SYNCHRONIZATION OVER RAPIDLY TIME-VARYING MULTIPATH CHANNELS FOR CDMA DOWNLINK RECEIVER IN TIME-DIVISION MODE

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ABSTRACT

In this paper, we consider a Time-Division CDMA system operating in downlink mode in a multi-user multipath channel scenario. In order to implement RAKE reception, the delays and the phases of the different paths have to be estimated and the estimates have to be updated. In the context of burst transmissions, the delays do not vary during a burst and only the phases have to be tracked. In this paper we investigate the issue of phase tracking over rapidly time varying multipath channels. We propose a new version of a conventional phase recovery loop that can cope with unresolvable paths. The optimization is based on the concept of prefiltering. The analysis shows improved tracking performance in comparison to the standard phase recovery algorithm.

1. INTRODUCTION

The conventional receiver for DS-CDMA communications is the RAKE receiver. The RAKE receiver is a matched filter, matched to the operations of spreading, pulse shape filtering and channel filtering. It requires the knowledge of the delays, the phases and the amplitudes of multipath components. We assume that estimates of those channel parameters have been obtained during an aquisition procedure. In this paper, we will focus on the tracking step, which consists in updating the channel parameters.

Typically, phases and magnitudes of the channel coefficients vary considerably faster than the channel delays. Time-Division CDMA system is a burst structured system where we can assume constant delays during one burst (*i.e.* one time-slot).

For instance in TDD mode of UMTS, the carrier radio wave frequency f_0 is around $2GHz$, the chip rate $\frac{1}{T_c}$ is around $4Mchip/s$ and the slot lasts about $T_{slot} = 666\mu s$. In the case of a deterministic Doppler model, the maximum variation of the phase $(2\pi\Delta f_d T_{slot})$ is around 55^o where $\Delta f_d = \frac{v_m}{v_0} f_0$ is the doppler spread, v_m is the mobile speed and v_0 is the wave celerity. On the other hand, the maximum variation of the path delay $(\frac{v_m}{v_0} T_{slot})$ is negligible during a slot, about $3 10^{-4}$ chip duration. Therefore in burst mode, it is just necessary to track the phases and the magnitudes.

We will focus in this paper on the problem of phase tracking. This task is usually assessed by mean of a conventional Phase Error Detector (PED), the remodulator loop [1][2]. The conventional PED operates correctly in the case of a single path channel, but in the case of a multipath channel, the output of the PED relative to one path becomes strongly influenced by the additional paths. In addition with the multi-user interference, this will have a large impact on the performance of the overall system. It is thus of paramount interest to mitigate the effect of the adjacent paths on the task of phase recovery. A new version of the conventional PED is presented. The modification is based on the concept of prefiltering. This concept has been studied by D'Andrea and Luise in [3] and by the authors in [4] for timing recovery in a single path scenario. It consisted in inserting a prefilter in the timing recovery loop and computing the optimal coefficients which minimize the timing variance. We propose to generalize this concept to the phase recovery for a multipath channel propagation. In this context, the prefilter coefficients are calculated in order to minimize the phase variance and to cope with adjacent and potentially unresolved multipath.

After a detailed description of the system model in section 2, we introduce in the following section the standard phase recovery loop. Its improved version including the prefilter is presented in Section 4. Numerical results are presented in Section 5, and conclusions are given in Section 6.

2. SYSTEM MODEL

The continuous-time baseband representation (complex envelope) of the transmitted signal is modeled as:

$$x(t) = T_s \sum_{k=1}^{K} \sum_{n} a_{k[n]} s_k(t - nT_s) \tag{1}$$

where $a_{k[n]}$ are the iid QPSK symbols with power A^2, transmitted by the k^{th} source at time nTs. K is the number of users. $s_k(t)$ is the signature of the k^{th} user, which results from the convolution between the k^{th} spreading code $\{c_k\}$ and the half-Nyquist filter h_e (square root raised cosine filter):

$$s_k(\tau) = \sum_{q=0}^{Q-1} c_{k[q]} h_e(\tau - qT_c) \tag{2}$$

where $T_c = T_s/Q$ is the chip duration and $c_{k[q]}, q = 0 \ldots Q-1$ are the chips. Let us assume that all K active codes are known. We suppose by convention one desired code only, the code number one. We note $\Gamma_{kj}(\tau)$ the cross-correlation function between user k and j: $\Gamma_{kj}(\tau) = \left(s_k * s_j^H\right)(\tau)$ [1]. The multi-user signal from the transmitter travels through a multipath propagation channel with L independant paths modeled as: $h(\tau,t) = \sum_{l=1}^{L} \alpha_l(t)\,\delta(\tau - \tau_l)$, with $\alpha_l(t) = \rho_l e^{j\theta_l(t)}$ being the l^{th} complex path tap and τ_l the respective path delay.

