# ROBUST TRANSMISSION OF VARIABLE-LENGTH ENCODED MARKOV SOURCES USING RATE-1 CHANNEL CODING AND EFFICIENT ITERATIVE SOURCE-CHANNEL DECODING

Ragnar Thobaben and Jörg Kliewer

University of Kiel
Institute for Circuits and Systems Theory
Kaiserstr. 2, 24143 Kiel, Germany
Phone: +49-431-880-6130, Fax: +49-431-880-6128
E-Mail: {rat, jkl}@tf.uni-kiel.de

## ABSTRACT

In this paper we present a novel low complexity bit-level soft-input/soft-output decoding approach for variable-length encoded packetized Markov sources transmitted over noisy communication channels. This approach has the advantage that all available residual source redundancy in form of transition probabilities of the Markov source can be exploited as additional a-priori information in the decoding process. When explicit redundancy from channel codes is additionally added to the interleaved variable-length encoded bit sequence, decoding can be carried out with an iterative source-channel decoding scheme. Furthermore, for reversible variable-length codes, which provide additional explicit source redundancy, good matching rate-1 channel codes are determined via an extrinsic information transfer chart analysis of the iterative decoder such that a robust transmission is possible even for channels with low signal-to-noise ratio.

## 1. INTRODUCTION

Motivated by the increasing demand for mobile access of multimedia data, joint source-channel decoding (JSCD) of variable-length codes (VLCs) has become an active research area. In order to achieve a reliable transmission of variable-length encoded sources some of the proposed techniques focus on iterative joint source-channel decoding, where soft-in/soft-out (SISO) VLC source decoders are used as constituent decoders in iterative decoding schemes. This is motivated by the fact that the encoding operation is similar to the one for serially concatenated channel codes, where the explicit redundancy from the outer encoder is replaced by implicit residual source redundancy.

Many SISO VLC source decoding approaches have been proposed in the literature. For example, in [1] the classical BCJR algorithm [2] is applied as a-posteriori probability (APP) decoder working on a symbol-level trellis for uncorrelated variable-length encoded data packets. In [3] an optimal symbol-based APP decoding approach for variable-length encoded first-order Markov sources is proposed considering a three-dimensional trellis. These methods have the drawback that the complexity increases strongly for larger source packet lengths. On the other hand, in [4] the BCJR algorithm is applied to a bit-level trellis proposed in [5] which has a moderate complexity even for increasing source packet lengths due to a linear increase of the overall number of trellis states. In [6] the authors propose a soft-output stack algorithm for low complexity iterative VLC decoding working on the corresponding code tree.

In this paper we present a joint source-channel decoding approach for packetized variable-length encoded correlated source data. It can be seen as an extension of the work in [7] where for a first-order Markov source a modified BCJR algorithm is proposed for SISO decoding on the bit-level trellis from [5]. In this approach the Markov property of the source is only partly exploited since it is only employed in the forward recursion of the BCJR-based decoder due to the causal definition of the source transition probabilities. As

a new result we show in the following that source statistics due to the Markov property can also be incorporated in the backward recursion with only a slight increase in complexity compared to the approach in [7]. When we furthermore assume additional error protection by channel codes the proposed SISO VLC decoder can be used as outer decoder in an iterative joint source-channel decoding scheme. In order to optimize the overall transmission system we apply an extrinsic information transfer (EXIT) chart analysis [8]. We demonstrate that by allowing additional source redundancy in form of reversible VLCs (RVLCs) [9] with a Hamming distance larger than one between equal-length codewords good recursive systematic convolutional (RSC) channel codes with a code rate of one and low memory can be found. Using the proposed APP VLC decoder instead of the one from [7] in the iterative source-channel decoding scheme the resulting transmission system exhibits a gain of 0.6 dB in channel signal-to-noise ratio (SNR) for the symbol error rate (SER) on an AWGN channel.

