

AN ADAPTED FILTER BANK FOR FREQUENCY ESTIMATION

El-Hadi Djermoune and Marc Tomczak

Centre de Recherche en Automatique de Nancy, CRAN-UMR CNRS 7039, Université Henri Poincaré Nancy 1
B.P. 239 – 54506 Vandoeuvre-lès-Nancy Cedex, France
phone: +33 3 83 68 44 61, fax: +33 3 83 68 44 62, web: www.cran.uhp-nancy.fr
email: {el-hadi.djermoune, marc.tomczak}@cran.uhp-nancy.fr

ABSTRACT

In this paper, a parametric spectral estimation method using an adapted filter bank is presented. The subband decomposition is performed classically through filtering and decimation stages. The decision about stopping or carrying on the decomposition of a given node is taken according to a new stopping rule. The latter uses a measure of whiteness of the residuals resulting from the node considered. Using Monte Carlo simulations, the results achieved with the proposed method are compared to those obtained with other methods performing fullband and subband estimations, in the case of noisy exponentials. The analysis points out the advantages of the proposed method.

1. INTRODUCTION

The advantages of a subband decomposition approach, in the framework of parametric spectral estimation, have been emphasized by several authors these last years. This idea enables one to transform a complex estimation problem into a set of subproblems, each much simpler than the original one. Moreover, since the early work of Quirk and Liu [1], it is known that subband decomposition enhances the performances of parametric estimators. More recently, approaching the problem from different points of view, Steedly et al. [2], Rao and Pearlman [3], and Tkacenko and Vaidyanathan [4], have pointed out the merits of subband decomposition.

The decomposition is achieved classically through filtering and decimation stages, but the question remains of how to perform the decomposition properly. In particular, a question which arises is the endpoint of the decomposition. At first, a tradeoff must be reached between two alternatives. To improve frequency resolution, it is necessary to increase the decimation factor, but the number of data samples reduces as the decimation gets deeper. Secondly, it would be desirable to stop the decomposition as soon as all the information is retrieved. These remarks suggest to use adaptive forms of decomposition rather than simple uniform ones. In this case, the decimation is carried out according to the spectral content of the subbands encountered, but the problem is then to establish a stop-criterion that determine an optimal (in some sense) decomposition tree.

For instance, in [5], the selection of the optimal decomposition is made by maximizing the number of modes over the whole decomposition tree. The number of modes lying in some band being unknown, it has to be estimated using, say, the minimum description length (MDL) criterion [6]. The problem which arises with such an approach is that it does not ensure that all the spectral information has been retrieved, because order criteria are not always reliable. As an alternative, we propose to use a stop-criterion that reflects the qual-

ity of the estimation in a given subband, that is a measure of whiteness of the corresponding residuals. Unlike adaptive decompositions using order criteria, the decision about stopping or following up the decomposition is made after the estimation process. This allows one to minimize the number of possible missed components. In addition, it will be shown that the number of terminal nodes is reduced, thus decreasing the computational load.

The paper is organized as follows. In section 2, after a brief recall about subband parameter estimation of complex exponential signals, the problem addressed in this paper is set forth. Then, in section 3, an adaptive subband decomposition using a test of whiteness is proposed. Simulation results are presented in section 4 to point out the advantages of the method. Finally, the conclusions are given in section 5.

2. PROBLEM STATEMENT

Consider the following complex signal composed of M complex sinusoids

$$x(n) = \sum_{k=1}^M h_k \exp(j\omega_k n) + e(n), \quad n = 0, \dots, N-1 \quad (1)$$

where $\{\omega_k = 2\pi f_k\}$ are the desired frequencies and $\{h_k\}$ are unknown complex amplitudes. $e(n)$ is an additive zero-mean complex Gaussian white noise.

The estimation of parameters h_k and ω_k may be achieved using different methods. For example, in this paper, we consider the well-known Tufts and Kumaresan (TK) method [7] although other approaches should lead to similar results. The TK method performs a reduced-rank pseudoinverse of the forward-backward data matrix to get the prediction coefficients, from which the frequencies ω_k may be obtained. Then, the amplitudes h_k may be computed by performing a least-squares estimation using equation 1.