The received signal $r(t) = (h * x)(t) + w(t)$ is perturbed by an additive white complex gaussian noise $w(t)$, with a two-sided power spectral density $2N_0$.

The receiver is a RAKE receiver. The received signal $r(t)$ is passed through an anti-aliasing filter (AAF) before being

[1] By convention, the exponent $(.)^H$ represents hermitian transformation *i.e.* $f^H(t) = f^*(-t)$ for a given function f and hermitian transposition for vectors

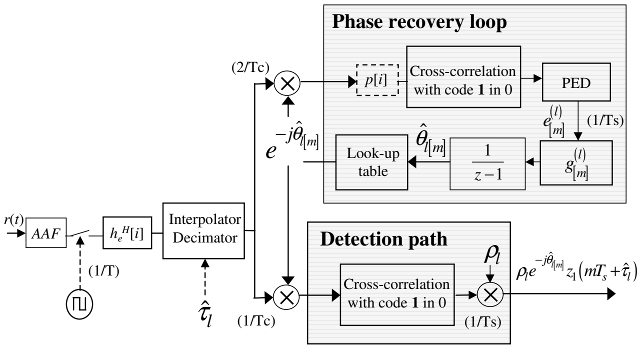

Figure 1: l^{th} RAKE finger with the standard phase recovery loop (without the prefilter $p_{[i]}$) or the improved version (with the prefilter)

sampled at a rate $1/T$, which must be at least equal to $2/T_c$ to guarantee sufficient statistics for detection and synchronization [2]. The RAKE receiver consists in a bank of L RAKE fingers, one per path. In each finger, an interpolator-decimator provides the samples at the desired interpolation instants ($\hat{\tau}_l$ for the l^{th} finger). Those samples are fed to the "detection path" and to the phase recovery loop. The phase recovery loop will be the topic of the next section. In the "detection path", the interpolated samples are despreaded by computing the cross-correlation with the code of user #1 . These despread signals are then combined using combining coefficients (*i.e.* the complex conjugate of $\hat{\alpha}_l$ for the l^{th} finger, where $\hat{\alpha}_l$ is the estimate of the channel coefficient α_l) to form a decision statistic. The l^{th} RAKE finger with its phase recovery loop is depicted in Figure 1.

3. THE CONVENTIONAL PHASE LOOP

We first consider the standard phase recovery loop for the l^{th} RAKE finger (cf Fig.1 without the prefilter). Its purpose is to estimate the channel phase θ_l. The estimate of θ_l is updated at a symbol rate by a phase error signal $e^{(l)}_{[m]}$ filtered by the loop filter $g^{(l)}_{[m]}$. The recursive equation of the phase recovery loop is thus defined as:

$$\hat{\theta}_{l[m+1]} = \hat{\theta}_{l[m]} + (g^{(l)} * e^{(l)})_{[m]} \tag{3}$$

where $\hat{\theta}_{l[m]}$ is the estimate of θ_l at instant mT_s.

The Phase Error Detector (PED) computes $e^{(l)}_{[m]}$ as follows

$$e^{(l)}_{[m]} = \mathrm{Im}\left\{e^{-j\hat{\theta}_{l[m]}}\hat{a}^*_{1[m]} z_1\left(mT_s+\tau_l\right)\right\} \tag{4}$$

where $\hat{a}_{1[m]}$ is the estimate of $a_{1[m]}$ and $z_1(t)$ is the output of the matched filter $s_1^H(\tau)$ when $r(t)$ is applied, *i.e.* $z_1(t) = (r * s_1^H)(t)$. By using (1), it can be written as:

$$z_1(t) = T_s \sum_{l_t=1}^{L} \alpha_{l_t} \sum_{k=1}^{K} \sum_n a_{k[n]} \Gamma_{k1}(t - nT_s - \tau_{l_t}) + n(t) \tag{5}$$

with $n(t)$ the filtered version of $w(t)$.