## 2. TRANSMISSION MODEL

The transmission system under consideration is shown in Fig. 1, where the transmission of a packet of K correlated source symbols given by  $\mathbf{U} = [U_1, U_2, \dots, U_K]$  is assumed. After (vector-) quantization of source symbols  $U_k$  indices  $I_k \in \mathcal{I}$  from the finite alphabet  $\mathcal{I} = \{0, 1, \dots, 2^M - 1\}$  represented with M bits are obtained. Due to delay and complexity constraints for the quantization stage, a residual index correlation remains in the index vector  $\mathbf{I} = [I_1, I_2, \dots, I_K]$ , which is modeled as a firstorder (stationary) Markov process with index transition probabilities  $P(I_k = \lambda | I_{k-1} = \mu)$  for  $\lambda, \mu \in \mathcal{I}$ . The quantization stage is followed by a VLC encoder which maps a fixed-length index  $I_k$ to a variable-length bit vector  $\mathbf{c}(\lambda) = \mathcal{C}(I_k = \lambda)$  of length  $l(\mathbf{c}(\lambda))$  using the VLC with codetable  $\mathcal{C}$ . The output of the VLC encoder is given by the binary sequence  $\mathbf{b} = [b_1, b_2, \dots, b_N]$  of length N where  $b_n$  represents a single bit at bit index n. Finally, the interleaved bit sequence  $\mathbf{b}'$  is input to a rate-R channel encoder, generating the sequence of code bits v. Under consideration of a binary transmission of code bits  $v_m$  at code bit index m the memoryless channel is characterized by the p.d.f.  $p(\hat{v}_m | v_m)$  leading to a softbit vector  $\hat{\mathbf{v}} = [\hat{v}_1, \hat{v}_2, \ldots]$  with  $\hat{v}_m \in \mathbb{R}$  at the channel output.

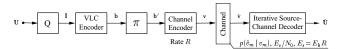


Figure 1: Model of the transmission system

## 3. APP SOURCE DECODING

In the following the derivation of the proposed APP VLC decoder is presented. For the purpose of convenience we consider an uncoded

transmission of the VLC bits throughout this section, i.e.  $\mathbf{v} = \mathbf{b}$ .

#### 3.1 Trellis representation

A quite simple trellis representation is proposed in [5], where the VLC trellis is derived from the corresponding code tree by mapping each node to a specific trellis state. The root node and all leaf nodes are mapped to the same trellis state, since in a sequence of codewords every leaf becomes the root of the tree for the next codeword. Each transition from state  $S_{n-1}$  to state  $S_n$  is caused by a single bit  $b_n \in \{0,1\}$  at time instant  $n=1,\ldots,N$  at the output of the VLC encoder. In Fig. 2 an example of a trellis segment considering the reversible variable-length code (RVLC)

$$C = \{\mathbf{c}(0) = [11], \mathbf{c}(1) = [00], \mathbf{c}(2) = [101], \\ \mathbf{c}(3) = [010], \mathbf{c}(4) = [1001], \mathbf{c}(5) = [0110]\}$$
(1)

is shown. In the following we denote the set of all  $N_s$  trellis states as  $\mathcal{S} = \{s_0, \dots, s_{N_s-1}\}$  where the  $s_k, k=0,\dots,N_s-1$ , represent the individual state hypotheses. Furthermore, the bit position for the root state or, equivalently, for the leaf state is denoted with  $\nu$ ,  $\nu \in \{0,\dots,N\}$ , and for the sake of brevity its hypothesis  $S_{\nu} = s_0$  is written as  $S_{\nu} = 0$  in the remainder of this paper.

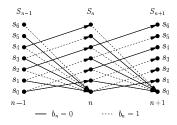


Figure 2: Trellis representation for the RVLC codetable in (1)

Due to the unique relationship between VLC tree and VLC trellis every non-leaf or non-root state  $S_n = \sigma, \ \sigma \in \mathcal{S} \setminus s_0$ , is associated with a certain codeword prefix written as  $\mathbf{c}_{[0...\sigma]}$ , or equivalently, with a state sequence  $[S_{\nu},\ldots,S_n]=[0,\ldots,\sigma]$ . This corresponds to a certain subset of codewords  $\mathcal{P}_{\sigma}=\{\lambda \in \mathcal{I} \mid S_n=\sigma \in \mathbf{c}(\lambda), \ \sigma \in \mathcal{S} \setminus s_0\}$  with the common prefix  $\mathbf{c}_{[0...\sigma]}$ .

