LINEAR OFDM PRECODER DESIGN FOR MULTIUSER WIRELESS COMMUNICATIONS USING CUTOFF RATE OPTIMIZATION

Yue Rong[1] *Sergiy A. Vorobyov*[1] *Alex B. Gershman*[1,2]

[1] Department of Communication Systems, University of Duisburg-Essen
Bismarckstr. 81, D-47057, Duisburg, Germany
email: rong@sent5.uni-duisburg.de, svor@sent5.uni-duisburg.de
[2] Department of Electrical and Computer Engineering, McMaster University
1280 Main St. West, Hamilton, L8S 4K1 Ontario, Canada
email: gershman@ieee.org

ABSTRACT

Multiuser wireless communications based on orthogonal frequency division multiplexing (OFDM) technique have two pronouncing advantages. First of all, the equalizer design at the receiver is facilitated by converting the frequency selective fading channel into parallel flat fading channels. Moreover, by providing each user with a non-intersecting fraction of the available number of subcarriers, multiple-access interference (MAI) can be mitigated. However, a serious drawback in this communication scheme is that some subcarriers may be subject to deep fading in the frequency domain. In this paper, a linear precoding technique is proposed in order to solve this problem. The design of our linear precoder is based on the cutoff rate criterion and, in contrast to other existing precoding techniques, only the knowledge of the average relative power and the multipath delays is required at the transmitter.

1. INTRODUCTION

Multiuser wireless communications based on orthogonal frequency division multiplexing (OFDM) is a very promising multiuser communication scheme. By providing each user with a non-intersecting fraction of the available number of subcarriers, multiple-access interference (MAI) is mitigated. Therefore, a larger system capacity can be achieved [1]. Moreover, due to the inverse fast Fourier transform (IFFT) at the transmitter and the fast Fourier transform (FFT) at the receiver, the frequency selective fading channel is converted into parallel flat fading channels [2]. This greatly facilitates the equalizer design at the receiver. However, a well known disadvantage of OFDM scheme is that, in each frequency subcarrier, the channel may be subject to a deep fading. This makes a reliable detection of the information-bearing symbols at this subcarrier very difficult. Therefore, the overall performance of the system may degrade in such a case.

A popular recent approach to solve this problem is to use linear precoding at the transmitter [3]. In [4], another linear precoder has been proposed which is referred to as the maximum diversity advantage precoder. This work has been followed by [5], where a linear precoder has been designed based on the pairwise error probability (PEP) criterion. In our paper, we also adopt the idea of linear precoder, but in contrast to [4] and [5], our linear precoder is designed based on the channel cutoff rate criterion, which can be viewed as a sort of lower bound on the Shannon channel capacity. To design our linear precoder, only the knowledge of the average relative power and delay of the multipath channels is required at the transmitter. This knowledge can be obtained in practical communication systems through a low-rate feedback channel[1].

2. SYSTEM MODEL

We consider a cellular communication system with M mobile stations (MS) in a certain cell. The frequency selective wireless channel between the base station (BS) and the mth MS at time t can be modelled as

$$h_m(t,\tau) = \sum_{l=1}^{L_m} h_{m,l}(t)\delta(\tau - \tau_{m,l})$$

where $h_{m,l}(t)$ and $\tau_{m,l}$ ($l = 1,\cdots,L_m$) are, respectively, the channel gain and delay of the lth path, and L_m is the total number of paths. We assume that $h_{m,l}(t)$, $l = 1,\cdots,L_m$ are independent but not necessarily identically distributed zero-mean complex Gaussian random variables. We also assume that N subcarriers are used. Let the mth MS use N_m subcarriers and no subcarriers are shared between different MSs, i.e., $N = \sum_{m=1}^{M} N_m$. Let the subcarriers assigned to the mth MS be denoted as $f_m^1,\cdots,f_m^{N_m}$ where f_m^n is the nth subcarrier used by mth MS.