The loop error signal can be decomposed into the conditional expectation $\mathrm{E}\left\{e^{(l)}_{[m]} \,|\, \hat{\theta}_{l[m]}\right\}$ and a zero-mean disturbance $N^{(l)}_{[m]}$. The first term, called the S-curve, is a function of the phase error: $\mathrm{E}\left\{e^{(l)}_{[m]} \,|\, \hat{\theta}_{l[m]}\right\} = S\left(\theta_l - \hat{\theta}_{l[m]}\right)$ and the second term, called the loop noise, is defined as: $N^{(l)}_{[m]} = e^{(l)}_{[m]} - S\left(\theta_l - \hat{\theta}_{l[m]}\right)$. The computation of the error signal expectation conditioned on fixed values of phase estimation ($\hat{\theta}_{l[m]} = \hat{\theta}_l$) provides the expression of the S-curve for the l^{th} finger:

$$\begin{aligned} S^{(l)}\left(\theta_l - \hat{\theta}_l\right) &= \mathrm{Im}\left\{T_s A^2 \rho_l e^{j\left(\theta_l - \hat{\theta}_l\right)} \Gamma_{11}(0)\right\} \\ &\underbrace{+\,\mathrm{Im}\left\{T_s A^2 \sum_{l_1 \neq l} \rho_{l_1} e^{j\left(\theta_{l_1} - \hat{\theta}_l\right)} \Gamma_{11}\left(\tau_l - \tau_{l_1}\right)\right\}}_{bias} \end{aligned} \tag{6}$$

Hence, depending on the delays and the coefficients of the other paths, the output of the PED appears to be biased.

In the context of small fluctuations of the phase error, it is possible to linearize the S-curve around its stable equilibrium point [5]:

$$S^{(l)}\left(\theta_l - \hat{\theta}_l\right) = D^{(l)}.\left(\theta_l - \hat{\theta}_l\right) - D^{(l)}.\left(\theta_l - \theta_{l_{eq}}\right) \tag{7}$$

where $D^{(l)}$ is the slope of the S-curve at this point and $\theta_{l_{eq}}$ is the stable equilibrium point. We recall that a necessary condition for an equilibrium point $(\theta_{l_{eq}})$ is that $S\left(\theta_l - \theta_{l_{eq}}\right) = 0$.

The computation of the phase variance with the linearized model described above [5] gives:

$$\sigma^2_{\theta_l} = 2B_P T_s \frac{\Gamma_{N^{(l)}[0]}}{{D^{(l)}}^2} \tag{8}$$

where $\Gamma_{N^{(l)}}$ is the autocorrelation of the noise $N^{(l)}$ and $B_P T_s$ is the loop bandwidth. The phase variance is strongly affected by the user interference and by the adjacent paths.

4. IMPROVEMENT OF THE PHASE LOOP

The conventional PED is well suited for a single path channel, but in the context of a multipath channel, we have seen that the output of the PED is badly influenced by the additional paths. Moreover the presence of other users have a significant impact on the performance of the loop in term of variance. So we propose in this section a new version of the PED which is better suited for a multi-user system with a multipath channel. The improvement is based on the concept of prefiltering. We insert a prefilter of finite impulse response, $p_{[i]}$, $i = -N, \ldots, N$, in the phase loop as illustrated in Fig 1. The prefilter works at a rate of two samples per chip in entrance. The following notation will be used in this section:

$$\tilde{f}(t) \;=\; \textstyle\sum_{i=-N}^{N} p_{[i]} f(t - i\frac{T_c}{2}) \tag{9}$$

where f represents any desired function.

The purpose of such a prefilter is to correct the shape of the correlation Γ_{11} in (6) so as to force it to zero at the locations of adjacent paths:

$$\begin{array}{ll}\mathrm{Re}\left\{\tilde{\Gamma}_{11}\left(\tau_l-\tau_{l_1}\right)\right\}=0 & l_1=1\ldots L,\ l_1\neq l\\ \mathrm{Im}\left\{\tilde{\Gamma}_{11}\left(\tau_l-\tau_{l_1}\right)\right\}=0 & l_1=1\ldots L,\ l_1\neq l\end{array} \quad (10)$$

Thus, we make the S-curve independant of the other paths but in order to cancel totally the bias, we need to add the following constraint: $\mathrm{Im}\left\{\tilde{\Gamma}_{11}(0)\right\}=0$. We choose the values of the prefilter coefficients that minimize the phase variance under those constraints. In order to avoid the zero solution, we add a constraint which normalizes the slope of the S-curve at $\theta_l-\hat{\theta}_l=0$. The next step is the computation of the phase variance as a function of the prefilter coefficients.