In order to reduce the problem complexity [8] and increase the resolution, a uniform subband approach may be considered. Using filtering and decimation operations, it allows one to decompose the original signal $x(n)$ into a set of subsignals, each being representative of a particular frequency band. If a total decimation factor $d = 2^i$ is used, where i is the decomposition depth, each subsignal $x^{(q)}(n)$, $q = 1, \dots, d$, contains or not some components (m) and thus may be modelled as

$$x'(n) = \sum_{k=1}^m h'_k \exp(j\omega'_k n) + e'(n), \quad n = 0, \dots, N'-1 \quad (2)$$

where x' denotes $x^{(q)}$ for simplicity. The subband parameters h'_k and ω'_k may be estimated as in the fullband using

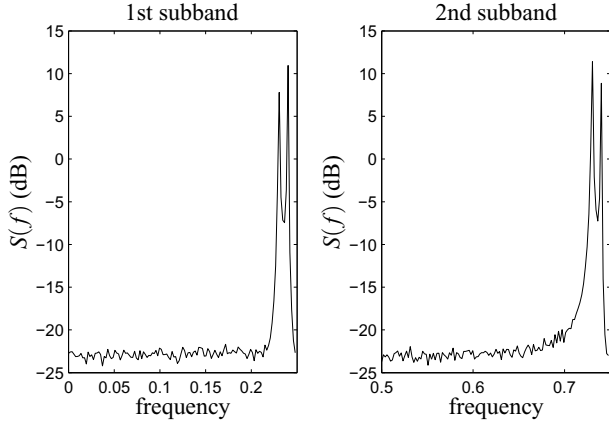


Figure 1: Problems related to spectral overlapping with a 2-channel wavelet filter bank (D_{10} filter).

the TK method. Once all subband parameters \hat{h}'_k and $\hat{\omega}'_k$ are obtained, simple transformations may be used to get the full-band values (see [2, 4]).

2.1 Filter bank structure

Here it is important to recall that, in the context of frequency estimation, the decimation filters have to be chosen carefully because of the overlapping and the mode attenuation they can introduce [4, 8]. In particular, the use of wavelet-based decimation filters without some preprocessing of the data may cause ambiguities about the true positions of modes, due to spectral overlapping [9]. This is illustrated by the following example. Consider a signal containing two cisoids:

$$x(n) = he^{j2\pi f_1 n} + he^{j2\pi f_2 n} + e(n) \quad (3)$$

where $f_1 = 0.24$, $f_2 = 0.73$ and h is fixed so that $SNR = 9.5$ dB. Figure 1 shows the averaged pseudospectra of the two subband signals using 100 numerical simulations. As can be seen, after a decimation by 2, the first mode appears in the two resulting subbands at $f_1 = 0.24$ and $f'_1 = 0.74$. This is also true for the second mode. So this particular example shows that two drawbacks may be observed. At first, it is rather difficult to distinguish between the true modes and their aliases, thus resulting in an ambiguity about the frequency that has to be converted to the fullband [4]. Secondly, a problem of resolution occurs (e.g. between $f_1 = 0.24$ and $f'_2 = 0.23$) when it was not the case before decimation.

So, for frequency estimation, we prefer to use the structure presented in figure 2 (designed for a decimation factor of 2). The decimation filters are frequency-shifted versions of a unique low-pass filter with passband edge $f_p = 0.125$ and stopband frequency $f_s = 0.25$. These values are chosen so that the successive passbands are contiguous. Using this structure ensures that all modes are not attenuated on the whole frequency interval $f \in [0, 1]$. In addition, the effects of overlapping are strongly reduced. However, the number of subbands is increased by a factor 2 (four bands are generated when the signal is decomposed by a factor 2).

The pseudospectra corresponding to the first and third subband signals are shown in figure 3. We observe that each peak appears once in the two subbands, which confirms that the overlapping phenomenon is reduced. This structure is re-

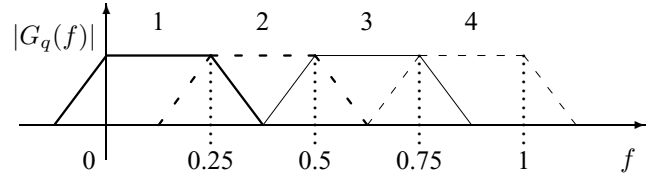


Figure 2: The uniform filter bank used for frequency estimation.