## 3.2 A-posteriori probabilities

Considering the VLC trellis representation from above we now derive a SISO decoding algorithm, which provides a-posteriori probabilities (APPs)  $P(b_n=i\mid \hat{\mathbf{b}})$  of the source bits  $b_n=i, i\in\{0,1\}$ . Analog to the BCJR algorithm [2] the APPs  $P(b_n=i\mid \hat{\mathbf{b}})$  can be decomposed as

$$P(b_{n} = i \mid \hat{\mathbf{b}}) = \frac{1}{p(\hat{\mathbf{b}})} \sum_{\sigma_{1} \in S} \sum_{\sigma_{2} \in S} \underbrace{p(\hat{\mathbf{b}}_{n+1}^{N} \mid S_{n} = \sigma_{2})}_{\beta_{n}(\sigma_{2})} \cdot \underbrace{p(\hat{b}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\gamma_{n}(i, \sigma_{2}, \sigma_{1})} \cdot \underbrace{p(S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = \sigma_{2} \mid S_{n} = \sigma_{2}, \hat{\mathbf{b}}_{1}^{n-1})}_{\alpha_{n-1}(\sigma_{1})}, \underbrace{p(\hat{\mathbf{b}}_{n}, b_{n} = i, S_{n} = i$$

with the well-known BCJR forward and backward recursions  $\alpha_n(\sigma_2)$  and  $\beta_n(\sigma_2)$ , and  $\hat{\mathbf{b}}_{n_1}^{n_2}$  specifying the subsequence  $\hat{\mathbf{b}}_{n_1}^{n_2} = [\hat{b}_{n_1}, \dots, \hat{b}_{n_2}]$ . In the following the derivations for both recursions will be presented where as a new result the first-order Markov property of the quantization indices is exploited as a-priori information in the derivation of *both* the forward and the backward recursion.

## 3.2.1 Forward recursion

According to [2] the forward recursion  $\alpha_n(\sigma_2)$  can be obtained as

$$\alpha_n(\sigma_2) = \sum_{\sigma_1 \in \mathcal{S}} \sum_{i=0}^1 \gamma_n(i, \sigma_2, \sigma_1) \, \alpha_{n-1}(\sigma_1), \quad \alpha_0(0) = 1, \quad (3)$$

where the  $\gamma$ -term can be stated as

$$\gamma_n(i, \sigma_2, \sigma_1) = p(\hat{b}_n \mid b_n = i) \cdot P(b_n = i, S_n = \sigma_2 \mid S_{n-1} = \sigma_1, \hat{\mathbf{b}}_1^{n-1}).$$
(4)

The term  $p(\hat{b}_n \mid b_n = i)$  in (4) corresponds to the soft-output of the transmission channel associated with bit  $b_n = i$ . The second term specifies the a-priori probability of a transition from state  $S_{n-1} = \sigma_1$  to state  $S_n = \sigma_2$  due to bit  $b_n = i$ . It also depends on the previously received soft-bits  $\hat{\mathbf{b}}_1^{n-1}$ , which in contrast to the BCJR algorithm in its classical form allows to exploit the transition probabilities of the Markov source model as additional a-priori knowledge.

In [7] it is shown that the a-priori term on the right-hand side of (4) can be obtained as a marginal distribution of conditional probabilities of source indices affected with the state transition from  $S_{n-1} = \sigma_1$  to  $S_n = \sigma_2$  according to

$$P(b_{n} = i, S_{n} = \sigma_{2} | S_{n-1} = \sigma_{1}, \hat{\mathbf{b}}_{1}^{n-1}) = \frac{1}{C(\sigma_{1})} \sum_{\lambda \in \mathcal{P}_{\sigma_{2}}} P(I_{0} = \lambda | S_{\nu} = 0, \hat{\mathbf{b}}_{1}^{\nu}) \quad (5)$$

with the normalization factor  $C(\sigma_1)$ . For the sake of clarity the source indices  $I_k$  are referenced relatively to the currently considered trellis path segment from bit position  $\nu$  to  $\nu+l(\mathbf{c}(\lambda))$  (which corresponds to k=0). Since  $P(I_0=\lambda\,|\,S_\nu=0,\,\hat{\mathbf{b}}_1^\nu)$  in (5) can be written as