The new error signal with the prefilter can be expressed as:

$$e_{[m]}^{(l)}=\mathrm{Im}\left\{e^{-j\hat{\theta}_{l[m]}}\hat{a}_{1[m]}^{*}\tilde{z}_1\left(mT_s+\tau_l\right)\right\} \quad (11)$$

Now, remembering that if $\hat{\theta}_{l[m]}=\theta_{l_{eq}}$, then $e_{[m]}^{(l)}=N_{[m]}^{(l)}$; the loop noise autocorrelation takes thus the form:

$$\Gamma_{N^{(l)}[n]}=\mathrm{E}\left\{e_{[m]}^{(l)}e_{[m-n]}^{(l)*}\right\} \quad (12)$$

By substituting (11) in the above expression and after some calculations we obtain the expression of the loop noise autocorrelation $\Gamma_{N^{(l)}}$ for $n=0$:

$$\begin{array}{l}\Gamma_{N^{(l)}[0]}=\frac{1}{2}T_s^2A^4\sum_{l_1=1}^{L}\rho_{l_1}^2\left[\sum_{n_1\neq 0}\left|\tilde{\Gamma}_{11}(n_1T_s+\tau_l-\tau_{l_1})\right|^2\right.\\ \left.+\sum_{k\neq 1}\sum_{n_1=-\infty}^{+\infty}\left|\tilde{\Gamma}_{k1}(n_1T_s+\tau_{l_{eq}}-\tau_{l_1})\right|^2\right]+\frac{1}{2}A^2\Gamma_{\tilde{n}[0]}\end{array} \quad (13)$$

where

$$\Gamma_{\tilde{n}[i]}=2N_0\sum_{m=-N}^{N}\sum_{q=-N}^{N}\Gamma_{11}\left((m-q+i)\frac{T_c}{2}\right)p_{[m]}p_{[q]}^{*} \quad (14)$$

is the expression of the autocorrelation of $\tilde{n}(t)$ sampled at a rate of two samples per chip.

We propose now to use matrix notations to perform the minimization under constraints of the phase variance. Let us define the following vectors:

$$\underline{p}=\left[p_{[-N]}\cdots p_{[N]}\right]^T \quad (15)$$

$$\underline{u}_{k,n}^{l,l_1}=\begin{bmatrix}\Gamma_{k1}(nT_s+N\frac{T_c}{2}+\tau_l-\tau_{l_1})\\ \vdots\\ \Gamma_{k1}(nT_s-N\frac{T_c}{2}+\tau_l-\tau_{l_1})\end{bmatrix} \quad (16)$$

The expression of the phase variance given in (13) can be written as a quadratic form:

$$\Gamma_{N^{(l)}[0]}=\underline{p}^H\,\underline{\underline{\Gamma}}\,\underline{p} \quad (17)$$

where $\underline{\underline{\Gamma}}=\underline{\underline{\Gamma_{ISI}}}+\underline{\underline{\Gamma_{MAI}}}+\underline{\underline{\Gamma_{TN}}}$ is the matrix containing the three disturbance terms, the intersymbol interferences, the multiple access interferences and the thermal noise perturbation. $\underline{\underline{\Gamma_{ISI}}}$ and $\underline{\underline{\Gamma_{MAI}}}$ are defined as:

$$\begin{array}{l}\underline{\underline{\Gamma_{ISI}}}=\frac{1}{2}T_s^2A^4\sum_{l_1=1}^{L}\rho_{l_1}^2\sum_{n\neq 0}\underline{u}_{1,n}^{l,l_1}\underline{u}_{1,n}^{l,l_1\,H}\\ \underline{\underline{\Gamma_{MAI}}}=\frac{1}{2}T_s^2A^4\sum_{l_1=1}^{L}\rho_{l_1}^2\sum_{k=1}^{K}\sum_{n}\underline{u}_{k,n}^{l,l_1}\underline{u}_{k,n}^{l,l_1\,H}\end{array} \quad (18)$$

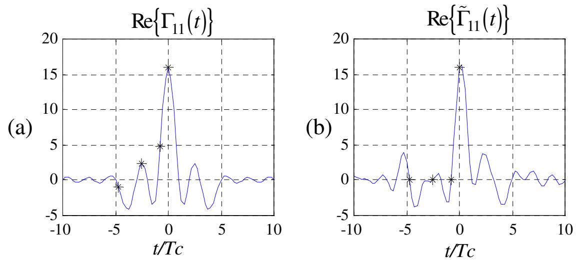

Figure 2: (a) correlation Γ_{11} before prefiltering (b) prefiltered correlation $\tilde{\Gamma}_{11}$

Now let us consider the following matrix: $\underline{\underline{B}}=[b_{mq}]$ with

$$b_{mq}=\Gamma_{11}\left((m-q)\frac{T_c}{2}\right) \quad (19)$$

The expression of $\underline{\underline{\Gamma_{TN}}}$ is given by: $\underline{\underline{\Gamma_{TN}}}=A^2N_0\underline{\underline{B}}$.

