SUBBAND ADAPTIVE FILTERING USING A MULTIPLE-CONSTRAINT OPTIMIZATION CRITERION

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ABSTRACT

In this paper we propose a new design criterion for subband adaptive filters (SAFs). The proposed multiple-constraint optimization criterion is based on the principle of minimal disturbance, where the multiple constraints are imposed on the updated subband filter outputs. Compared to the classical fullband least-mean-square (LMS) algorithm, the subband adaptive filtering algorithm derived from the proposed criterion exhibits faster convergence under colored excitation. Furthermore, the recursive tap-weight adaptation can be expressed in a simple form comparable to that of the normalized LMS (NLMS) algorithm. We also show that the proposed criterion is related to another known weighted criterion. The efficacy of the proposed criterion and algorithm are examined and validated via mathematical analysis and simulation.

1. INTRODUCTION

Among various adaptive filtering algorithms, the LMS algorithm is the most popular and widely used because of its simplicity and robustness. However the LMS algorithm suffers from slow convergence when the input signal is highly correlated. Adaptive filtering in subbands has been proposed to improve the convergence behavior of the LMS algorithm under colored excitation [1-5]. In subband adaptive filtering, the input signal and desired response are band-partitioned into almost mutually exclusive subband signals. This feature of the SAF permits the manipulation of each subband, in such a way that their properties (e.g., variance) can be exploited [5] allowing each subband to converge almost separately on various modes [3], and thus improving the overall convergence behavior. Yet, band edge effects limit the convergence rate of the conventional SAFs [2, 7].

The principle of minimum disturbance [5] states that: from one iteration to the next, the tap weights of an adaptive filter should be changed in a minimal manner, subject to a constraint imposed on the updated filter output. Based on this principle, we formulate a novel design criterion for the SAF as a constrained optimization problem involving multiple constraints imposed on the updated subband filter outputs. From one iteration to the next, these multiple subband constraints force each of the almost mutually exclusive subbands to converge almost independently without any influence from other subbands. Furthermore, by virtue of critical subsampling, the computational load of the proposed algorithm remains almost unchanged when the number of subbands is increased. Hence, the subband algorithm derived from this design criterion, called the normalized SAF (NSAF) algorithm, is expected to possess attractive convergence behavior under colored excitation, and remain computationally efficient.

Note that the major issue considered in this paper is to improve the convergence behavior of the LMS algorithm rather than to reduce its computational complexity as emphasized in the conventional SAF structure [1, 2]. Another unique characteristic of the proposed NSAF algorithm is that the error signals are estimated in subbands, whereas the coefficients that are explicitly adapted are the fullband tap weights of the modeling filter. This adaptive weight-control mechanism is different from that in the conventional SAF structure where each subband has its own sub-filter and adaptation loop.

The paper is organized as follows. In section 2, we establish proper definitions for various subband signals, which facilitate the formulation of the proposed criterion and derivation of the NSAF algorithm in Section 3. In Section 4, we relate the proposed criterion with a known weighted criterion [3, 4]. In Section 5, we present some simulation results to demonstrate the convergence behavior of the NSAF algorithm. Finally, Section 6 concludes the paper.

2. DECIMATED SUBBAND SIGNALS

Fig. 1 shows a subband structure where the desired response and filter output, $d(n)$ and $y(n)$, are partitioned into N subbands by mean of analysis filters $H_0(z),\ldots,H_{N-1}(z)$. The subband signals, $d_i(n)$ and $y_i(n)$ for $i=0,\ldots,N-1$, are critically sub-sampled to a lower rate commensurate with the corresponding bandwidth of the subband signals. Note that we use the variable n to index the original sequences, and k to index the decimated sequences. Sampling rate of the decimated sequences are N times slower than the original sequences. Assuming that the modeling filter $\hat{\mathbf{w}}(k)$ is stationary (i.e., adaptation of its tap weights is frozen), we can transpose it to follow the analysis filter bank as shown in Fig. 2. The $N:1$ decimators retain only those samples of $y_i(n)$ that occur at instants of time equal to multiples of N. Hence, the decimated filter output at each subband can be written as

$$y_{i,\mathrm{D}}(k)=\sum_{m=0}^{M-1}\hat{w}_m(k)u_i(kN-m)=\hat{\mathbf{w}}^T(k)\mathbf{u}_i(k), \tag{1}$$