$$P(I_0 = \lambda \mid S_{\nu} = 0, \hat{\mathbf{b}}_1^{\nu}) = \sum_{\mu=0}^{2^{M}-1} P(I_0 = \lambda \mid I_{-1} = \mu) \cdot P(I_{-1} = \mu \mid S_{\nu} = 0, \hat{\mathbf{b}}_1^{\nu}) \quad (6)$$

we can now utilize the index transition probabilities  $P(I_0=\lambda\,|\,I_{-1}=\mu)$  as a-priori information for APP decoding. The last missing term to be expressed with known quantities is the conditional probability  $P(I_{-1}=\mu\,|\,S_\nu=0,\hat{\mathbf{b}}_1^\nu)$  in (6). Due to the fact that the trellis branch specified by the triple  $(S_{\nu-1}=\sigma_0,S_\nu=0,b_\nu=i),\sigma_0\in\mathcal{S},$  uniquely identifies the trellis path associated with the source index  $I_{-1}=\mu,\,\mu\in\mathcal{I},$  at the previous index position, the hypothesis  $I_{-1}=\mu$  may be replaced with  $b_\nu=i(\mu)$  and  $S_{\nu-1}=\sigma_0(\mu)$  which now depend on  $\mu$ . We thus obtain

$$P(I_{-1} = \mu \mid S_{\nu} = 0, \hat{\mathbf{b}}_{1}^{\nu}) = \frac{1}{\alpha_{\nu}(0)} \cdot \gamma_{\nu}(i(\mu), 0, \sigma_{0}(\mu)) \cdot \alpha_{\nu-1}(\sigma_{0}(\mu)), \quad (7)$$

where the term  $\alpha_{\nu}(0)$  is a normalization factor only depending on the bit position  $\nu$  for the root state. Finally, combining (4), (5), (6), and (7) leads to a modified expression for the  $\gamma$ -term which allows us to exploit the residual source correlation as a-priori information in the forward recursion (3).

## 3.2.2 Backward recursion

Due to the causal definition of the source transition probabilities from time instant k=-1 to k=0 (relatively to the root state index  $\nu$ ) and the forward-directed character of the VLC trellis it is not possible to incorporate this additional a-priori information in the standard BCJR-style backward recursion. Therefore, in the following an alternative approach is proposed.

Since a certain trellis state  $S_n = \sigma_2$  can be associated with the subset  $\mathcal{P}_{\sigma_2}$  of codewords having the same prefix  $\mathbf{c}_{[0...\sigma_2]}$  we can reformulate the  $\beta$ -term according to

$$\beta_n(\sigma_2) = p(\hat{\mathbf{b}}_{n+1}^N \mid I_0 \in \mathcal{P}_{\sigma_2}, S_{\nu} = 0),$$
 (8)

where again  $\nu=n-l(\mathbf{c}_{[0...\sigma_2]})$  refers to the bit position of the affected root state  $S_{\nu}=0$ . The expression in (8) can now be decom-

posed as

$$\beta_{n}(\sigma_{2}) = \frac{1}{C'(\sigma_{2})} \cdot \sum_{\mu \in \mathcal{P}_{\sigma_{2}}} \underbrace{p(\hat{\mathbf{b}}_{n+1}^{N} \mid I_{0} = \mu, S_{\nu} = 0)}_{=:\beta_{n}(\mu, \nu)} \cdot P(I_{0} = \mu \mid S_{\nu} = 0), \quad (9)$$

with the normalization factor  $C'(\sigma_2)$ . As we can see from (9) the term  $\beta_n(\sigma_2)$  can be obtained by performing a marginal distribution over the p.d.f.  $\beta'_n(\mu,\nu) := p(\hat{\mathbf{b}}^N_{n+1} \mid I_0 = \mu, S_\nu = 0)$  and the term  $P(I_0 = \mu \mid S_\nu = 0)$  representing the probability of a codeword corresponding to index  $I_0 = \mu$  starting at position  $\nu$ . The conditional probability  $P(I_0 = \mu \mid S_\nu = 0, \hat{\mathbf{b}}^\nu_1)$  from (6) may be taken as a local estimate for  $P(I_0 = \mu \mid S_\nu = 0)$  since  $\hat{\mathbf{b}}^N_{n+1}$  and  $\hat{\mathbf{b}}^\nu_1$  can be considered as independent for a memoryless transmission channel.