In this paper, the downlink mode is considered [2]. The block of information-bearing symbols $\mathbf{s}_m = [s_m(t),\cdots,s_m(t+N_m-1)]^T$ of the mth MS corresponding to the time slot $t,\ldots,(t+N_m-1)$ is first precoded by a square matrix $\mathbf{T}_m$ (by using a square precoding matrix we do not sacrifice the data rate). Then the precoded symbols are passed through a subcarrier group selection matrix $\mathbf{\Theta}_m = [\mathbf{e}_m^1, \mathbf{e}_m^2,\cdots,\mathbf{e}_m^{N_m}]$, where $\mathbf{e}_m^n$, $n = 1,\cdots,N_m$ denotes the $N \times 1$ vector which has one in the entry that corresponds to the nth subcarrier assigned to the mth MS and zeros elsewhere [2]. After this, N symbols for M MSs are IFFT-modulated and the cyclic prefix (CP) is inserted to form one OFDM symbol. We assume that the length of the CP is longer than the maximum path

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[1] Although the channel state variations can be very fast due to small-scale fading, the power and multipath delay variations are typically much slower [1].

[2] Our results can be extended to the uplink mode as well.

delay among $\tau_{m,l}$, $l = 1, \cdots, L_m$, $m = 1, \cdots, M$. Finally, each symbol is pulse-shaped and transmitted through the channel. We also assume that the channel remains fixed during one OFDM symbol time. For the simplicity of our subsequent consideration, hereafter, the time dependence of $h_{m,l}$ and $\mathbf{s}_m$ is omitted.

After removing the CP, the received signal block $\mathbf{y}_m$ at the mth MS can be written as

$$\mathbf{y}_m = \mathbf{H}_m \mathbf{F}^H \sum_{i=1}^{M} (\boldsymbol{\Theta}_i \mathbf{T}_i \sqrt{E_i} \mathbf{s}_i) + \tilde{\mathbf{n}}_m \tag{1}$$

where E_i denotes the transmit power of the symbol for the ith MS, $\mathbf{F}$ is the $N \times N$ FFT matrix, $\mathbf{H}_m$ is the circulant channel matrix between the mth MS and BS with its (k,l)-th entry given by $h_{m,(k-l+1)\mathrm{mod}N}$, and $\tilde{\mathbf{n}}_m$ is the additive white Gaussian noise (AWGN) at the mth MS whose variance is σ_m^2. After the FFT operation, the corresponding subcarrier group of mth MS is selected by matrix $\boldsymbol{\Theta}_m^T$. The resulting output symbol vector is given by

$$\mathbf{r}_m = \boldsymbol{\Theta}_m^T \mathbf{F} \mathbf{y}_m \tag{2}$$

Inserting (1) into (2) and using the fact that $\boldsymbol{\Theta}_i^T \boldsymbol{\Theta}_l = \mathbf{0}_{N_i \times N_l}$ $\forall\ i \neq l$, we obtain

$$\mathbf{r}_m = \sqrt{E_m} \mathbf{F} \mathbf{H}_m \mathbf{F}^H \mathbf{T}_m \mathbf{s}_m + \mathbf{n}_m \tag{3}$$

where $\mathbf{n}_m = \boldsymbol{\Theta}_m^T \mathbf{F} \tilde{\mathbf{n}}_m$. Obviously, $\mathrm{E}\{\mathbf{n}_m \mathbf{n}_m^H\} = \sigma_m^2 \mathbf{I}_{N_m}$.

From (3), we can see that the signals received at the mth MS do not contain the signal components from other MSs. Therefore, the MAI is mitigated. Due to the circulant structure of matrix $\mathbf{H}_m$, $\mathbf{F}\mathbf{H}_m\mathbf{F}^H$ is a diagonal matrix [2]. We denote it as $\mathbf{D}_m = \mathrm{diag}(d_1, \cdots, d_{N_m})$. Then, equation (3) can be rewritten as

$$\mathbf{r}_m = \sqrt{E_m} \mathbf{D}_m \mathbf{T}_m \mathbf{s}_m + \mathbf{n}_m$$

In this paper, we assume that the maximum likelihood (ML) receiver is used to restore the vector $\mathbf{s}_m$ (the block of information-bearing symbols), and we design the precoder matrices $\mathbf{T}_m$, $m = 1, \cdots, M$ which maximize the channel cutoff rate.