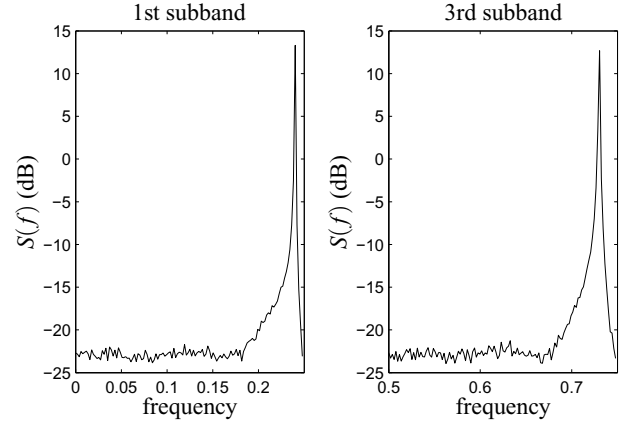


Figure 3: Pseudospectra of the first and third subband signals.

tained for the adaptive method that will be described later.

2.2 Why an adaptive decomposition?

Adaptive schemes are preferable because they tend to achieve a compromise between the decimation level and the signal complexity. Thus, at each node (or band), the decomposition may be stopped or continued according to the spectral content of the subband considered (see figure 4).

Several kinds of stop-criteria may be considered, like power-based criteria, order criteria, spectral flatness measures, etc. For example, in [5], the criterion used is based upon the number of components determined by the MDL. The decomposition is stopped if the estimated number of modes in a particular node is greater than that obtained in its children. The problem which arises with such an approach is that it does not ensure that all the spectral information has been retrieved, because order criteria are not always reliable and what is more the decision is taken before the subband estimation. Thus, the resulting decomposition does not take into account the fact that an isolated mode may be estimated without needing a deeper decomposition. The method proposed in the next section tries to overcome this problem.

3. ADAPTIVE SUBBAND DECOMPOSITION

The adaptive subband decomposition method we propose here aims to satisfy two constraints. Firstly, if in some subband, the estimation procedure cannot resolve two modes, the decomposition should continue. Secondly, if all spectral information has been retrieved accurately in a given subband, there is no need to carry on the decomposition, and thus it should be stopped in order to reduce the complexity

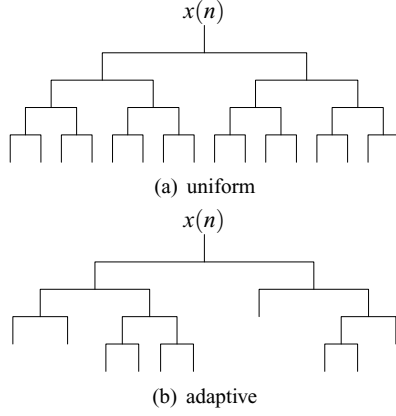


Figure 4: Uniform and adaptive decompositions. Each division represents a filtering/decimation stage.

and the risk of false detections. For this purpose, we propose a stop-criterion that uses a measure of whiteness [10] of the residuals.

The residuals are defined as the difference between a sub-signal and the estimated subsignal containing m' modes

$$\varepsilon(n) = x'(n) - \hat{x}'(n) = x'(n) - \sum_{k=1}^{m'} \hat{h}'_k \exp(j\hat{\omega}'_k n) \quad (4)$$

for $n = 0, \dots, N' - 1$. Ideally, if all subsignal modes have been retrieved (i.e. $m' = m$), the residuals are close to a white noise. If one or more modes are missed, then the sequence $\varepsilon(n)$ is no more white.