The symbol-based quantity  $\beta_n'(\mu,\nu)$  can be regarded as a modified  $\beta$ -term conditioned on the source hypothesis  $I_0 = \mu$  and the root state  $S_{\nu} = 0$ . Let  $\nu' = \nu + l(\mathbf{c}(\mu))$  denote the bit position which marks the end of the codeword  $\mathbf{c}(\mu)$  of the considered quantization index  $I_0 = \mu$ . Let furthermore  $\nu'' = \nu' + l(\mathbf{c}(\lambda))$  be the end position of the ascending codeword  $\mathbf{c}(\lambda)$  corresponding to the quantization index  $I_1 = \lambda$  starting at the bit position  $\nu'$ . The term  $\beta_n'(\mu, \nu)$  can then be written as

$$\underbrace{p(\hat{\mathbf{b}}_{n+1}^{N} \mid I_{0} = \mu, S_{\nu} = 0)}_{= \beta_{n}'(\mu, \nu)} = p(\hat{\mathbf{b}}_{n+1}^{\nu'} \mid \mathbf{c}_{[\sigma_{2} \dots 0]}(\mu)) \cdot \underbrace{p(\hat{\mathbf{b}}_{\nu'+1}^{N} \mid I_{0} = \mu, S_{\nu} = 0)}_{= \beta_{\nu'}'(\mu, \nu)}. \quad (10)$$

The channel term  $p(\hat{\mathbf{b}}_{n+1}^{\nu'} | \mathbf{c}_{[\sigma_2...0]}(\mu))$  is given by the product of the channel p.d.f.s  $p(\hat{b}_{n+\eta} | c_{\eta}(\mu))$  corresponding to the  $\eta$ -th bit  $c_{\eta}(\mu)$  of the codeword *postfix*  $\mathbf{c}_{[\sigma_2...0]}(\mu)$  according to

$$p(\hat{\mathbf{b}}_{n+1}^{\nu'} \mid \mathbf{c}_{[\sigma_2...0]}(\mu)) = \prod_{\eta=1}^{l(\mathbf{c}_{[\sigma_2...0]}(\mu))} p(\hat{b}_{n+\eta} \mid c_{\eta}(\mu)).$$
 (11)

It can be shown that  $\beta'_{\nu'}(\mu,\nu)$  in (10) can be expressed as an indexbased recursion defined on the root states of the underlying VLC trellis, as it is depicted in Fig. 3 for the RVLC example in (1). This recursion utilizes the residual source index correlation as a-priori information and writes

$$\underbrace{p(\hat{\mathbf{b}}_{\nu'+1}^{N} \mid I_{0} = \mu, S_{\nu} = 0)}_{= \beta_{\nu'}'(\mu,\nu)} = \sum_{\lambda \in \mathcal{I}} \underbrace{p(\hat{\mathbf{b}}_{\nu''+1}^{N} \mid I_{1} = \lambda, S_{\nu'} = 0)}_{= \beta_{\nu'}'(\lambda,\nu')} \cdot \underbrace{p(\hat{\mathbf{b}}_{\nu''+1}^{\nu''} \mid I_{1} = \lambda) \cdot P(I_{1} = \lambda \mid I_{0} = \mu)}_{=: \gamma'_{.I}(\lambda,\mu)}. \quad (12)$$

As in the bit-based BCJR-style backward recursion the  $\gamma$ -term  $\gamma'_{\nu'}(\lambda,\mu)$  in (12) both contains a-priori source statistics, now in form of the source index transition probabilities  $P(I_1=\lambda\,|\,I_0=\mu)$ , and soft information in form of  $p(\hat{\mathbf{b}}^{\nu''}_{\nu'+1}\,|\,I_1=\lambda)$  at the output of the transmission channel. The latter expression can be calculated by multiplying bit-based channel p.d.f.s analog to (11). The desired relation for calculating the  $\beta_n(\sigma_2)$  in (2) is now obtained by combining (12), (10), and (9).