3. LINEAR PRECODER BASED ON THE CUTOFF RATE CRITERION

The channel cutoff rate R_0 is a lower bound on the Shannon channel capacity, beyond which the sequential decoding becomes impractical [6], [7]. R_0 also specifies an upper bound of the error rate of an optimal (ML) decoder. The cutoff rate has been frequently used as a practical coding limit because it can be calculated in a simpler way than the channel capacity. Due to these facts, cutoff rate represents a proper criterion for the design of linear precoders.

We assume that the constellation used at the BS is discrete. The MSs know the channel state information (CSI) perfectly and the unquantized demodulation is used. The conditional probability density function of the received signal can be written as

$$\begin{aligned} & f(\mathbf{r}_m | \mathbf{s}_m^{(i)}, \mathbf{T}_m, \mathbf{D}_m) \\ & = \frac{1}{(\pi \sigma_m^2)^{N_m}} \exp\left(- \frac{\|\mathbf{r}_m - \sqrt{E_m} \mathbf{D}_m \mathbf{T}_m \mathbf{s}_m^{(i)}\|^2}{\sigma_m^2} \right) \end{aligned} \tag{4}$$

where $\mathbf{s}_m^{(i)}$ is the ith member of the transmit vector constellation used for the mth MS, and $\|\cdot\|$ denotes the Frobenius norm of a matrix or the Euclidean norm of a vector. It can be seen that only the index i of the block of information-bearing symbols in $f(\mathbf{r}_m | \mathbf{s}_m^{(i)}, \mathbf{T}_m, \mathbf{D}_m)$ is essential for the following derivations. Hence, for the sake of simplicity, in the sequel we denote $f(\mathbf{r}_m | \mathbf{s}_m^{(i)}, \mathbf{T}_m, \mathbf{D}_m)$ as $f(i)$. The cutoff rate can be calculated as [6, p. 361]

$$\begin{aligned} R_0 &= -\log \mathrm{E}_{\mathbf{D}_m} \left\{ \int_{\mathbf{r}_m} \left[\frac{1}{A^{N_m}} \sum_{i=1}^{A^{N_m}} \sqrt{f(i)} \right]^2 d\mathbf{r}_m \right\} \\ &= -\log \left[\frac{1}{A^{N_m}} + \frac{1}{A^{2N_m}} \sum_{i=1}^{A^{N_m}} \sum_{l=1;l\neq i}^{A^{N_m}} \mathrm{E}_{\mathbf{D}_m} \left\{ \int_{\mathbf{r}_m} \sqrt{f(i)f(l)}\, d\mathbf{r}_m \right\} \right] \end{aligned}$$

where A stands for the constellation size and $\mathrm{E}\{\cdot\}$ is the statistical expectation. Inserting (4) into the latter expression, after some manipulation we obtain

$$\begin{aligned} R_0 = & -\log \left[\frac{1}{A^{N_m}} + \frac{1}{A^{2N_m}} \right. \\ & \left. \cdot \sum_{i=1}^{A^{N_m}} \sum_{l=1;l\neq i}^{A^{N_m}} \mathrm{E}_{\mathbf{D}_m} \left\{ \exp\left(- \frac{E_m \|\mathbf{D}_m \mathbf{T}_m (\mathbf{s}_m^{(i)} - \mathbf{s}_m^{(l)})\|^2}{4\sigma_m^2} \right) \right\} \right] \end{aligned} \tag{5}$$

Using the theorem from [9], the expectation in (5) can be explicitly calculated as

$$\begin{aligned} & \mathrm{E}_{\mathbf{D}_m} \left\{ \exp\left(- \frac{E_m \|\mathbf{D}_m \mathbf{T}_m (\mathbf{s}_m^{(i)} - \mathbf{s}_m^{(l)})\|^2}{4\sigma_m^2} \right) \right\} \\ & = \prod_{k=1}^{r\{\mathbf{E}_{i,l}\}} \left(1 + \frac{E_m}{4\sigma_m^2} \lambda_k \right)^{-1} \end{aligned} \tag{6}$$

where $\mathbf{E}_{i,l} = \mathrm{E}_{\mathbf{D}_m}\{\mathbf{D}_m \mathbf{T}_m \mathbf{e}_{i,l} \mathbf{e}_{i,l}^H \mathbf{T}_m^H \mathbf{D}_m^H\}$, $\mathbf{e}_{i,l} = \mathbf{s}_m^{(i)} - \mathbf{s}_m^{(l)}$, λ_k is the kth eigenvalue of the matrix $\mathbf{E}_{i,l}$, and $r\{\cdot\}$ denotes the rank of a matrix.