For notation simplicity and without loss of generality, the desired path is the first path: $l=1$. Let $\underline{C}$ be the following vector:

$$\begin{array}{l}\underline{C}=\left[\left(\mathrm{Re}\left\{\underline{p}^T\underline{u}_{1,0}^{1,1}\right\}-\frac{1}{\rho_1T_sA^2}\right),\ \mathrm{Re}\left\{\underline{p}^T\underline{u}_{1,0}^{1,2}\right\}\cdots\right.\\ \left.\quad\mathrm{Re}\left\{\underline{p}^T\underline{u}_{1,0}^{1,L}\right\},\ \mathrm{Im}\left\{\underline{p}^T\underline{u}_{1,0}^{1,1}\right\}\ldots\mathrm{Im}\left\{\underline{p}^T\underline{u}_{1,0}^{1,L}\right\}\right]^T\end{array} \quad (20)$$

The constraints may be expressed in the form:

$$\underline{C}=0 \quad (21)$$

This problem of minimization under constraints may be solved with the Lagrange-multiplier method. We form the Lagrange combination:

$$F\left(\underline{p},\underline{\lambda}\right)=\frac{1}{2}\underline{p}^H\,\underline{\underline{\Gamma}}\,\underline{p}+\underline{C}^T\underline{\lambda} \quad (22)$$

where $\underline{\lambda}$ is the vector containing the Lagrange multipliers. We compute the complex gradient of F with respect to $\underline{p}$ and we find the values of $\underline{p}$ which make the gradient zero:

$$\underline{\nabla}_pF\left(\underline{p},\underline{\lambda}\right)=\underline{\underline{\Gamma}}\,\underline{p}+\underline{\underline{G}}\,\underline{\lambda}=0 \quad (23)$$

where

$$\underline{\underline{G}}=\left[\underline{u}_{1,0}^{1,1^*}\cdots\underline{u}_{1,0}^{1,L^*},\ j\underline{u}_{1,0}^{1,1^*}\cdots j\underline{u}_{1,0}^{1,L^*}\right] \quad (24)$$

Substituting $\underline{p}$ by $(-\underline{\underline{\Gamma}}^{-1}\underline{\underline{G}}\,\underline{\lambda})$ in (21) yields the expression of $\underline{\lambda}$. We finally subsitute this expression in (23) and we obtain the result:

$$\underline{p}_{opt}=\frac{1}{T_sA^2\rho_1}\underline{\underline{\Gamma}}^{-1}\,\underline{\underline{G}}\,\underline{\underline{M}}^{-1}\underline{1} \quad (25)$$

where $\underline{1}^T=[1,0\ldots,0]$ and:

$$\underline{\underline{M}}=\begin{bmatrix}\mathrm{Re}\left\{\underline{u}_{1,0}^{1,1}\underline{\underline{\Gamma}}^{-1}\underline{\underline{G}}\right\}\\ \vdots\\ \mathrm{Re}\left\{\underline{u}_{1,0}^{1,L}\underline{\underline{\Gamma}}^{-1}\underline{\underline{G}}\right\}\\ \mathrm{Im}\left\{\underline{u}_{1,0}^{1,1}\underline{\underline{\Gamma}}^{-1}\underline{\underline{G}}\right\}\\ \vdots\\ \mathrm{Im}\left\{\underline{u}_{1,0}^{1,L}\underline{\underline{\Gamma}}^{-1}\underline{\underline{G}}\right\}\end{bmatrix} \quad (26)$$

