EFFICIENT GROUP DELAY EQUALIZATION OF DISCRETE-TIME IIR FILTERS

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ABSTRACT

A novel methodology for estimating the optimum pole allocation of allpass group delay equalizers with real coefficients is introduced. A computer-aided algorithm, whose input is this initial estimate, is developed. Results verifying the effectiveness of the proposed approach are presented and compared to other procedures described in the literature.

1. INTRODUCTION

Group delay (or phase) equalization is a viable alternative to compensate non-linear phase response effects, an issue of increasing interest, for instance, in modern high-speed telecommunication signal processing, where a number of narrow channels are accommodated in the available frequency band. IIR elliptic filters have the drawback of possessing highly non-linear phase response, despite their advantage in terms of computational complexity, when compared to linear-phase FIR filters. On the other hand, as approximately linear phase characteristic may satisfy system requirements in many applications, phase equalized IIR elliptic filters may find an edge over competing linear phase FIR counterparts.

The problem of the phase response non-linearity is well known, and an approach to overcome it in the case of analog filters was mentioned in [1]. In fact, there are several methodologies for designing group delay equalizers for analog filters. Unfortunately, these methods cannot be extended to discrete-time filters through an s-to-z mapping, such as the bilinear transformation, because of the warping effect introduced in the phase frequency response of the equalized analog filters [2], [3].

Many authors have developed different design procedures to achieve the optimum group delay response, including genetic algorithms [4], adaptive filters [5], quasi-allpass filters [6] and allpass based equalizers [3], [7]-[12]. Allpass filters are a powerful signal processing building block applied to several applications [7], [13]. One of those applications is on group delay equalization, which is the focus of this paper.

Some results presented in the literature have indicated that achieving optimum high order equalization is a difficult task, because of the large number of parameters (allpass filter coefficients) involved [3], [8], [11]. The initial choice of the parameters of the optimization procedures described so far does not appropriately exploit the knowledge about the distortions in the group delay to be equalized [8], [11], and in some instances ends up with more sections than what would be necessary [3]. This paper proposes a novel methodology for finding initial estimates and applies them to a computer-aided optimization algorithm, to accomplish after a small number of iterations a fine tuning of the parameters of the problem [14].

The problem of phase response non-linearity is illustrated in Section 2. A methodology for initially allocating the equalizer poles is introduced in Section 3. Results are presented and discussed in Section 4. Concluding remarks are made in Section 5.

2. PHASE RESPONSE NON-LINEARITY

The distortion introduced by the non-linear phase response of IIR discrete-time filters may drastically compromise the performance of the whole system. Examples of unwanted effects are inter-symbolic interference (ISI) and amplitude losses. To illustrate the problem, let us consider a signal comprising the sum of 4 sinusoids having the frequencies of 20, 25, 40 and 45 Hz, sampled at f_s = 1 kHz. The signal was applied to a fourth-order lowpass elliptic filter, whose group delay response is shown in Fig. 1(a). Figure 1(b) depicts the group delay response obtained after cascading the filter with a sixth-order allpass equalizer, as in Fig. 2. Observe that all the signal components are inside the filter passband, which extends from 0 up to 100 Hz. The input and output are displayed superposed in Fig. 3(a) for easiness of comparison. The largest amplitude losses are pointed out by arrows. Plotted in Fig. 3(b), the output of the equalized filter is practically identical to the input, because the dispersals have been attenuated.

Let us now compare the computational complexity of the IIR equalized design with that of the equivalent linear FIR filter. If we consider the same specifications of the elliptic filter in the above example, then a 97th order FIR filter would be required, performing 98 multiplications, 1 addition and 97 delays per output sample. On the other hand, the fourth-order elliptic filter requires 9 multiplications, 2 additions and 4 delays, while each second order allpass section employs 5 multiplications, 2 additions and 2 delays. Therefore, the overall filter (IIR and equalizer) performs 24 multiplications, 8 additions and 10 delays. This represents a reduction of 75% in the number of multiplications, a critical operation in digital filtering implementations. Alternatively, in switched-capacitor filters it would lead to a reduction in

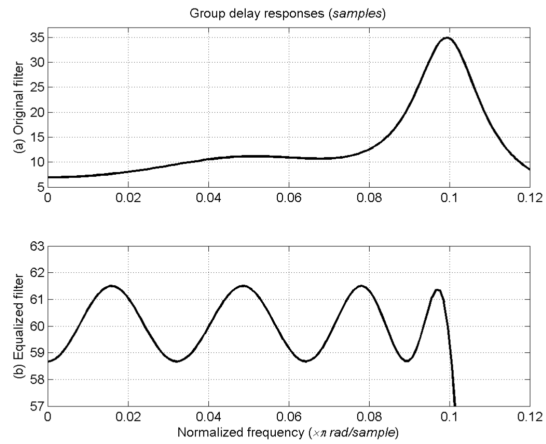

Figure 1: Group delay responses of a fourth-order lowpass elliptic filter without (a) and with (b) a sixth-order equalizer.

the number of operational amplifiers, consequently saving power consumption and silicon area.