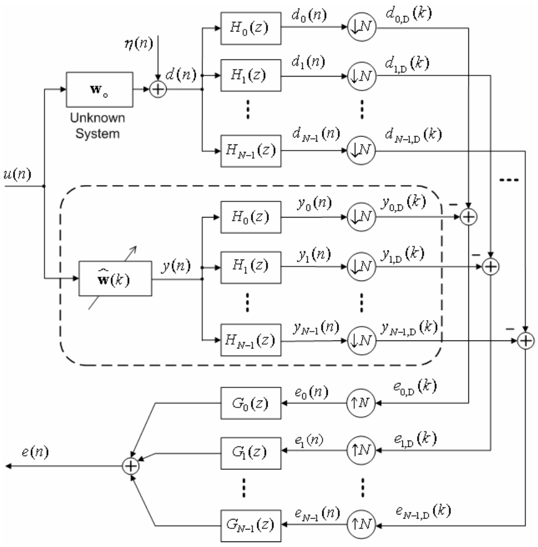

Fig. 1. A subband structure showing the subband desired responses, subband filter outputs, and subband error signals.

where

$$\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \ldots, u_i(kN-N+1), \\ u_i(kN-N), \ldots, u_i(kN-M+1)]^T \tag{2}$$

is the input data vector for the ith subband, $\widehat{\mathbf{w}}(k) = \left[\widehat{w}_0(k), \widehat{w}_1(k), \ldots, \widehat{w}_{M-1}(k)\right]^T$ is the fullband adaptive tap-weight vector, and the superscript T denotes matrix transposition. Now, we define the decimated subband error signal $e_{i,\mathrm{D}}(k)$ as the difference between the decimated subband desired response $d_{i,\mathrm{D}}(k)$ and filter output $y_{i,\mathrm{D}}(k)$:

$$e_{i,\mathrm{D}}(k) = d_{i,\mathrm{D}}(k) - \widehat{\mathbf{w}}^T(k)\mathbf{u}_i(k), \quad i = 0,\ldots,N-1. \tag{3}$$

Equations (1) and (2) indicate that, at every time instant k each data vector $\mathbf{u}_i(k)$ is packed with N new samples and $M-N$ old samples to produce a single sample of $y_{i,\mathrm{D}}(k)$. In this and subsequent sections, we assume that all signals and filter coefficients are real.

3. THE PROPOSED NSAF ALGORITHM

Based on the principle of minimum disturbance [5], we formulate the design criterion as a multiple-constrained optimization problem as follows:

Minimize the squared Euclidean norm of the change in the tap-weight vector

$$f\left[\widehat{\mathbf{w}}(k+1)\right] = \left\|\widehat{\mathbf{w}}(k+1) - \widehat{\mathbf{w}}(k)\right\|^2, \tag{4}$$

subject to the set of N constraints imposed on the decimated filter output

$$\widehat{\mathbf{w}}^T(k+1)\mathbf{u}_i(k) = d_{i,\mathrm{D}}(k) \quad \text{for} \quad i = 0,\ldots,N-1. \tag{5}$$