Now we have derived all terms in order to calculate the APPs  $P(b_n=i\,|\,\hat{\mathbf{b}})$  from (2) with known quantities, where besides the source statistics only the number of transmitted bits N is used as side information in the APP calculation. Finally, a source symbol packet estimate  $\hat{\mathbf{U}}$  can be obtained via a MAP sequence estimation where the bit-based APPs  $P(b_n=i\,|\,\hat{\mathbf{b}})$  are used to calculate the corresponding path metrics.

# 4. ITERATIVE DECODING AND EXIT CHARTS

Let us now consider the case where additional explicit redundancy from channel codes is introduced to the interleaved output of the

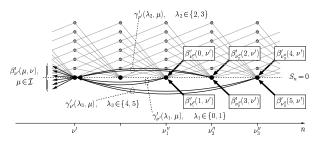


Figure 3: Symbol-based backward recursion defined on the root states of the underlying VLC trellis for the RVLC in (1)

VLC encoder according to Fig. 1. This allows us to utilize the proposed APP VLC source decoder from Section 3.2 in an iterative decoding scheme which can be optimized by performing an EXIT chart analysis.

#### 4.1 Iterative joint source-channel decoding

Due to the serial concatenation of source and channel encoding the iterative source-channel decoding scheme from [7] can be used for calculating APPs for the decoded information bits. In the decoding process the (inner) APP channel decoder calculates a-posteriori log-likelihood-ratios (LLRs)  $L^{(C)}(\mathbf{b}')$  for the interleaved information bit vector  $\mathbf{b}' = [b_1', b_2', \dots, b_N']$ . The quantity  $L_e^{(C)}(\mathbf{b}') = L^{(C)}(\mathbf{b}') - L_a^{(C)}(\mathbf{b}')$  is used after deinterleaving by the (outer) APP source decoder as a-priori information  $L_a^{(S)}(\mathbf{b})$  in order to obtain the a-posteriori LLRs  $L^{(S)}(\mathbf{b})$ . For completing the iteration the APP source decoder generates the extrinsic information  $L_{\text{extr}}^{(S)}(\mathbf{b}) = L^{(S)}(\mathbf{b}) - L_a^{(S)}(\mathbf{b})$  which after interleaving can be exploited as additional a-priori information  $L_a^{(C)}(\mathbf{b}')$  by the inner channel decoder at the beginning of the next iteration. Finally, a MAP sequence estimation is performed on the LLRs  $L^{(S)}(\mathbf{b})$  in order to obtain an estimate  $\hat{\mathbf{U}}$  of the transmitted source sequence

## 4.2 EXIT charts

In order to analyze the iterative decoding process we apply an EXIT chart analysis [8] to the problem of iterative joint source-channel decoding. Denoting the mutual information I between transmitted bit sequence  $\mathbf{b}$  and the inputs/outputs of the inner (channel) and outer (source) decoder as

$$I_{A_i} = I(L_a^{(C)}(\mathbf{b}'); \mathbf{b}')$$
 and  $I_{E_i} = I(L_e^{(C)}(\mathbf{b}'); \mathbf{b}'),$ 
 $I_{A_o} = I(L_a^{(S)}(\mathbf{b}); \mathbf{b})$  and  $I_{E_o} = I(L_{\text{extr}}^{(S)}(\mathbf{b}); \mathbf{b}),$ 

the (extrinsic) information transfer characteristics of the constituent decoders are given by  $I_{E_i}=T_i(I_{A_i},E_s/N_0)$  and  $I_{E_o}=T_o(I_{A_o}),$  respectively, which can easily be obtained by the method proposed in [10]. The function  $T_i$  for the inner decoder is parameterized with the channel SNR  $E_s/N_0$  since a-priori information and channel observation  $\hat{\mathbf{v}}$  are employed in the decoding process, while the transfer characteristics  $T_o$  of the outer constituent decoder only depends on  $I_{A_o}$ . An EXIT chart can be obtained by plotting both mappings  $T_i$  and  $T_o$  into a single diagram.

