MOVING THE PAR REDUCTION CRITERION INTO THE LINE DRIVER

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ABSTRACT

Traditionally, Peak to Average Ratio (PAR) reduction in digital subscriber line (DSL) transmitters focuses on a digital-domain signal, either at the output of the baseband processing block or at the input of the digital-to-analogue converter (DAC). However, analysis of a DSL transceiver shows that the power dissipation is highly dependent on the PAR at a certain node inside the line driver. Thus, in order to be fully effective, the algorithm design should include the power amplifier dynamics. A typical, actively terminated, line driver is analysed and a model is constructed for PAR reduction purposes. The PAR reduction algorithm is then extended to take advantage of the model. Simulations show that algorithms which are designed to reduce PAR at the new, physically motivated, node obtain about 0.5 dB lower PAR evaluated at this node compared to methods that focus on the PAR of the DAC input.

1. MOTIVATION

Enormous investments have been made in the copper-based infrastructure during the last century. Discrete Multitone (DMT) technology such as ADSL enables high-speed data transmission over the existing telephone lines. Operators, in particular alternative providers, who often have to rent facilities closely located to highly populated areas for their equipment, try to pack DMT transceivers for as many customers as possible into the available space. An important limiting factor with this respect is power dissipation in the transmitter, i.e., the part of the consumed power that is not transmitted on the line.

The line driver is the circuit that provides the interface of the transceiver to the physical line. It is usually an amplifier of class AB. Typically, 70% of the power dissipation of a DMT transceiver occurs in the line driver [1]. To avoid distortion of the transmitted signal, the line driver should provide linear amplification over the amplitude span of the signal. Typical line drivers have a power dissipation that is approximately proportional (see [2]) to the span of the linear amplification region at a certain node inside the line driver. It is thus desirable to keep the signal swing at this point as small as possible.

With DMT based transmission, the sample amplitude distribution of the transmitted signal is approximately Gaussian,

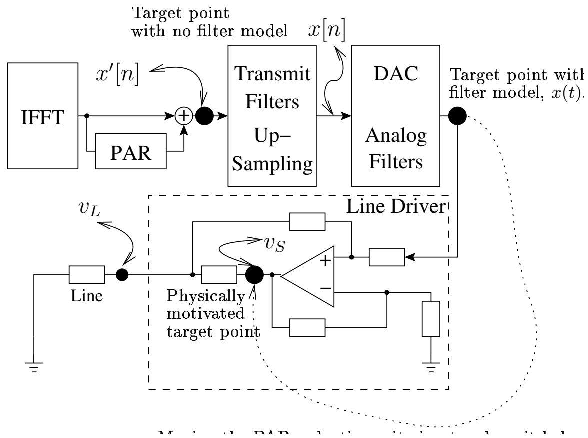

Figure 1: Block diagram of a DMT-based transmitter showing three different points a PAR reduction criterion can focus on. The node inside the line driver labeled v_S is the physically motivated point. The PAR at this point directly influences the power consumption of the system.

thus there is a probability of very high amplitude for some samples [3]. A number of methods have been suggested [3-6] for the reduction of the signal's *Peak to Average Ratio*, PAR[1]. Usually, PAR reduction methods concentrate on a signal in the digital domain, before or after the transmit filters (see Figure 1). In the following, focus of the PAR reduction is moved to a point inside the line driver, where the PAR directly influences the power dissipation. The signal at this point does not coincide with the signal input to the line driver and the analysis and algorithm design therefore require detailed knowledge of the line driver.

A model that is generally applicable to actively terminated line drivers is developed in Section 2 and used to extend an important PAR reduction algorithm in Section 3. Simulations evaluating the performance of the idea for an ADSL downstream link, as described in annex A of the corresponding standardisation document [7], are presented in Section 4.

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[1] The PAR of a signal $x(\tau)$ in the interval $\mathscr{T}$ is defined as $\mathrm{PAR} = \frac{\max_{\tau \in \mathscr{T}} |x(\tau)|^2}{\sigma^2}$, where σ^2 is the average symbol energy *before* PAR reduction. Normalising by the energy after PAR reduction would cause a bias towards lower PAR values.