To reach the desired phase response with IIR filters, however, computationally expensive optimization algorithms are usually necessary to allocate the equalizer poles and zeros. In the next section a new methodology to estimate the poles and zeros of the optimum equalizer is presented. It can be used as the starting point of the optimization routine, so that the desired result is obtained within a few iterations. The proposed approach can be applied to sampled-data filters in general, including digital, switched-capacitor and switched-current filters.

3. INITIAL ALLOCATION

A known effect of the filter poles is that the closer they are to the unit circle, the more distorted the phase response will be. Hence, the purpose of the allpass equalizing filter is to introduce poles and zeros to compensate the influence of the filter poles. The transfer function of a second-order equalizer is given by

$$A(z) = \frac{b + az^{-1} + z^{-2}}{1 + az^{-1} + bz^{-2}} \tag{1}$$

where a and b are here assumed real. The methodology described here consists of starting the allocation of the equalizer poles and zeros in the *z*-plane, such as to partition the angle 2θ between the pair of outermost poles of the IIR filter into equal angle parts, as illustrated in Fig.4(a). These outermost poles are mostly responsible for setting the filter cut off frequency. To equalize with n allpass sections, the $2n$ allpass poles may be allocated according to two different

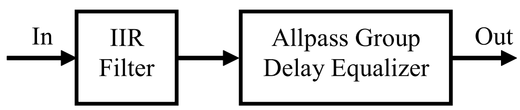

Figure 2: Group delay equalization of IIR filters.

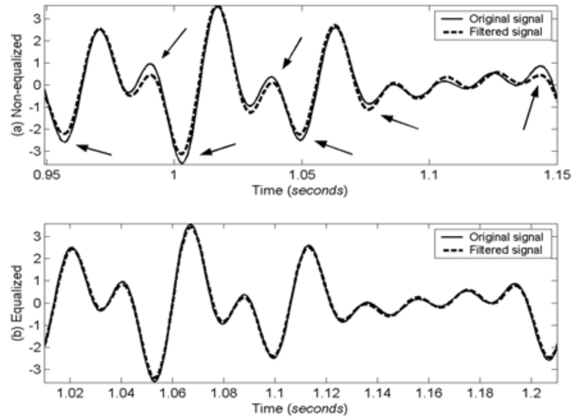

Figure 3: Input (original) and output signals of a fourth-order lowpass elliptic filter without (a) and with (b) a sixth-order equalizer.

strategies, as shown in Fig. 4(b) and 4(c):

A. generate $2n+1$ partitions of 2.24θ;
B. generate $2n-1$ partitions of 2θ.

Observe that the angle partitioned in strategy *A* is somewhat larger than in strategy *B*. Therefore, the angles between two consecutive poles of the equalizer are, respectively, for choices *A* and *B*

$$A. \quad \Delta\theta_a = 2.24\theta \big/ 2n+1 \qquad B. \quad \Delta\theta_b = 2\theta \big/ 2n-1$$

Accordingly, the pole closer to the real axis has angle equal to $\varphi_n = \Delta\theta_{(a,b)}/2$. The following poles are allocated at

$$\varphi_i = \varphi_{i+1} + \Delta\theta_{(a,b)}, \tag{2}$$

with $i = n$-1, ..,1, and $\Delta\theta_{(a,b)}$ is as in Fig.4. Next, we must choose the radii of each pair of poles, which can be accomplished by setting three parameters, q_0, q_1 and q_2, according to the chosen strategy, *A* or *B*, as follows:

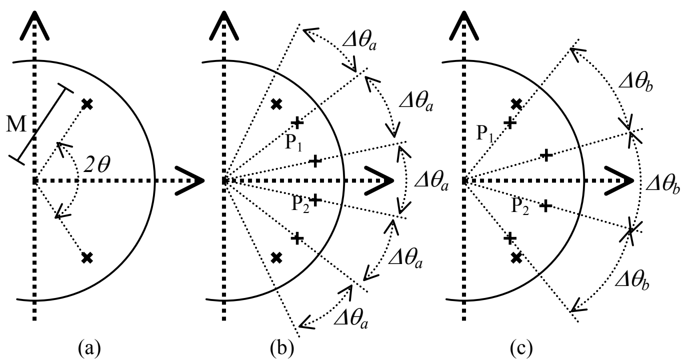

Figure 4: (a) Outermost poles of the IIR filter to be equalized; (b) Type A fourth-order equalization; (c) Type B fourth-order equalization. ("**x**": filter poles; "+": equalizer poles).

$$A. \quad q_0 = 0.975 \times 0.999^{2n} \qquad q_1 = {0.98}\big/{0.999^{2n}} \qquad q_2 = 4n$$
$$B. \quad q_0 = 0.92 \qquad q_1 = 1.05 \qquad q_2 = 4n + 2$$

Then, the radius of the equalizer outermost pair of poles, P_1, is calculated as q_0M, where M is the radius of the filter outermost poles. For equalizers with orders higher than two, P_2 is calculated as q_1P_1, and the magnitudes of the next equalizer poles – starting from the outermost ones – are calculated as

$$P_i = P_{i-1} \cdot \sqrt[q_2]{1.1}\ , \tag{3}$$

for i = 3,..,n. Having all the necessary information for allocating the equalizer parameters been established at this point, either strategy A or B must then be chosen. This choice depends on the cutoff frequency and the sharpness characteristics of the original group delay of the filter to be equalized. The first step consists in designing the equalizer according to strategy A. Then, denoting by $\tau_{E,\theta}$ the group delay response of the equalizer evaluated at the cutoff frequency θ, and by $\tau_{max,F}$ the maximum group delay of the filter, then strategy A is maintained if

$$1.07^{n+1} \tau_{E,\theta} < 0.85 \times 1.04^n \tau_{F,\max}\ , \tag{4}$$

otherwise strategy B is selected. The purpose of this test is to verify the influence of the filter original group delay response in the final one.

The procedure presented in this section simultaneously allocates the zeros of the equalizer, in view of the existing pole-zero symmetry of allpass transfer functions with real coefficients. It should also be noticed that this is an initial approximation to the equalization problem, and a tuning routine is necessary to achieve the optimum equiripple delay frequency response.

As the number of allpass sections is increased, the group delay of the equalized filter approaches the constant group delay response. However, as the equalizer complexity increases so do the number of circuit components, power consumption and noise. These tradeoffs must be carefully evaluated to suite the current application. In addition, high equalizer orders complicate the equalizer design, making it necessary the use of sophisticated and computationally expensive computer-aided optimization algorithms. The initial pole allocation described above substantially alleviated the complexity of the optimization routine derived in this work.

4. SIMULATIONS RESULTS

The robustness and efficiency of the proposed methodology of initial allocation of the equalizer poles and zeros was verified by applying it to a large variety of non-linear phase responses. Some results are shown next.
Other equalization examples can be found in [14]. The method was used as the initial estimate of an optimization procedure, in which the cost function was defined as

$$F = \left| \tau_E + \tau_F - \overline{(\tau_E + \tau_F)} \right| \tag{5}$$

where τ_E and τ_F are the equalizer and filter group delays, respectively. Different IIR filtering approximations were considered, such as the elliptic filters and the ones proposed in [15], [16].

As a first example, let us consider the fourth-order elliptic filter applied in Section 2 (see Fig. 1). This filter has $0.1f_s$ normalized cut-off frequency, where f_s is the sampling frequency, *40dB* attenuation in the stopband and *1dB* ripple in the passband. The poles and zeros of the elliptic filter are presented in Fig. 5(a), and the poles and zeros of the initially

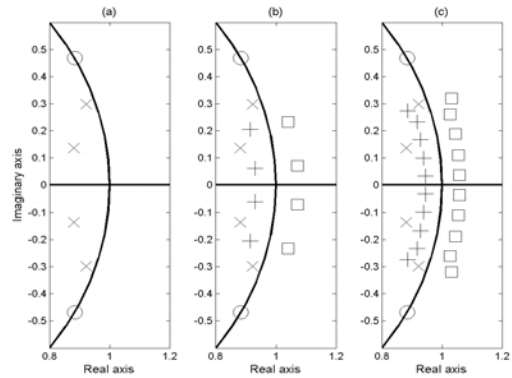

Figure 5: (a) elliptic filter poles (x) and zeros (o); (b) fouth-order equalizer poles (+) and zeros ([]); (c) tenth-order equalizer poles (+) and zeros ([]).

