# TRANSMIT DIVERSITY OVER FREQUENCY-SELECTIVE FADING CHANNEL: A BLIND APPROACH

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## **ABSTRACT**

In this article, we introduce a novel blind space time processing allowing the transmission of digital communication over a frequency-selective fading channel. We show that it is relevant to use transmit diversity in such a context. Our scheme uses 2 transmit antennae and K ( $K \ge 2$ ) received antennae. We call our method Turbo deflation because it is based on results from the field of blind sources separation of convolutive mixture (i.e. deflation approach [1]) and from principle of iterative turbo decoding [2].

Furthermore it is a blind approach which means that the receiver does not need any training sequence. Our novel method could for instance be used for underwater communications where blind equalization scheme have already been introduced with really convincing performances [3].

# 1. INTRODUCTION

Let us consider the complex envelope v(t) of the continuous-time signal transmitted by a communication system using two transmit antennae and K received antennae. Denote by  $y_p$  the contribution in y of the signal emitted by antenna number p. Thus y can be seen as a linear mixture of 2 sources  $y = y_1 + y_2$ . In the telecommunication framework,  $y_p$  may be a linear process modeled as :  $y_p(t) = \sum_m s_p(m) h_p(t-mT)$  where  $(s_p(m))_m$  is an i.i.d. sequence of discrete Q-PSK symbols, T is the symbol period and  $h_n$  denotes the impulse response of the channel relative to source p, i.e. a filter stemming from the cascade of a band-limited pulse-shaping filter and multi-path transmission effects. At the receiver side, y is observed through an array of Ksensors and thus y(t) and  $h_p(t)$  are vector-valued. We assume that the symbols emitted by antenna 2 are an interleaved version of the one emitted by antenna 1.  $\mathbf{S}_1(n) = (s_1(nN), \dots, s_1(nN+N-1))^T$ and  $S_2(n) = (s_2(nN), \dots, s_2(nN+N-1))^T$  have the following relation:

$$\mathbf{S}_2(n) = \Pi_N \mathbf{S}_1(n) \tag{1}$$

where  $\Pi_N$  is a  $N \times N$  pseudo-random interleaver. We also assume that the channel is stationary over the duration NT.

We address here the following blind problem that consists in restoring the symbol sequence  $S_1(n)$ . Note that neither training sequences, nor prior knowledge about the filters are available at the receiver side. In fact, this problem has obvious connections with the framework of blind source separation. We remind that the standard requirements in this field are: the sources are mutually independent, centered, ergodic and stationary. In our context,  $S_1(n)$  and  $S_2(n)$  do not strictly obey the independence assumption. However for large enough interleavers, we can assume mutual independence between the sources.

In order to extract the sources, we use the deflation approach [1] that is based on the minimization of a contrast function [4]. We recall that a contrast function is a function of the statistics of the received signal and its minimization allows the extraction of one source. Section 2.1 reviews the deflation procedure in details.

By re-itering the deflation procedure and exploiting the relation (1) between the sources, we can use the information obtained from the estimate of the first extracted source as *a priori* information for the second one. This allows us to improve the estimate of the sequence of symbols  $S_1(n)$ .

## 2. THE TURBO DEFLATION

#### 2.1 Description of the iterative procedure

First of all, we describe the deflation procedure for two sources which is the key-stone of our reception scheme. This procedure can be splitted into two stages:

- Stage 1:
- 1. The observation y is sampled at rate 1/T and passed through a digital  $K \times 1$  vector-filter  $G_1(z)$ . The coefficients of  $G_1(z)$  are adapted by minimization of a contrast function. The minimum is reached iif  $z_1(n)$  the output of  $G_1(z)$  equals the symbols of one of the source up to a delay and a complex multiplicative factor.
- 2. By an adaptive subtraction procedure, we compute from  $z_1(n)$  the contribution on each captor of the source currently estimated and we subtract this contribution from the observed signal. More precisely, we search  $t_1(z)$  of size  $K \times 1$  that minimizes

$$\mathbb{E}[||y(n) - [t_1(z)]z_1(n)||^2].$$

- $t_1(z)$  is an estimate of the channel impulse response corresponding to the first estimated source. And  $y'(n) = y(n) [t_1(z)]z_1(n)$  is a signal of dimension K corresponding to the contribution of source 2.
- Stage 2: We iterate stage 1 but on the signal y'(n) in order to adapt a filter  $G_2(z)$ , to estimate the second source and the corresponding channel impulse response.

The existence of filters  $G_k(z)$ , k = 1,2 allowing the extraction of the sources of the mixture is based on results presented in [1].