Applying the method of Lagrange multipliers [5] on the proposed criterion, we obtain the recursive relation for updating the tap-weight vector:

$$\widehat{\mathbf{w}}(k+1) = \widehat{\mathbf{w}}(k) + \frac{1}{2}\sum_{i=0}^{N-1}\lambda_i \mathbf{u}_i(k), \tag{6}$$

where the λ_i are the Lagrange multipliers pertaining to the multiple constraints described in (5). Substitute (6) into the N constraints of (5) and solve for the Lagrange multipliers, we get

$$\boldsymbol{\lambda} = 2\left[\mathbf{U}^T(k)\mathbf{U}(k)\right]^{-1}\mathbf{e}_{\mathrm{D}}(k), \tag{7}$$

where $\boldsymbol{\lambda} = [\lambda_0, \lambda_1, \ldots, \lambda_{N-1}]^T$ is the $N\times 1$ Lagrange vector, $\mathbf{U}(k) = [\mathbf{u}_0(k), \mathbf{u}_1(k), \ldots, \mathbf{u}_{N-1}(k)]$ is the $M\times N$ data matrix, and $\mathbf{e}_{\mathrm{D}}(k) = \left[e_{0,\mathrm{D}}(k), e_{1,\mathrm{D}}(k), \ldots, e_{N-1,\mathrm{D}}(k)\right]^T$ is the $N\times 1$ error vector. It is shown in Section 3.1 that if the subband input signals are orthogonal at zero lag, the off-diagonal elements of the matrix $\mathbf{U}^T(k)\mathbf{U}(k)$ are negligible. With this diagonal assumption, (7) essentially reduces to a simple form:

$$\lambda_i = 2\frac{e_{i,\mathrm{D}}(k)}{\left\|\mathbf{u}_i(k)\right\|^2} \quad \text{for} \quad i = 0,\ldots,N-1. \tag{8}$$

Combining the results in (6) and (8), we obtain the tap-weight adaptation equation of the NSAF algorithm:

$$\widehat{\mathbf{w}}(k+1) = \widehat{\mathbf{w}}(k) + \mu\sum_{i=0}^{N-1}\frac{\mathbf{u}_i(k)}{\left\|\mathbf{u}_i(k)\right\|^2}e_{i,\mathrm{D}}(k). \tag{9}$$

It is clear that, this tap-weight adaptation equation is in a simple form comparable to that of the NLMS algorithm.

A positive step-size parameter μ is introduced in the recursive relation to exercise control over the change in the tap-weight vector. Following the procedure outlined in [5], we analyze the convergence behaviour of the proposed subband algorithm based on mean-square deviation. In the absence of the disturbance, i.e., the $\eta(n)$ term in Fig. 1 is negligible, the necessary and sufficient condition for the convergence in the mean-square is that the step size parameter μ must satisfy the double inequality:

$$0 < \mu < 2. \tag{10}$$

Note that the constrained optimization criterion defined above involves N equality constraints; thus, the number of subbands N (i.e., number of constraints) must be smaller than the length of the adaptive tap-weight vector M. This requirement sets an upper limit on the number of allowable subbands in the proposed NSAF algorithm.

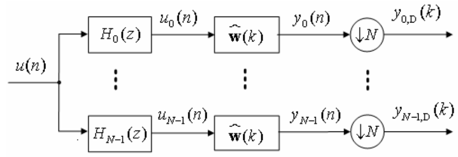

Fig. 2. The input signal is split into subband signals before going through the filters $\widehat{\mathbf{w}}(k)$ and decimators.

3.1 The Diagonal Assumption

Consider the subband structures in Fig. 2. For two arbitrary subband signals $u_i(n)$ and $u_p(n)$, where $i, p = 0,\ldots,N-1$ and $i \neq p$, their cross-correlation at zero lag $l = 0$ can be formulated as

$$\gamma_{ip}(0) = \frac{1}{2\pi}\int_{2\pi}\left|H_i(e^{j\omega})\right|\left|H_p(e^{j\omega})\right|\Gamma_{uu}(e^{j\omega})e^{j\phi_{ip}(\omega)}\,d\omega, \tag{11}$$

where $\left|H_i(e^{j\omega})\right|$ and $\left|H_p(e^{j\omega})\right|$ are the magnitude responses of the ith and pth analysis filters, respectively, $\phi_{ip}(\omega) = \phi_i(\omega) - \phi_p(\omega)$ is the phase difference between the analysis filters, and $\Gamma_{uu}(e^{j\omega})$ is the power spectrum of the