estimated fourth- and tenth-order equalizers are, respectively, in Figs. 5(b) and 5(c). These two pole-zero configurations represent, respectively, the strategies A and B in the initial allocation method described in Section 3. Plotted in Fig. 6 are the group delay responses of the elliptic filter (original), the elliptic filter equalized with the fourth-order allpass filter obtained with the initial pole-zero allocation strategy (initial estimate), and the elliptic filter equalized with the fourth-order allpass filter obtained by tuning the initial estimate with an optimization algorithm. Since, as indicated in Fig. 6, the initial pole-zero configuration was quite close to the optimal one, the optimization routine converged in only 6 iterations.

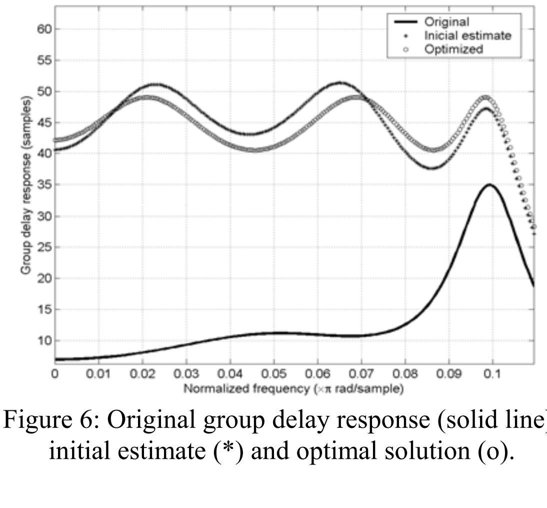

Figure 6: Original group delay response (solid line), initial estimate (*) and optimal solution (o).

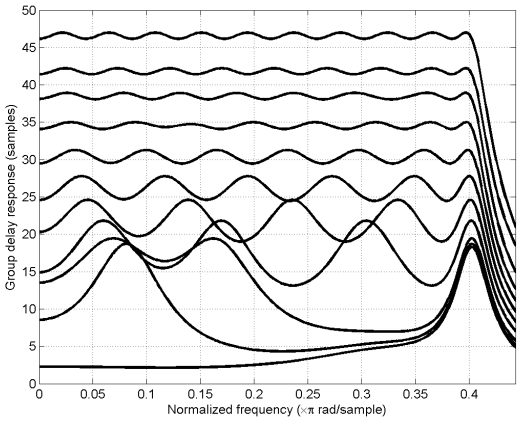

Figure 7: Group delay equalization of a fifth-order wideband elliptic filter; from bottom to top: original response and equalized responses using 1 up to 10 second-order allpass sections.

The results of the optimum group delay equalization of a fifth-order wideband elliptic filter are depicted in Fig. 7 for different numbers of allpass sections. The convergence of the optimization routine was quite rapidly achieved in less than 150 iterations in all cases. Similarly efficient, as indicated by the results in Fig.8, was the proposed procedure in equalizing the unconventional analog discrete-time filtering approach proposed in [16].

5. CONCLUDING REMARKS

A new design methodology of group delay equalization was introduced for IIR discrete-time filters, in which an initial estimate of the allocation of the equalizer poles and zeros was incorporated to an optimization algorithm. The approach proved effective and robust in avoiding local minima, since, in contrast with other design procedures described in the literature, equiripple group delay frequency response was obtained after few iterations, including high-order equalizer designs. The method was applied to the equalization of a large variety of transfer function approximations, and several results were presented.

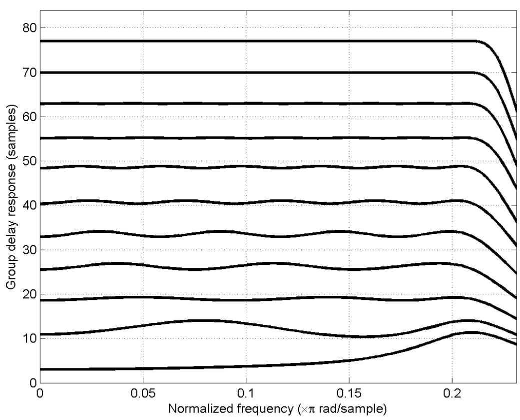

Figure 8: Group delay equalization of the filtering structure proposed in [18]; from bottom to top: original response and equalized responses using 1 up to 10 second-order allpass sections.

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