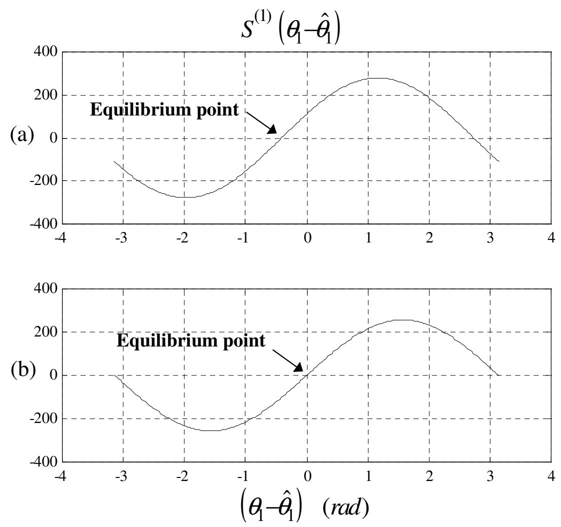

Figure 3: (a) conventional PED S-curve (b) Improved PED S-curve

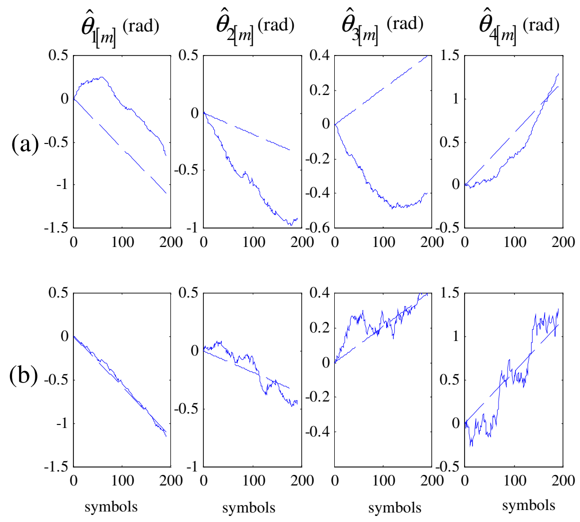

Figure 4: (a) Estimated phases with the conventional loop (b) Estimated phases with the improved loop with a 11 coefficient prefilter

5. NUMERICAL RESULTS

In this section, we present a numerical analysis of the results obtained in the previous section. We use Hadamard codes of length $Q = 16$ with a scrambling sequence. The chip shaping filter is a square-root raised cosine with the roll-off factor of 0.22. The mobile speed is $v_m = 120Km/h$, *i.e.* $\Delta f_d = 220Hz$ with the transmission parameters given in introduction. We choose second-order loops with a damping factor $\zeta = 0.7$ and a natural frequency $f_n = 1KHz$. Those loops are suitable for a random Doppler of $220Hz$ but the simulations are realized with a deterministic Doppler. We consider a downlink communication with $K = 5$ users. Here, 4 paths have been assumed at relative delays of 0, $0.75T_c$, $2.5T_c$ and $4.75T_c$ with powers of $0dB$, $-0.9dB$, $-4.9dB$ and $-8dB$. The prefilter has 11 coefficients ($N = 5$).

Fig 2 illustrates the effect of the prefilter on the real part of the correlation Γ_{11}. The current path is the first path. The prefilter forces to zero the correlation at $\tau_1 - \tau_2$, $\tau_1 - \tau_3$ and $\tau_1 - \tau_4$ (located by a '*' on the figure). The resulting S-curve will not be distorded by the additional paths.

Fig 3 shows the S-curves of the conventional and the optimized PED for the first RAKE finger ($l = 1$). In the first case, it is noted that the zero-crossing of the S-curve is not situated at 0 ($\theta_1 - \theta_{1_{eq}} \approx -0.5\ rad$). It results in a bias on the estimated phase, due to the presence of the adjacent paths. Fig 3(b) shows the S-curve of the optimized PED. The zero-crossing is shifted at the origine, yielding a non-biased estimation.

Fig 4 illustrates tracking performance of both structures with a signal to noise ratio $E_b/N_0 = 20dB$. The four paths present a Doppler of $220Hz$, $70Hz$, $-80Hz$ and $-230Hz$. The tracked phases and the channel phases are shown for each path for the conventional and the improved phase recovery loop in Fig 4(a) and Fig 4(b) respectively. In the latter case, the improved loop is able to follow the channel phases. The improved tracking loop outperforms the conventional tracking loop.

6. CONCLUSION

In this paper, a new performant phase tracking technique in downlink Time-Division CDMA system appropriate for rapidly time-varying multipath channel is proposed. The new structure is based on a conventional phase recovery loop, the "remodulator loop", in which a prefilter has been inserted. It is explained how to design the optimum prefilter in order to minimize the phase variance and to cancel the bias due to the additional paths. An analytical solution is provided for calculating the prefilter coefficients. Linear analysis and computer simulations have been employed to evaluate the performance of the new tracking loop. The numerical results show that the new tracking loop outperforms the conventional loop.

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