Table I: Summary of the NSAF algorithm

Parameters and variables	Computation	multiplications/T_s
M = filter length	For $n=0,1,2,\ldots$, at $1/T_s$ processing rate	
N = number of subbands	*Band-partitioning*:	
μ = step-size parameter, $0<\mu<2$	$u_i(n)=\mathbf{h}_i^T\mathbf{u}(n), \quad i=0,\ldots,N-1$	NL
L = length of the analysis filters $\mathbf{h}_i$ and synthesis filters $\mathbf{g}_i$	$d_i(n)=\mathbf{h}_i^T\mathbf{d}(n), \quad i=0,\ldots,N-1$	NL
	Synthesizing:	
δ = positive constant	$e(n)=\sum_{i=0}^{N-1}\mathbf{g}_i^T\mathbf{e}_i(n)$	NL
$\mathbf{u}(n)=[u(n), u(n-1), \ldots, u(n-L+1)]^T$		
$\mathbf{d}(n)=[d(n), d(n-1), \ldots, d(n-L+1)]^T$	For $k=0,1,2,\ldots$, at $1/NT_s$ processing rate	
$\mathbf{e}_i(n)=[e_i(n), e_i(n-1), \ldots, e_i(n-L+1)]^T$	*Error estimation*:	
$\mathbf{u}_i(k)=[u_i(kN), \ldots, u_i(kN-M+1)]^T$	$e_{i,\mathrm{D}}(k)=d_{i,\mathrm{D}}(k)-\hat{\mathbf{w}}^T(k)\mathbf{u}_i(k), \quad i=0,\ldots,N-1$	$\frac{(M\times N)}{N}=M$
$d_{i,\mathrm{D}}(k)=d_i(kN)$	*Tap-weight adaptation*:	
$e_i(n)=\begin{cases} e_{i,\mathrm{D}}(k/N), & n=0,\pm N,\pm 2N,\ldots,\\ 0, & \text{otherwise.}\end{cases}$	$\hat{\mathbf{w}}(k+1)=\hat{\mathbf{w}}(k)+\mu\sum_{i=0}^{N-1}\frac{\mathbf{u}_i(k)}{\delta+\Vert\mathbf{u}_i(k)\Vert^2}e_{i,\mathrm{D}}(k)$	$\approx\frac{(2M\times N)}{N}=2M$

input signal $u(n)$ which is assumed to be stationary. The magnitude response $|H_i(e^{j\omega})||H_p(e^{j\omega})|$ and phase difference $\phi_{ip}(\omega)$ play important roles in characterizing the cross-correlation function (11). Particularly, if the analysis filter bank is paraunitary [6], the subband input signals are orthogonal at zero lag, $\gamma_{ip}(0)=0$. Now, with the assumption that the input signal $u(n)$ is ergodic, the cross-correlation at zero lag $\gamma_{ip}(0)$ can be approximated with the time average $\hat{\gamma}_{ip}(0)=\mathbf{u}_i^T(k)\mathbf{u}_p(k)/M$. So, if the subband input signals are orthogonal at zero lag, the off-diagonal elements $\mathbf{u}_i^T(k)\mathbf{u}_p(k)=M\hat{\gamma}_{ip}(0)$ of the matrix $\mathbf{U}^T(k)\mathbf{U}(k)$ are much smaller than its diagonal elements $\mathbf{u}_i^T(k)\mathbf{u}_i(k)=M\hat{\gamma}_{ii}(0)$ implying that $\mathbf{U}^T(k)\mathbf{U}(k)$ can be well approximated by a diagonal matrix.