Let us introduce a new vector $\mathbf{d}_m = [d_m(1), \cdots, d_m(N_m)]^T$, which is formed by stacking all the diagonal elements of $\mathbf{D}_m$ into a column vector. Then $\mathbf{E}_{i,l}$ can be rewritten as

$$\begin{aligned} \mathbf{E}_{i,l} &= (\mathrm{E}_{\mathbf{d}_m}\{\mathbf{d}_m \mathbf{d}_m^H\}) \diamond (\mathbf{T}_m \mathbf{e}_{i,l} \mathbf{e}_{i,l}^H \mathbf{T}_m^H) \\ &= \mathbf{R}_{\mathbf{d}_m} \diamond (\mathbf{T}_m \mathbf{e}_{i,l} \mathbf{e}_{i,l}^H \mathbf{T}_m^H) \end{aligned} \tag{7}$$

where $\mathbf{R}_{\mathbf{d}_m} = \mathrm{E}_{\mathbf{d}_m}\{\mathbf{d}_m \mathbf{d}_m^H\}$, and $\diamond$ stands for the Schur-Hadamard (element-wise) matrix product. Since $d_m(n)$, $n = 1, \cdots, N_m$ is the channel gain at the nth subcarrier, we have

$$d_m(n) = \sum_{l=1}^{L_m} h_{m,l} \exp\left(- \frac{j2\pi n \tau_{m,l}}{NT} \right)$$

where $j = \sqrt{-1}$ and T is the sample time interval of the OFDM symbol. Consequently, the (n,k)th entry of $\mathbf{R}_{\mathbf{d}_m}$ can be calculated as

$$\begin{aligned} \mathbf{R}_{\mathbf{d}_m}(n,k) &= \mathrm{E}\{d_m(n) d_m(k)^*\} \\ &= \sum_{l=1}^{L_m} P_{m,l} \exp\left(- \frac{j2\pi (n-k) \tau_{m,l}}{NT} \right) \end{aligned} \tag{8}$$

where $P_{m,l}$ is the average power of the lth path relative to the first path.

Inserting (6) into (5) we obtain

$$R_0 = -\log\left[\frac{1}{A^{N_m}} + \frac{1}{A^{2N_m}}\sum_{i=1}^{A^{N_m}}\sum_{l=1;l\neq i}^{A^{N_m}}\prod_{k=1}^{r\{\mathbf{E}_{i,l}\}}\left(1+\frac{E_m}{4\sigma_m^2}\lambda_k\right)^{-1}\right] \tag{9}$$

With all the necessary quantities at hand, our task now is to design matrix $\mathbf{T}_m$ to maximize R_0 in (9), subject to the unit power constraint $\|\mathbf{T}_m\| = 1$. In principle, the precoder matrix $\mathbf{T}_m$ can be any full rank matrix provided that the power constraint is satisfied. However, the objective R_0 in (9) is a very complex nonlinear function of $\mathbf{T}_m$, which makes the optimization with respect to arbitrary $\mathbf{T}_m$ intractable. In our design, we constrain $\mathbf{T}_m$ to be a unitary matrix because unitary precoders have the advantage that they do not alter the Euclidian distance between the entries of the block $\mathbf{s}_m$ of information-bearing symbols [5]. We can parameterize the unitary matrix $\mathbf{T}_m$ in the following way

$$\mathbf{T}_m = \frac{1}{\sqrt{N_m}}\prod_{i=1}^{N_m}\prod_{l=i+1}^{N_m}\mathbf{T}_m^{i,l} \tag{10}$$

where $\mathbf{T}_m^{i,l}$ differs from the identity matrix $\mathbf{I}_{N_m}$ only in four elements which are located at the intersections of the ith and lth rows with ith and lth columns. These four elements are parameterized as