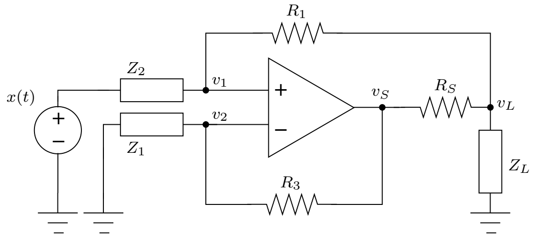

Figure 2: A simplified block diagram of a line driver with active synthesis of the output impedance.

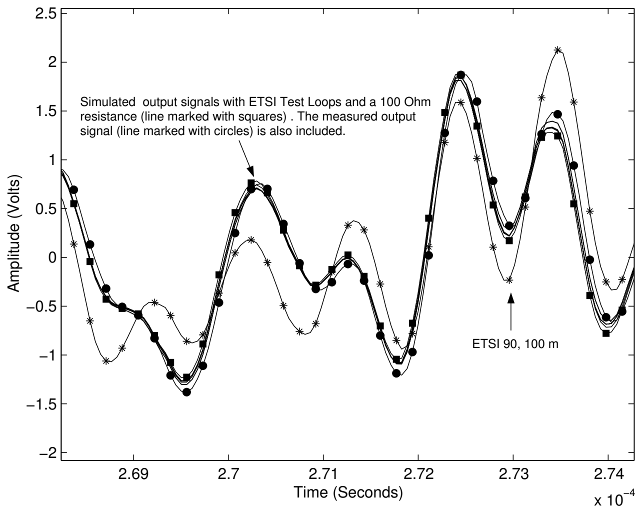

Figure 3: Waveforms of $v_S(t)$: Simulated waveforms for the ETSI loop test cases defined in [8] and for a purely resistive load $Z_L = 100\,\Omega$ (line marked with squares); Measured waveform (line marked with circles) for ETSI40 3.5 km. Significant deviations occur only for the extreme (100m of the ETSI90 cable) loop. The input was identical for all cases.

2. MODELLING THE LINE DRIVER

A typical, simplified line driver structure is shown in Figure 2. Line drivers are often built around two identical amplifiers such as the one in Figure 2, where one of them acts in counter phase. Each amplifier thus provides half the required voltage and half the output resistance. Since the two amplifiers are identical, PAR reduction considerations only need to focus on one of them. Apart from producing the required voltage amplification and current driving capabilities, the line driver should also minimise the receive signal energy loss. This implies that the output impedance should match the impedance of the load, over the frequency band of interest. In traditional design, this is achieved by connecting a resistor, here referred to as R_S, with the same impedance as the load in series with the line driver and the line. This corresponds to $R_1 = \infty$ in Figure 2. Such a design has the drawback of high power dissipation; half of the power of the line driver is wasted in R_S.

In modern designs the output resistance is actively synthesised, and R_S can be made smaller without sacrificing the quality of the received signal. The line driver treated in the following uses this kind of active termination.

2.1 Selecting a target point

The main goal of PAR-reduction is to lower the power consumption of the line driver. For the class of amplifiers treated here that means lowering the span of the linear amplification region. This is only possible if the signal swing is made smaller, which means reducing the highest peaks in the signal.

The maximum, in terms of voltage, of the signal $v_S(t)$ measured at the point v_S in the line driver is limited by the supply voltage, and it is thus the voltage span at this node that sets the limits on the linear amplification region. PAR reduction should aim at keeping the signal swing of $v_S(t)$ as low as possible. This insight, gained from an investigation of parts of the transceiver that are out of scope in most of the work done on PAR reduction, may be valuable for the study of algorithms and future research.

Line drivers are designed to provide linearity of the signal $v_L(t)$ measured at the point v_L, *not* $v_S(t)$, while PAR reduction should aim at $v_S(t)$, *not* $v_L(t)$. For the passively terminated design, cable dynamics would affect the transfer function to v_L but not to v_S. In actively terminated designs, which are considered here, cable dynamics enter both transfer functions, although in slightly different ways.