As the two sources send the same symbols but interleaved, we compute from the first source an extrinsic information that is used as a priori information for the estimate of the second source. Following the turbo principle and by iterating this procedure, the extrinsic information computed from the second source can be used as a priori information for the estimate of the first extracted source at the next iteration. The first iteration of our turbo deflation procedure consists in the above depicted deflation procedure with the computation of the respective extrinsic information. At the end of this first iteration we obtain the signal  $e^{(1)}(n)$  (the upperscript  $^{(m)}$  stands for iteration number m):  $e^{(1)}(n) = y(n) - [t_1^{(1)}(z)]z_1^{(1)}(n) - [t_2^{(1)}(z)]z_2^{(1)}(n)$  that can be considered as a residual signal containing the part of the source signal not exploited during the first iteration. In order to start the next iteration and to re-extract the first source, we add to  $e^{(1)}(n)$  the estimated contribution of source 1 at the previous iteration (i.e.  $[t_1^{(1)}(z)]z_1^{(1)}(n)$ ). Once the symbols of the source are estimated (using the *a priori* information of the previous iteration, see section 2.2 for details) we estimate  $[t_1^{(2)}(z)]$  and subtract the contribution of this source and add the previous estimate of source number two (as shown by figure 1) before starting stage 2 of iteration 2.

By such a procedure, we expect an improvement of the source and channel  $(t_1^{(m)}(z))$  and  $t_2^{(m)}(z)$  estimates. This will be confirmed by simulations in section 3.

# 2.2 Estimation of the symbols sequence

At the output of  $G_k^{(m)}(z)$ , we obtain an estimate up to a multiplicative factor and a delay of the symbols of one of the sources. In order to estimate the sequence of symbols, we need the following assumptions: (i) The delay and the multiplicative factor are known by the receiver. (ii) The receiver knows the source currently estimated. Without restriction let us assume that the first source extracted is the one sending the sequence  $S_1(n)$ . (iii) Let  $z_k^{(m)}(n)$  for k=1,2 be the output of  $G_k^{(m)}(z)$  at iteration m. We assume that:

$$z_k^{(m)}(n) = s_k(n) + b_{k,m}(n)$$
 (2)

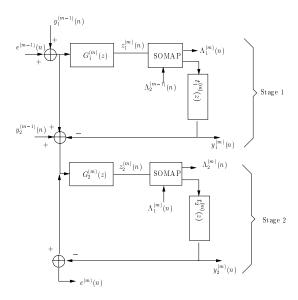


Figure 1: Iteration *m* of the Turbo Deflation

where  $b_{k,m}(n)$  is white Gaussian noise with zero mean and unknown variance  $v_{k,m}^2$ 

Assumption (i) and (ii) are not too restrictive, we discuss this point in section 2.3. The third assumption is also realistic. Indeed, if we know the delay and the multiplicative factor, we can compensate their effect. Furthermore as  $G_k(z)$  inverses the channel of the source number k, it is pertinent to assume that  $z_k^{(m)}(n)$  satisfies equation (2). Note that the variance  $v_{k,m}^2$  of the noise is empirically estimated by :

$$\hat{v}_{k,m}^2 = \frac{1}{N} \sum_{n=1}^{N} \left( z_k^{(m)}(n) - \hat{s}_k^{(m)}(n) \right)^2.$$
 (3)

where  $\hat{s}_k^{(m)}(n)$  is the hard decision of  $s_k(n)$  obtained from  $z_k^{(m)}(n)$ .  $\hat{v}_{k,m}$  measures the reliability of the symbols extraction.

We use the principle of turbo decoding to estimate the sequence of symbols. Indeed at each iteration, we have two different estimations of the same information : one obtained at the output of  $G_1(z)$  and the other at the output of  $G_2(z)$ . We now describe in details the estimation of  $\mathbf{S}_1(n)$  that we call "SOMAP" in figure 1 for soft mapping. We consider Q-PSK symbols in this paper. Therefore we treat separately their real and imaginary part. The problem now consists in estimating a vector  $\mathbf{U}_1(n) = [\mathcal{R}(\mathbf{S}_1(n)), \mathcal{I}(\mathbf{S}_1(n))]^T$  of size  $2N \times 1$  with value in  $\{+1, -1\}$ , where  $\mathcal{R}(x)$  (respectively  $\mathcal{I}(x)$ ) denotes the real (respectively imaginary) part of x. Let us consider the iteration m. The estimate  $\hat{\mathbf{U}}_1^{(m)}(n)$  of  $\mathbf{U}_1(n)$  is obtained by (for details please