$$\mathbf{T}_m^{i,l} = \begin{array}{cc} \left[\begin{array}{cc} \cos\phi_{il} & \exp(-j\varphi_{il})\sin\phi_{il} \\ -\exp(j\varphi_{il})\sin\phi_{il} & \cos\phi_{il} \end{array}\right] & \begin{array}{c}\leftarrow i\\ \leftarrow l\end{array} \\ \begin{array}{cc}\quad\uparrow\quad & \quad\quad\uparrow\\ \quad i\quad & \quad\quad l\end{array} & \end{array}$$

where $\phi_{il} \in [-\pi, \pi]$ and $\varphi_{il} \in [-\pi/2, \pi/2]\ \forall\ i,l$. Exhaustive search, Monte Carlo optimization, or alternative projections can be performed to obtain the $\mathbf{T}_m$ ($m = 1,\ldots,M$) which give the maximum R_0. If each MS uses a moderate number of subcarriers (not more than 3 subcarriers per MS), and since the precoding matrices can be designed for each user independently, the total number of real parameters for the mth user is $N_m(N_m-1) \le 6$ and the precoder design becomes practically feasible.

From (7)-(9), it can be seen that only the information of the average relative power and the delay of the dispersive wireless channel is required for the design of our linear precoder. Although the channel state variations can be very fast due to small-scale fading, the power and multipath delay variations are typically much slower [1]. Therefore, a low-rate feedback can be used to convey this information to the BS.

4. SIMULATIONS

In this section, we study the performance of the proposed linear precoder through numerical simulations. The simulation scenario is based on the ETSI "Vehicular A" channel environment, which is defined by ETSI for the evaluation of UMTS radio interface proposals [10]. The total available bandwidth is divided into $N = 64$ subcarriers. We provide each user with 2 or 3 subcarriers depending on simulation scenario. BPSK modulation is used. For the optimization of the precoder matrix $\mathbf{T}_m$, we carry out 10^5 Monte Carlo trials and pick up the parameters which maximize R_0.

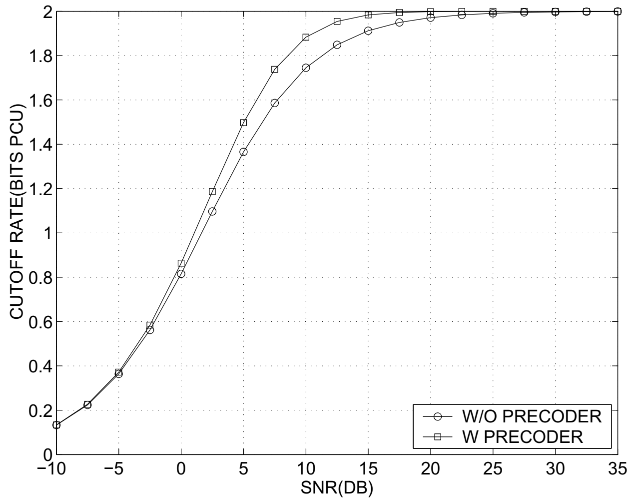

Figure 1: Cutoff rate versus SNR, 2 sub-carriers per user.

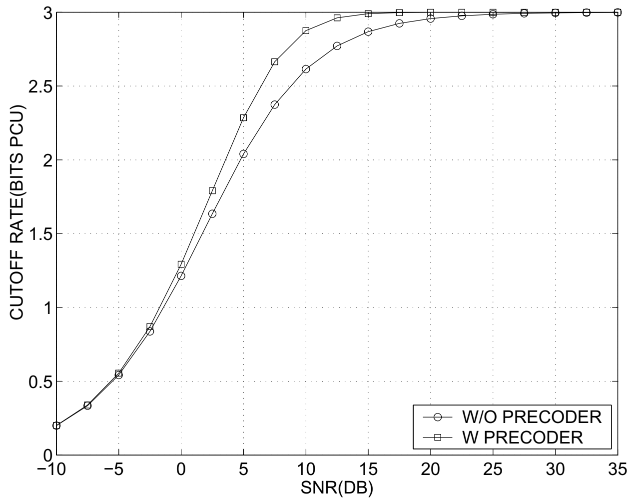

Figure 2: Cutoff rate versus SNR, 3 sub-carriers per user.