The signal $v_S(t)$ differs both from the input signal $x(t)$ of the line driver and from the signal $v_L(t)$ measured at the load. A model of the signal transformation from the input to this node must be included in the PAR reduction algorithm.

With an ideal operational amplifier, the transfer function $H(Z_L)$ of the system with input $x(t)$ and output $v_S(t)$ is given by

$$H(Z_L) = \frac{V_S(f)}{X(f)} = \frac{(R_3+Z_1)(Z_L R_S + R_1 R_S + Z_L R_1)}{Z_L(Z_1 R_S + Z_1 R_1 - Z_2 R_3) + Z_1 R_S (R_1 + Z_2)}, \tag{1}$$

where $V_S(f)$ and $X(f)$ are the Fourier transforms of $v_S(t)$ and $x(t)$, respectively. The impedances Z_L, Z_1 and Z_2 are complex and frequency dependent. The dynamics of the loop and the transformer enter the transfer function[2] (1) via the impedance Z_L. A similar linear analysis on more detailed circuit schemes for real line drivers yields models that give excellent matches to measured data [2]. These measurements further show that non-linear effects can be neglected in this application.

2.2 Cable dependency

According to (1), the transfer function $H(Z_L)$ depends on the load impedance Z_L. The characteristics of the load vary with the actual cable attached to the line driver. If the model used for PAR reduction purposes is sensitive to variations of the load impedance, that has to be addressed.

Figure 3 shows simulated waveforms of $v_S(t)$ for different testloops and for a purely resistive load of $Z_L = 100\,\Omega$, as well as a measured waveform. Variation of the cable impedance does not cause large model deviations, except for the comparably short (100m) and thick ETSI90 cable, which causes a more oscillating signal. Simulations and measurements show that, given an accurate model of the transformer,

[2]The transfer function to v_L is dependent on the load characteristics also in the passively terminated design, but as previously stated, $v_S(t)$ is the signal of interest for PAR reduction.

a $100\,\Omega$ resistance approximates the cable impedance with a negligible error, as long as no cables with extreme parameters are used. Hence, as the PAR reduction algorithm can be designed for a single representative load impedance, there is no need to adapt the algorithm to the actual load when installing a system in a new environment.

2.3 Discrete time filter representation

If the impedances Z_L, Z_1 and Z_2 can be approximated as rational functions of $s = j2\pi f$, a closed-form discrete-time representation of (1) can be found using the bilinear transformation $s' = 2(8F_{\text{Nyquist}}) \cdot \frac{z-1}{z+1}$, where $z = e^{j2\pi f/(8F_{\text{Nyquist}})}$ and F_{Nyquist} is the sampling frequency at the IFFT output. The oversampling factor of eight is recommended in [3] and [4] for a sufficiently accurate representation of an analogue signal for PAR reduction purposes. For the line driver considered here and for the approximation $Z_L = 100\,\Omega$, the resulting discrete-time filter has an IIR form with 9 poles and 9 zeros. We approximate this IIR form with a 20-tap FIR filter $\mathbf{h} = [h[0] \cdots h[19]]$. The filter operation can be represented by the convolution matrix $\mathbf{A}_{ld}$, whose first row is $\mathbf{h} = [h[0]\ 0 \cdots 0]$. This form will be used in the next section.

The transmit filters, including an increase in sampling rate, and the DAC are represented by the matrix $\mathbf{A}_{tx}$. The entire transformation from the space of critically sampled signals just after the IFFT ($x'[n]$ in Figure 1) to the space of oversampled signals in the v_S domain can now be described by the matrix $\mathbf{A} = \mathbf{A}_{ld}\mathbf{A}_{tx}$, which has size 512×4352 for our setup. This representation can be used to extend existing PAR reduction methods in the sense that the PAR reduction criterion is defined for $v_S(t)$ instead of $x'[n]$. In practice, implementations may use the IIR or FIR structure instead.