refer to [5]):

$$\hat{\mathbf{U}}_{1}^{(m)}(n) = \mathbb{E}[\mathbf{U}_{1}(n)/L_{1}^{(m)}(\mathbf{U}_{1}(n))]$$

$$= \tanh\left(\frac{L_{1}^{(m)}(\mathbf{U}_{1}(n)|\mathbf{Z}_{1}^{(m)}(n))}{2}\right)$$

where  $L_1^{(m)}(\mathbf{U}_1(n))$  is the information *a priori* of  $U_1(n)$  and  $L_1^{(m)}(\mathbf{U}_1(n)|\mathbf{Z}_1^{(m)}(n))$  is the *a posteriori* Log-likelihood ratio of  $U_1(n)$ . From  $z_1^{(m)}(n)$ , we estimate  $L_1^{(m)}(\mathbf{U}_1(n)|\mathbf{Z}_1^{(m)}(n))$  as follows:

$$L_1^{(m)}(\mathbf{U}_1(n)|\mathbf{Z}_1^{(m)}(n)) = L_1^{(m)}(\mathbf{U}_1(n)) + \mathbf{\Lambda}_1^{(m)}(n)$$
(4)

where  $\Lambda_1^{(m)}(n)$  is the extrinsic information estimate of the sequence  $U_1(n)$ . By equation (2),  $z_1^{(m)}(n)$  may be viewed as the output of an equivalent AWGN channel. Thus we have:

$$\mathbf{\Lambda}_{1}^{(m)}(n) = \left[\frac{2\Re(z_{1}^{(m)}(n))}{\hat{\mathbf{v}}_{1,m}^{2}}, \frac{2\Im(z_{1}^{(m)}(n))}{\hat{\mathbf{v}}_{1,m}^{2}}\right]^{T}.$$
 (5)

 $L_1^{(m)}(\mathbf{U}_1(n))$  is the estimate at iteration m of the a priori information of  $\mathbf{U}_1(n)$ . At stage 2 of the previous iteration, we computed  $\boldsymbol{\Lambda}_2^{(m-1)}(n)$  from  $z_2^{(m)}(n)$ . As  $S_2(n)$  is assumed independent of  $S_1(n)$ , we can use the extrinsic information  $\boldsymbol{\Lambda}_2^{(m-1)}(n)$  of source 2 as a priori information for source 1. Using equation (1) we have:

$$L_1^{(m)}(\mathbf{U}_1(n)) = \begin{pmatrix} \Pi_N^{-1} & \mathbf{0}_N \\ \mathbf{0}_N & \Pi_N^{-1} \end{pmatrix} \mathbf{\Lambda}_2^{(m-1)}(n)$$

where  $0_N$  is a  $N \times N$  matrix of zeros.

At the next stage, we procide the same way for the estimation of  $S_2(n)$ .

## 2.3 The identification problem...

We assumed in the previous section that  $z_1(n)$  (respectively  $z_2(n)$ ) is an estimate of  $s_1(n)$  (respectively  $s_2(n)$ ). In fact  $z_1(n)$  is an estimate of  $\lambda_1 s_k(n-n_1)$  where  $k \in \{1,2\}$  and  $z_2(n)$  is an estimate of  $\lambda_2 s_l(n-n_2)$  where  $l \neq k$  and  $l \in \{1,2\}$ . In order to apply the proposed method, we need to estimate blindly  $\lambda_1$ ,  $\lambda_2$ , k, l and  $n_2-n_1$ . The estimate of  $\lambda_1, \lambda_2$  is out of the scope of this paper but there exists methods to do it [6].

We present an algorithm that estimates blindly, k, l and  $n_2 - n_1$ . This algorithm uses the relation (1) linking  $s_1(n)$  and  $s_2(n)$ . Let's take  $l \in \{-L, ..., L\}$ , for each l we estimate the intercorrelation coefficient  $\hat{\gamma}_2(l)$  between the sequence  $z_1(n)$  and the  $\Pi_N$ -interleaved version of the delayed sequence  $z_2(n+l)$ :  $\hat{\gamma}_2(l) = \frac{1}{N} \mathbf{Z}_1(n)' \Pi_N \mathbf{Z}_2(n+l)$  where  $\mathbf{Z}_i(n) = [z_i(n), ..., z_i(n+N-1)]^T$  and l stands for transposed and conjugated. In the same way, we estimate the intercorrelation coefficient  $\hat{\gamma}_1(l)$  between

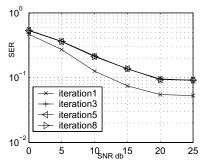


Figure 2: SER vs SNR: Iterative deflation - 1st Source

the sequence  $z_1(n)$  and the  $\Pi_N^{-1}$  -interleaved version of the delayed sequence  $z_2(n+l)$ :  $\hat{\gamma}_1(l) = \frac{1}{N}\mathbf{Z}_1(n)'\Pi_N^{-1}\mathbf{Z}_2(n+l)$ . It is easily seen that we have:  $[k, n_2 - n_1] = \operatorname{ArgMax}_{i,l} |\hat{\gamma}_i(l)|$ .