Figures 1 and 2 compare the channel cutoff rate with and without linear precoding versus signal-to-noise ratio (SNR). We can see that with linear precoding technique, the SNR gain around 6 dB is achieved at the cutoff rate of 1.9 bits compared with the system without linear precoding if each user is provided with 2 subcarriers. If each user is provided with 3 subcarriers, then the SNR gain around 4 dB is achieved at the cutoff rate of 2.8 bits.

In Figures 3 and 4 we compare the proposed linear precoder in terms of the symbol error rate (SER) versus SNR with the Vandermonde precoder [5] in the cases of 2 subcarriers per user and 3 subcarriers per user, respectively. A total of 1000 Monte Carlo runs are used to obtain each simulated point. Since both precoders are designed based on the ML receiver, we show their performance when this receiver is used. Moreover, we also display the performance of both precoders when ML receiver is not affordable and computationally efficient linear minimum mean square error (MMSE) receiver is used instead.

It can be seen that our linear precoder outperforms the Vandermonde precoder of [5]. The performance improve-

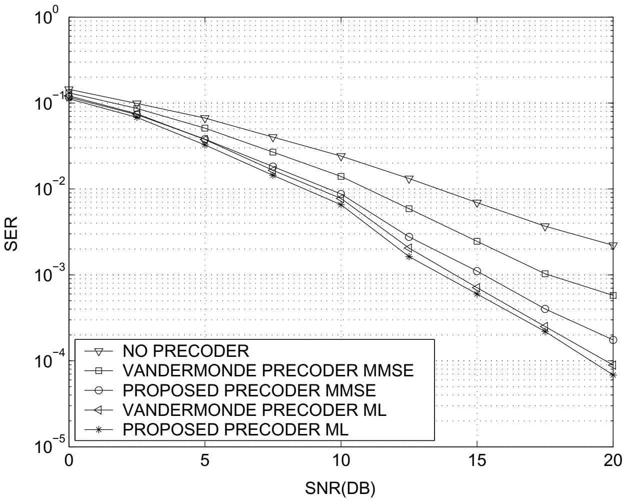

Figure 3: SER versus SNR, 2 sub-carriers per user.

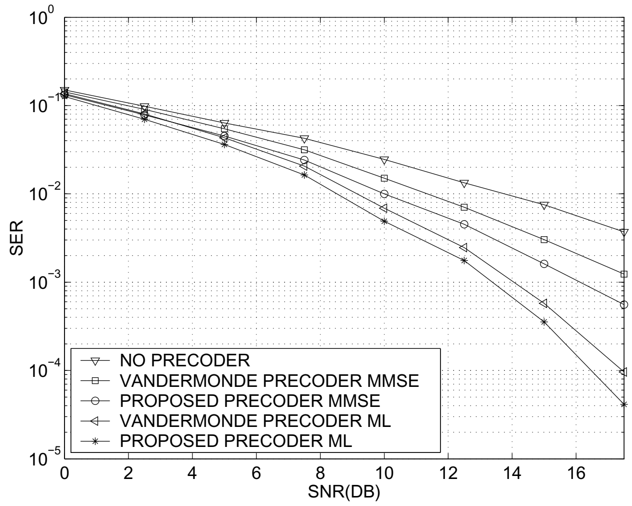

Figure 4: SER versus SNR, 3 sub-carriers per user.

ments are especially pronounced at high SNRs and in the case of 3 subcarries per user. This conclusion is true not only for the ML receivers but for the MMSE receivers as well.

5. CONCLUSION

In this paper, a new linear precoder for multiuser OFDM wireless communications has been proposed. This linear precoder maximizes the channel cutoff rate and mitigates deep fading which occurs in the frequency domain in OFDM systems. To design the linear precoder, only the knowledge of the average relative power and multipath delays is required. The implementation of the proposed precoder makes use of a parameterization of the precoder matrix using Givens rotation matrices. Simulation results show an improved performance of the proposed linear precoder compared to the Vandermonde precoder of [5].

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