3.1 The criterion

The principle of the test for whiteness used here is now briefly recalled. First, assume that $\varepsilon(k)$ is a wide sense stationary Gaussian sequence. Its power spectral density is denoted by $P(\omega)$. Let us define $W(P)$ as

$$W(P) = \log \frac{1}{2\pi} \int_{-\pi}^{\pi} P(\omega) d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} \log P(\omega) d\omega \quad (5)$$

Drouiche [10] showed that $W(P) = 0$ if and only if $P(\omega)$ is constant, and $W(P) > 0$ otherwise. In practice, the power spectral density is estimated by the periodogram defined by

$$\hat{P}_{N'}(\omega) = \frac{1}{2\pi N'} \left| \sum_{n=0}^{N'-1} \varepsilon(n) e^{jn\omega} \right|^2 \quad (6)$$

The test that has to be constructed is intended to decide whether or not $\varepsilon(n)$ is white. In other words, it should verify the null hypothesis $H_0 : P = c$ if $\varepsilon(n)$ is white and the non-null hypothesis $H_1 : P \neq c$ if it is not the case. Here c is a positive constant. Consequently, we consider the following quantity

$$\hat{W}_{N'} = \log \frac{1}{2\pi} \int_{-\pi}^{\pi} \hat{P}_{N'}(\omega) d\omega - \frac{1}{2\pi} \int_{-\pi}^{\pi} \log \hat{P}_{N'}(\omega) d\omega - \gamma \quad (7)$$

where $\hat{W}_{N'}$ is an estimate of $W(P)$. The parameter γ denotes the Euler constant ($\gamma = 0.57721$) and has been introduced to prevent bias under the null hypothesis, i.e. to ensure that

$\mathbb{E}_{H_0} \hat{W}_{N'} = 0$. The quantity $\hat{W}_{N'}$ is a measure of whiteness : $\hat{W}_{N'} \approx 0$ under H_0 , and $\hat{W}_{N'} \rightarrow \infty$ if $\varepsilon(n)$ is maximally correlated. In practice, we reject H_0 if $\hat{W}_{N'} > t_\alpha$, where t_α is a threshold obtained using significant level α (which defines a false alarm rate). According to [10], $\sqrt{N'} \hat{W}_{N'}$ and $\sqrt{N'} (\hat{W}_{N'} - W)$ converge in law to normal distributions as $N' \rightarrow \infty$, under H_0 and H_1 respectively. Using this result, the threshold is given by

$$t_\alpha = \frac{\sqrt{2}v_0}{\sqrt{N'}} \text{erf}^{-1}(1 - 2\alpha) \quad (8)$$

where $v_0 = \sqrt{\pi^2/6 - 1}$, and $\text{erf}^{-1}(x)$ is the inverse of the standard error function

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \quad (9)$$

The significance level α is a free parameter. In practice, it is often taken inferior to 10%. In this paper, we fixed $\alpha = 1\%$.

3.2 The complete algorithm

The method proposed is now briefly summarized.

1. Choose decimation filter and decimation factor.
2. Perform a subband decomposition of the signal.
3. For each resulting node do the following
 - (a) obtain the subband parameters h'_k and ω'_k ,
 - (b) generate the residuals,
 - (c) compute t_α and test for whiteness,
 - (d) if residuals are white, this node is a terminal one, else the decomposition must be carried on.
4. Search for a decomposable node from the tree and obtain its children by further decimation.
5. Repeat step 3, until no decomposable node is found.
6. Convert the subband parameters to their fullband values.

4. SIMULATION RESULTS

In this section, we present an experiment made on a simulated signal. The basic decimation filter is a low-pass equiripple FIR filter with both passband and stopband ripples fixed to 0.01. The simulation signal used here consists in two sinusoids in zero-mean white noise:

$$x(n) = h e^{j2\pi f_1 n} + h e^{j2\pi f_2 n} + e(n) \quad (10)$$

where $f_1 = 0.11$ and $f_2 = 0.15$, as in [5], and the number of samples is $N = 160$. The signal-to-noise ratio is varying between -10 and 20 dB. At each SNR, multiple simulations have been performed using 200 realizations of the additive noise. The results achieved using the suggested adaptive subband decomposition are compared to those obtained with three other estimation methods : The first considers the adaptive decomposition proposed in [5], the second uses a uniform decomposition with a total decimation factor $d = 8$, and the third performs a fullband estimation. All methods are compared on the base of their estimation variances, their miss ratios and their percentage of terminal nodes relatively to the uniform decomposition (see figure 5).