3. EXTENSION OF THE TONE RESERVATION METHOD

The tone reservation method, a promising PAR reduction scheme suggested by Tellado [3], uses a selected set of tones to modify the signal such that the PAR of the resulting signal is lowered. Finding a low-PAR solution is an optimisation problem, which can be formulated as a linear program. Modifying the algorithm so that it takes into account the line driver dynamics, we obtain the linear program

$$\begin{aligned} &\underset{\check{\mathbf{C}}}{\text{minimise}} \quad \gamma \\ &\text{subject to} \quad \begin{bmatrix} \mathbf{A}\check{\mathbf{Q}} & -\mathbf{1} \\ -\mathbf{A}\check{\mathbf{Q}} & -\mathbf{1} \end{bmatrix} \begin{bmatrix} \check{\mathbf{C}} \\ \gamma \end{bmatrix} \le \begin{bmatrix} -\mathbf{A}\mathbf{x} \\ \mathbf{A}\mathbf{x} \end{bmatrix}, \end{aligned} \tag{2}$$

where $\gamma = \sqrt{\text{PAR}}$ is the crest factor. The vector $\check{\mathbf{C}}$ is a length $2U$ vector (where U is the number of tones reserved for PAR reduction) containing the PAR reduction symbols and the matrix $\check{\mathbf{Q}}$ represents a coordinate transform, mapping $\check{\mathbf{C}}$ onto the space of allowed PAR reduction signals. The transformation $\mathbf{A}$ maps signals from $x'[n]$ to the v_S domain in the line driver. With $\mathbf{A} = \mathbf{I}$, (2) corresponds the original problem considered by Tellado [3]. A similar approach to the one presented here considers the transmit filters [6]. Note that, while the criterion for PAR reduction is moved, the actual PAR reduction signal is still added to the critically sampled signal at the output of the IFFT block.

The extension of the model increases complexity significantly, which is mainly due to the oversampling. Even if no line driver model would be used in the algorithm, intersample behavior should still be taken into account. Thus, in a fair comparison, the increased complexity caused by the line driver modelling is modest.

The same problem formulation can be kept when using lower complexity active-set based algorithms, proposed in [4]. Here, a good but suboptimal solution will be achieved after a few iterations. Performance results of algorithms both based on linear programming and on the active-set method when focusing on v_S are provided in the next section.

4. SIMULATION RESULTS

A simulation environment was built around the IIR model of the actively terminated line driver and specifications for digital domain filters given in [9]. Three line driver transfer functions were used in the simulations, corresponding to three different ETSI test loops. The transfer function obtained for the $100\,\Omega$ test loop was used in the PAR reduction algorithm. This illustrates the dependency of the transfer function, and thus the PAR reduction algorithm, to the load connected to the line driver. The effect of these variations are illustrated in Figure 5. The symbols were created using random tone constellations. Ten tones, randomly selected from the transmission band, were reserved for PAR reduction. The tone selection problem is treated in [1].

Three distinct transmit path matrix models were tested. The first model is a pure upsampling $\mathbf{A} = \mathbf{U}_8$, the second model is the digital transmit filter matrix $\mathbf{A} = \mathbf{U}_2\mathbf{A}_{tx}$ (a four times increase in sampling rate is already included in the filter specifications), while the third represents the full model $\mathbf{A} = \mathbf{A}_{ld}\mathbf{U}_2\mathbf{A}_{tx}$. The matrix $\mathbf{U}_i$ is a convolution matrix describing an upsampling operation by a factor i using a sinc impulse response. Note that the comparison is fair in the sense that all three models include the effects of oversampling.

Figure 4 shows the performance of a linear programming based algorithm using the three different models in terms of clip probability evaluated at three different points in the system. The results illustrate an important point: When comparing PAR reduction algorithms, the point in which PAR is evaluated is critical. When PAR is evaluated after the IFFT, the original method with $\mathbf{A} = \mathbf{U}_8$ gives lower PAR than the extended method. Also, when PAR is evaluated at the DAC output, the method with the full line driver model performs worse. Since the goal is to reduce power dissipation in the line driver, the method with the best performance at v_S should be selected. A PAR criterion at the IFFT output or immediately after the DAC has limited physical motivation. Moving the criterion into the line driver, the PAR at v_S is lowered by approximately 0.5 dB, as shown in Figure 4.

