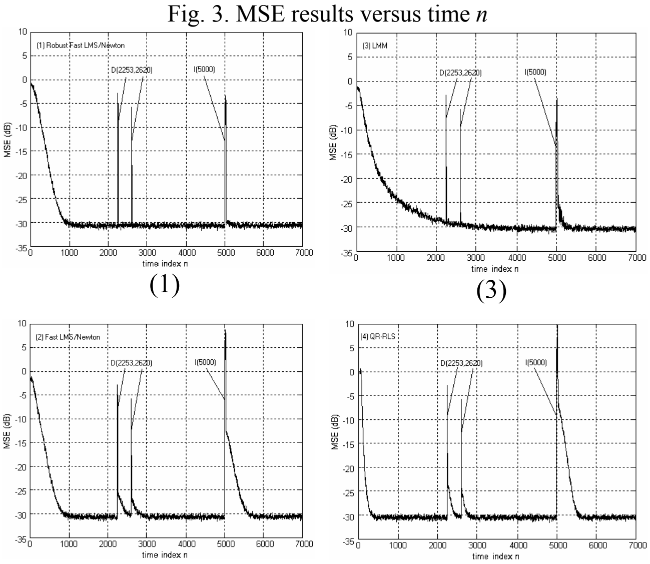


Fig. 3. MSE results versus time *n*



(1) (3)

(2) (4)

Fig. 4. Respective MSE results versus time *n*. (1)Robust Fast LMS/Newton; (2)Fast LMS/Newton; (3)LMM; (4)QR-RLS

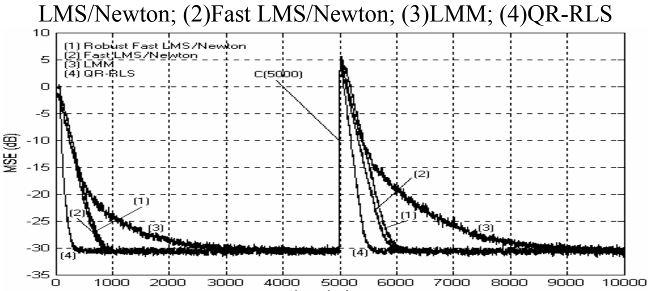


Fig. 5. MSE results versus time *n* upon system sudden change. (1)Robust Fast LMS/Newton; (2)Fast LMS/Newton; (3)LMM; (4)QR-RLS

Four algorithms, Robust and conventional Fast LMS/Newton, LMM, QR-RLS are tested. The MSE results averaged over 100 independent runs are plotted in Fig.3 as a whole and in Fig.4 respectively. It can be seen that the QR-RLS algorithm has the fastest initial convergence speed and the two Fast LMS/Newton algorithms also exhibit much faster initial convergence speed as compared to the LMS-based LMM algorithm. Fast LMS/Newton and QR-RLS, as being conventional algorithms, are both not robust to any of the impulses in either the desired or the input signal. In contrast, the LMM and our new robust Fast LMS/Newton algorithms can effectively suppress the impulses in both forms, and particularly, the latter reveals better performance than the former with respect to combating the impulse in input signal. We next examine the response speed of these algorithms to the sudden system change. With all the other parameters unchanged, the system is switched to $-\boldsymbol{w}$ from $\boldsymbol{w}$ at n=5000. The results shown in Fig. 5 illustrate that with the QR-RLS algorithm being the fastest of all, the new algorithm, together with the fast LMS/Newton algorithm, have a much faster convergence speed than the LMM algorithm. All the above simulation results indicate that aside from having a fast convergence behavior and a low request for computational complexity, the new algorithm is very robust to the impulse noise in both the desired and the input signal. Further results concerning its convergence analysis will appear in our future work.

V CONCLUSION

In this paper, a new LMS/Newton algorithm for impulse suppression is presented. Based on the non-linear filtering technique and the robust statistic approach, the new algorithm is developed with the AR-process input assumption. Besides possessing a low computational complexity, it also exhibits more robust and effective performance in suppressing the impulses in both desired and input signals in simulation results as compared to the convention fast LMS/Newton algorithm. So it may be a good alternative to the latter for the applications in impulse noise environment.

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