3.2 Computational Complexity

The proposed NSAF algorithm is summarized in Table I. Note that a small positive constant δ is introduced in the tap-weight adaptation equation to avoid numerical difficulties when the input signal level is too low. By virtue of critical sub-sampling, the tap-weight vector is adapted at a lower rate $1/NT_s$ compared to full-band sampling rate $F_s=1/T_s$. Hence, the number of multiplications incurred for error estimation and tap-weight adaptation during a single sampling period T_s is always $3M$ for an arbitrary number of subbands N (as shown in Table I). Apart from this, the NSAF algorithm requires an additional $3NL$ multiplications for the analysis and synthesis filter banks, giving us a total of $3M+3NL$ multiplications during a single sampling period T_s. Hence, compared to the fullband NLMS algorithm, the NSAF algorithm requires a slight number of extra multiplications (i.e., an additional $3NL$ multiplications) for the filter banks implementation.

4. SAF STRUCTURES

Recently, an innovative SAF structure has been proposed in [3, 4] where the subband algorithms are derived by optimizing a weighted criterion defined as

$$J=E\left[\sum_{i=0}^{N-1}\alpha_i\left|e_{i,\mathrm{D}}(k)\right|^2\right]=\sum_{i=0}^{N-1}\alpha_i E\left[\left|e_{i,\mathrm{D}}(k)\right|^2\right]. \tag{12}$$

The scaling factors α_i for $i=0,\ldots,N-1$, are proportional to the inverse of the power of the subband input signals. The adaptive weight-control mechanisms proposed in [3, 4] are similar to that proposed in this paper, where the fullband tap weights of the modeling filter $\hat{\mathbf{w}}(k)$ are adapted instead of sub-filters as in the conventional SAF structure (in [4], the fullband filter is decomposed into polyphase components). If we set the scaling factors α_i equal to $1/\Vert\mathbf{u}_i(k)\Vert^2=1/M\hat{\sigma}_i^2$, where $\hat{\sigma}_i^2$ is the variance estimate of the ith subband, the tap-weight adaptation equation of the NSAF coincides with that presented in [3, 4]. This implies that the multiple-constraint optimization criterion proposed in this paper is related to the weighted criterion (12).

Both criteria bring insight into subband adaptive filtering from different perspectives. Particularly, the proposed criterion reveals the necessary conditions (i.e., the subband input signals must be orthogonal at zero lag, and the number of subbands $N<M$) to be imposed on the filter banks, whereas the weighted criterion shows that the error performance surface is characterized by a weighted correlation matrix [3, 4]. From the proposed criterion, we also find that the appropriate scaling factor is $\alpha_i=1/M\hat{\sigma}_i^2$. With this choice of scaling factor, the SAF is stable as long as the step-size parameter is bounded within the double inequality $0<\mu<2$.

It is commonly known that the condition number of a correlation matrix is bounded by the spectral dynamic range of the signal. Fig. 3(a) shows the power spectrum of an AR(2) random process with coefficients $(1.0, -1.6, 0.81)$. The spectral dynamic range of the AR(2) process is 31.9 dB. It can be shown that the power spectrum characterizing the weighted correlation matrix can be formulated as

$$\Gamma_{\mathrm{w}}(e^{j\omega})=\sum_{i=0}^{N-1}\frac{1}{M\hat{\sigma}_i^2}\left|H_i(e^{j\omega})\right|^2\Gamma_{\mathrm{uu}}(e^{j\omega}). \tag{13}$$

Equation (13) indicates that the power spectrum of the input signal, $\Gamma_{\mathrm{uu}}(e^{j\omega})$, is partitioned into N overlapping subbands which are than normalized with their corresponding subband energies. Recombination of these normalized partitions yields a flatten power spectrum, $\Gamma_w(e^{j\omega})$, as shown in Fig. 3(b), for the same AR(2) process and $N = 8$ subbands (using the paraunitary filter bank described in the next section). The spectral dynamic range is greatly reduced to 6.6 dB yielding an improvement in convergence rate. Furthermore, the band edge effects which limit the convergence rate of the conventional SAFs [2, 7], are eliminated as well. Hence, the proposed SAF structure is also an improvement over the conventional SAF structure in term of convergence rate.