In the following we propose a transmission system where some explicit redundancy is also added for error protection in the VLC encoder by using a RVLC with free distance  $d_f = 2$  [11]. Since the channel code still can be arbitrarily chosen, good codes in terms of convergence behavior and decoding performance may be searched via an EXIT chart analysis. In this connection we restrict ourselves to rate-1 channel codes, which do not introduce further (explicit) redundancy into the variable-length encoded bitstream. Fig. 4 shows the EXIT chart of the resulting iterative joint source-channel decoder. The transfer characteristic  $T_o(I_{A_o})$  of the (outer) VLC source decoder is derived for an AR(1) input process with correlation coefficient a = 0.9, uniformly quantized with M = 4

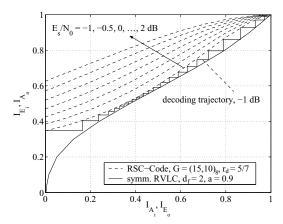


Figure 4: EXIT chart for the iterative source-channel decoder

bits. The mapping  $T_i(I_{A_i},E_s/N_0)$  is obtained for a rate-1 recursive systematic convolutional (RSC) channel code being punctured from a rate-1/2 memory-3 mother code with generator polynomials  $(g_0,g_1)_8=(15,10)_8$ . Here, the puncturing pattern with a systematic-to-coded bit ratio  $r_d=5/7$  is obtained from an EXIT-chart-based code search. In order to illustrate the iterative decoding process Fig. 4 also depicts a simulated decoding trajectory for  $E_s/N_0=-1$  dB where the decoder is able to pass the bottleneck region.

## 5. SIMULATION RESULTS

In order to verify the performance of the resulting transmission system for the constituent codes from Section 4.2 simulations were carried out for a BPSK-modulated AWGN channel and 200 simulated transmissions of a correlated uniformly quantized AR(1) source (a = 0.9, M = 4) of 20000 source symbols. Fig. 5 shows the SER plotted over the channel parameters  $E_s/N_0$  and  $E_b/N_0$ for the proposed method and also includes results from [7], where  $E_b = E_s/R$  denotes the transmitted energy per information bit. The overall code rate R is obtained as  $R = R_{RSC} \cdot H(I_k \mid I_{k-1}) / M_{RVLC}$ and contains all redundancy exploited for error protection: the conditional entropy  $H(I_k | I_{k-1})$  takes the redundancy due to the residual source correlation into account and the mean word length after VLC encoding  $\overline{M}_{RVLC}$  the explicit redundancy from the RVLC, respectively. Due to a rate-1 channel code we have  $R_{\rm RSC} = 1$ , leading to an overall code rate of R = 0.57. As we can see from Fig. 5, for channel SNRs  $E_s/N_0 \ge -0.95$  dB and  $E_b/N_0 \ge 1.49$  dB, resp., a reliable transmission is likely to be possible since no error event has occurred for all simulated channel realizations. This corresponds quite well with Fig. 4 where convergence for  $E_s/N_0 \ge -1$  dB is predicted. We can furthermore observe from Fig. 5 that compared to the APP VLC decoder from [7] a gain of 0.6 dB in channel SNR can be achieved using the proposed SISO VLC decoding algorithm within the iterative decoding framework.

# 6. CONCLUSION

We have presented a novel VLC APP decoding approach which extends the work from [7] and can be regarded as a modified bit-based BCJR algorithm adapted to variable-length encoded first-order Markov sources. As a new result we have shown that it is possible to exploit symbol-based source statistics as a-priori information on a bit-level trellis in both the forward and backward recursion of the VLC APP decoder. This leads to a significant reduction of complexity compared to symbol-based approaches due to the strongly reduced number of trellis states especially for long packets of source data. Considering an additional error protection by channel codes the proposed VLC APP decoder can be applied as (outer) constituent decoder in an iterative joint source-channel decoding scheme. An EXIT-chart-based analysis of the resulting transmission system reveals that by using RVLCs with distance constraints

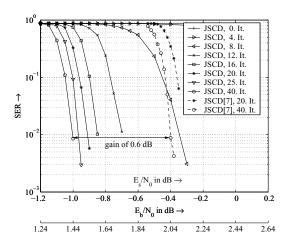


Figure 5: Simulation results for the AWGN channel, symbol error rate (SER) versus  $E_s/N_0$  and  $E_b/N_0$  (AR(1) source with a=0.9 and M=4, RVLC with  $d_f=2$ , rate-1 memory-3 RSC code, overall code rate R=0.57)

a reliable transmission with rate-1 convolutional codes is possible even at low SNR on the transmission channel.

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