The fullband prediction order is set to 32; it is then divided by 2 as soon as the decomposition gets deeper. This particular choice of the prediction order gives to the fullband

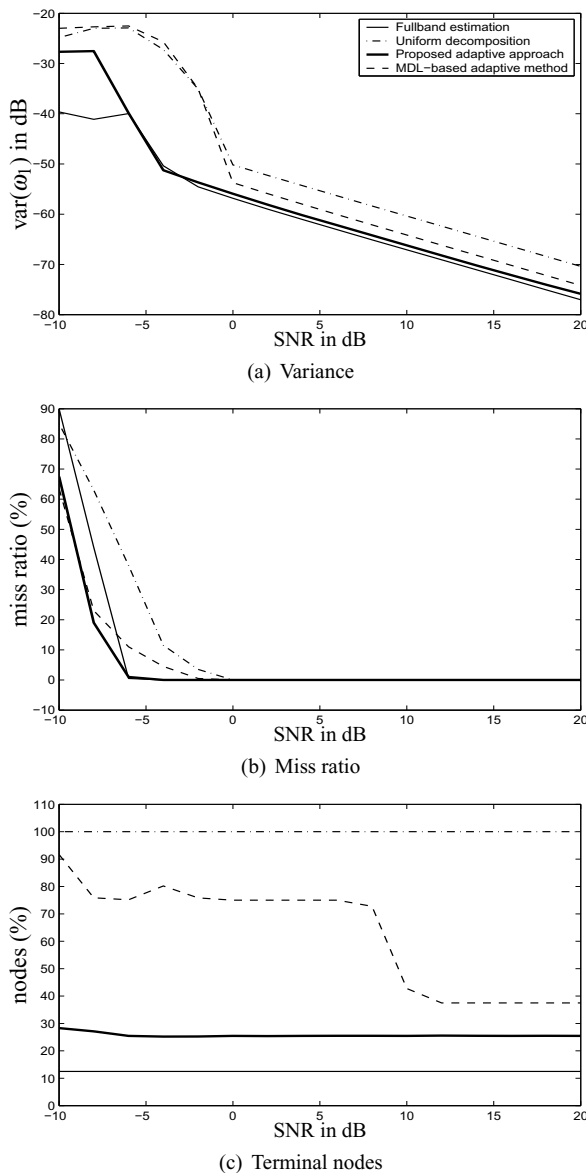


Figure 5: Results achieved on the simulation signal.

estimation an advantage in view of the complexity of the signal. Indeed, one can observe from figure 5(a) that the variance of the first component obtained in the fullband is the lowest one. The uniform decomposition leads to the poorest variance while the proposed criterion allows to the adaptive decomposition to reach approximately the same variance as the fullband estimation.

The miss ratio of the proposed method is the smallest one, and converges faster to zero (figure 5(b)). In particular, the miss ratio of the adaptive decomposition is smaller than that of the uniform one. This is because the a priori choice of the decimation factor is not suitable for the signal considered.

The great advantage of the proposed method over an MDL-based adaptive decomposition lies in the fact that the former leads to a much smaller number of terminal nodes over which the final estimation process is performed (figure 5(c)). Thus, in general, using the suggested method allows one to stop the decomposition at a lower decimation level.

The MDL-based decomposition leads to approximately 80% of the number of uniform decomposition nodes at low SNR and decreases to 40% at high SNR. The proposed method attains only 25% of the number of terminal nodes. This result is of high importance when analyzing very intricate signals composed of a large number of components and/or samples. This is the case for example for carbon-13 nuclear magnetic resonance spectroscopy signals [11].

5. CONCLUSION

This paper presents a new method for adaptive subband decomposition. First, a filter bank structure which is well suited to the problem of frequency estimation is presented. Then, a stop-criterion which is intended to allow the estimation procedure to retrieve all useful spectral information without a large additive computation complexity is introduced. Using Monte Carlo simulations, it is shown that the proposed method is advantageous over both a uniform decomposition and an MDL-based adaptive approach, in terms of the variance and the number of terminal nodes.

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