In Figure 5, the results of an active-set based algorithm evaluated at v_S are shown, now also with the unreduced signal. Also for this less complex algorithm, modelling the line driver clearly improves the performance. PAR is about 0.5 dB lower at a symbol clip rate of 10^{-4} when a line driver model is included in the algorithm. The influence of variations of the line driver transfer function due to the different testloops is negligible.

5. CONCLUSION

A typical line driver structure was analysed and it was concluded that an efficient PAR reduction algorithm should fo-

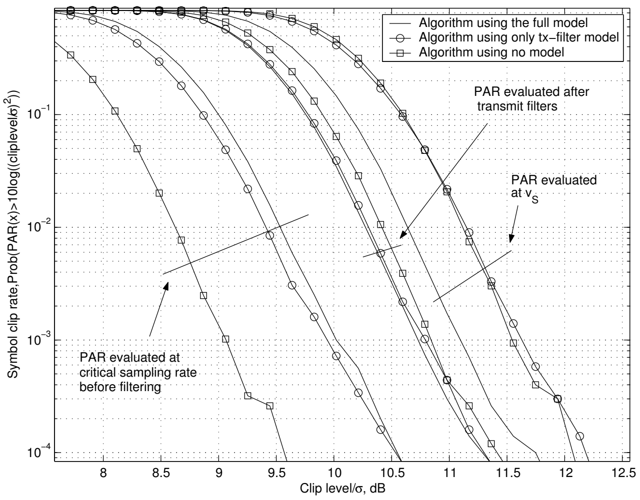

Figure 4: Clip probability as a function of clip level for a signal that is PAR reduced using the three linear programming based methods. The PAR was evaluated at three points in the system; corresponding to $x'[n]$, $x[n]$ and $v_S(t)$ in Figure 1. Evaluating PAR at the critical sampling rate (the three left-most curves), the best performance is achieved when using no filter model. However, when evaluating the physically motivated signal $v_S(t)$, the reduction scheme using the full model performs best.

cus on the signal at a specific node inside the line driver that coincides neither with the input signal nor with the output signal. A linear model of the line driver dynamics was developed. Measurements suggest that this model structure is accurate enough for the purpose.

The tone reservation method has been modified to use the new knowledge. Simulations show that improvements of around 0.5 dB are possible. However, note that the gain depends on the algorithm and the transmit filters. The key point of this work is the shift of focus towards a physically motivated PAR reduction criterion.

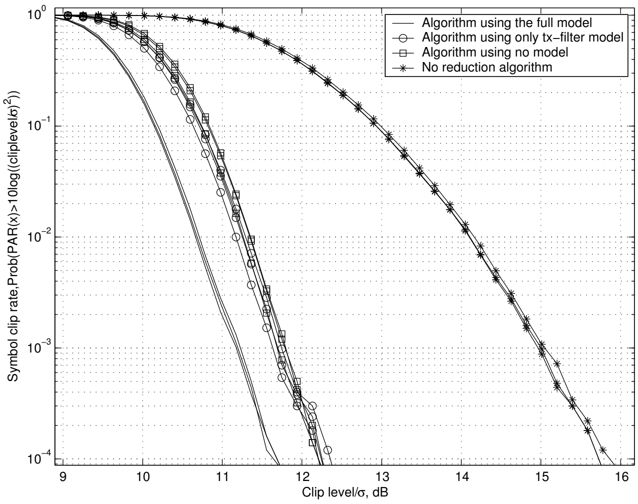

Figure 5: Clip probability as a function of clip level. Three different active set based methods were used. The first uses *no model* but only upsampling by a factor 8 ($\mathbf{A} = \mathbf{U}_8$). The second version has an accurate *transmit filter model*, i.e. $\mathbf{A} = \mathbf{A}_{tx}$, while the third version uses the *full model* including line driver and transmit filters. PAR is evaluated at v_S in all cases. Three line driver models were used in the simulations, corresponding to the three curves in the plot for each method. The line driver models differ only in the load Z_L that is used, corresponding to realistic cable deviations. One of these three models was also used in the design of the PAR reduction algorithm with the full model. The right-most curves correspond to the case when no PAR reduction is employed.

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