5. SIMULATIONS

In this section, we consider the system identification problem (as shown in Fig. 1) to examine the convergence behavior of the proposed NSAF algorithm. The unknown system to be identified is modeled with the acoustic response of a room with 300 ms reverberation time and truncated to 2048 taps. The length of the adaptive tap-weight vector M is set equal to 1024 taps. The unmodeled tail of the acoustic response forms a disturbance to the adaptive identification system. This disturbance limits the final misalignment to more than -45 dB. The excitation signal to the adaptive identification system is the AR(2) random process mentioned earlier.

For the subband structure, we use paraunitary cosine modulated filter banks [6] with the length of the prototype filter L chosen to be 16, 32, and 64, respectively, for $N = 2$, 4, and 8 subbands. The paraunitary property of the analysis filters ensures that the subband input signals $u_i(n)$ are orthogonal at zero lag. Recall that the diagonal matrix requirement is fulfilled if the subband signals are orthogonal at $l = 0$, $\gamma_{ip}(0) = 0$.

We examine the performances of both NSAF and NLMS algorithms by inspecting their normalized misalignment leaning curves. The normalized misalignment is defined as the norm of the weight-error vector $\|\boldsymbol{\varepsilon}(k)\|$ normalized by the norm of the optimum tap-weight vector $\|\mathbf{w}_o\|$. Fig. 4 shows the learning curves of the NLMS and the proposed NSAF (for $N = 2$, 4, and 8) algorithms. The learning rate for both algorithms is chosen to be at the middle of the stability bound given by the inequality (10), i.e., $\mu = 1$. This choice of step-size parameter setting enables a fair comparison between the NSAF and NLMS algorithms. From the learning curves in Fig. 4, it can be noted that the NSAF algorithm provides faster convergence than the NLMS algorithm. Furthermore, with an increased number of subbands, the convergence rate improves considerably. Hence, the proposed algorithm is an improvement over the fullband NLMS algorithm.

6. CONCLUSIONS

A novel multiple-constraint optimization criterion for the SAF is proposed in this paper. Compared to the fullband NLMS algorithm, the NSAF algorithm derived from this criterion exhibits faster convergence under colored excitation. Regarding computational complexity, the NSAF and NLMS algorithms require almost the same number of multiplications per sampling period. Furthermore, the proposed SAF structure is also an improvement over the conventional SAF structure in term of convergence rate.

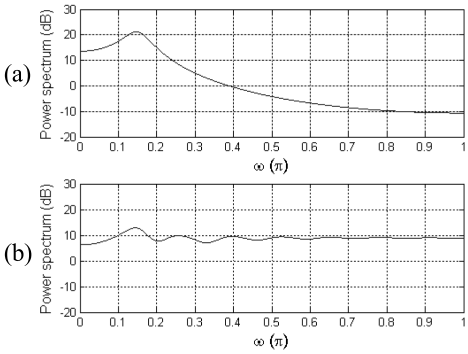

Fig. 3. Power spectrums. (a) The AR(2) random process. (b) The flatten power spectrum.

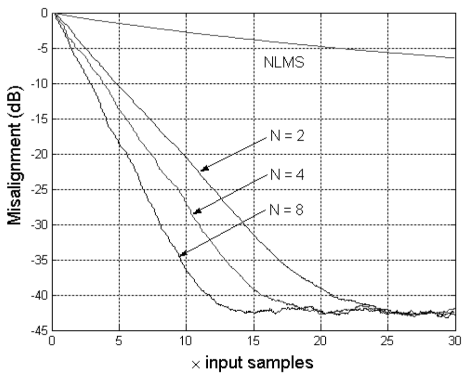

Fig. 4. Normalized misalignment learning curves for the NLMS and NSAF (for $N = 2$, 4, and 8) algorithms. The step-size is set at $\mu = 1.0$.

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