#### 3. SIMULATION RESULTS

In this section, we consider a mixture of 2 O-PSK sources. The considered contrast function is the Godard criterion and  $G_k^{(m)}(z)$  is obtained as follows  $G_k^{(m)}(z) = \operatorname{ArgMin} \frac{1}{N} \sum_{n=0}^{N-1} \left( |z_k^{(m)}(n)|^2 - 1 \right)^2$ . The filter G has a fixed number of taps set to 8 and the minimization is done by a Newton algorithm. We now specify the propagation model. The excess bandwidth factors are all equal to 0.2. The propagation channel results from the superposition of 3 paths: the delays and directions of arrival and directions of departure are uniformly chosen in respectively [0;3T] and  $[0;2\pi]$ . The attenuation of the path follows a Rayleigh distribution. The distance between the transmitted (respectively received) antennae is  $\lambda/2$  where  $\lambda$  is the wave length of the signal. The number of sensors is set to K = 3. Finally, duration of y is set to 1000T. All the following results have been averaged over 500 trials, where the channel and the symbols sequence are randomly generated for each new trial. The method is tested in a noisy context. The noise is assumed Gaussian. complex, temporally and spatially white with zero mean.

In order to illustrate the behaviour of our method, we first consider that the sources are mutually independent: information obtained from the extraction of source 1 can not be used for the estimation of source 2. Performance is measured by the residual Symbol Error Rate (SER) at the output of the iterative deflation procedure.

Figure 2 (respectively 3) represents the SER for the first (respectively second) extracted source at iteration 1, 3, 5 and 8.

We observe that the estimation of the second source is worst than the one of the first source. Furthermore the SER increases with the number of

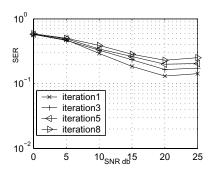


Figure 3: SER vs SNR: Iterative deflation - 2nd Source

iteration, this can be explained by the fact that in a noisy context, the deflation procedure introduces error propagation. Indeed at each deflation stage the subtraction of the contribution of the extracted source from the observation increases the noise level. Of course this affects on the performance of our method.

We now consider the case where the second source transmits an interleaved version of the symbols emitted by the first antenna. A pseudo-random interleaver of size N=1000 symbols is used. In order to evaluate the performance of our method, we measure symbols error rates (SER) computed from the sequence estimated at iteration 1,3, 5 and 8.

Results are presented in figure 4. The more itereations, the smaller the SER. Although the iterative deflation procedure also introduces error propagation, here the exploitation of transmit diversity allows us to improve the estimation of symbols from one iteration to another.

As we did not find in the litterature any blind transmit diversity method over frequency-selective fading channel and as we need a point of comparison, we compare our method with the following scenario: one single antenna transmitting the sequence  $S_1(n)$  over the same propagation channel (equivalent to a SIMO model) but with a power equal to the sum of powers used on each transmit antenna. Results are also presented in figure 4 and demonstrate that our method outperforms the performance obtained by a single antenna using twice as much power. Thus it is relevant to use transmit diversity in a blind communication scheme. Note that it is not obvious that our method performes better. Just keep in mind that the channel is a frequency-selective fading one and is unknown at the receiver. Furthermore our method has to deal with a convolutive mixture of two sources and needs to separate the sources before estimating the symbols sequence.

# 4. CONCLUSION

We describe in this paper a blind space time processing scheme that allows to transmit digital com-

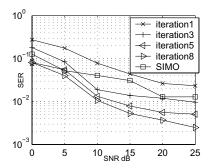


Figure 4: SER versus SNR: Turbo deflation

munication over a frequency-selective fading channel. Futhermore we provide simulations to illustrate the behaviour of our method and we show that it is relevant to consider transmit diversity for blind communications over a frequency-selective fading channel. For instance a potential application could be acoustic transmissions. Note that the method can be easely extended for more than two transmit antennae. At this point no codes have been used but this is currently under investigation, obviously this will give better performence and we hope a better turbo effect.

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