SEM BLIND IDENTIFICATION OF ARMA MODELS APPLICATION TO SEISMIC DATA

Benayad Nsiri, Thierry Chonavel, Jean-Marc Boucher

ENST de Bretagne, Signal and Communications Department,
Technopôle Brest-Iroise - CS 83818 - 29238 Brest Cedex 3, FRANCE
phone: +33 2 229001070, fax: +33 2 229001012,
email: Benayad.Nsiri@enst-bretagne.fr, JM.Boucher@enst-bretagne.fr,
Thierry.Chonavel@enst-bretagne.fr

ABSTRACT

In this paper, we address blind identification of an ARMA model convolved with an impulse sequence via Maximum Likelihood (ML) approach. A Stochastic Expectation Maximization (SEM) implementation of the criterion is considered. The problem of ARMA models with long impulse response is addressed as well as the SEM initialization problem. The model estimation is performed in two steps : First, a truncated estimate of the wavelet is obtained from a SEM algorithm. Then improved wavelet estimation is achieved by fitting an ARMA model to the initial MA wavelet using the Prony algorithm. Simulation results show the significant improvement brought by this approach in situations corresponding to seismic data deconvolution.

1. INTRODUCTION

In this paper, we address the blind identification of an ARMA impulse response convolved with an impulse signal. In particular, this situation arrives in seismic deconvolution, where one tries to recover the geological structure of the underground sedimentary layers from seismic data records [1]. As usually in this kind of situation, the seismic traces are modeled as the filter output, that represents the transmitted wavelet, whose input consists in a two components Gaussian mixture: the Gaussian component with high variance models the strong reflectivity at layers interfaces [2]. Recovering this sequence from the data enables detecting the layers position in the subsurface.

The wavelet estimation is an important step of the global deconvolution procedure, because the result of deconvolution is sensitive to this estimation. In some practical interesting experiments, the wavelet is quite long [1]. In such situations, the estimation of model parameters using classical algorithms generally yields a high variance of the wavelet estimator. The Higher Order Statistics (HOS) methods can solve this problem, but they often lack robustness when smal amounts of data are available [3, 4, 5]. In this paper, we consider an alternative approach that permits to overcome this problem within the framework of classical blind seismic deconvolution techniques [6, 7].

The contribution of this paper lies in the study of the wavelet initialization : a new criterion is proposed for accurate estimation of the wavelet impulse response maximum position, which is on important practical issue for accurate wavelet estimation. Also, we apply an improved wavelet estimation [8]. More precisely, a robust Maximum Likelihood MA estimate of a truncated version of the wavelet is obtained via a SEM approach, then an ARMA model is fitted to the initial MA wavelet by means of a Prony algorithm.

Deconvolution is then achieved efficiently from this new wavelet estimator by means of the MPM approach for single trace deconvolution while in the multi-channel case this step is no longer necessary.

The paper is organized as follows: Section 2 describes the problem, while section 3 is devoted to wavelet initialization for the SEM procedure. In section 4, the improved wavelet estimation is presented. The trace deconvolution is treated in section 5. In section 6, we check in a simulation, the significant improvement brought by this approach.

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2. PROBLEM FORMULATION

The observed signal is of the form

$$y_k = \sum_{i=0}^{L} h_i r_{k-i} + w_k, \tag{1}$$

where $\mathbf{h} = (h_k)_{k=0,L}$ is the wavelet finite impulse response of length L, $\mathbf{r} = (r_k)_{k=1,N}$ is the reflectivity sequence, and $\mathbf{w} = (w_k)_{k=1,N}$ is the observation noise sequence, with variance σ_w^2. Also, we note $\mathbf{y} = (y_k)_{k=1,N}$.

The reflectivity process $\mathbf{r}$ is described by a generalized Bernoulli-Gaussian process [2], characterized by an underlying state model $\mathbf{q} = (q_k)_{k=0,N}$, with $q_k = 1$ at high reflectivity points and $q_k = 0$ at low reflectivity points. The corresponding reflectivity r_k is distributed according to a zero mean Gaussian distribution with variance σ_1^2 if $q_k = 1$ or σ_0^2 if $q_k = 0$:

$$r_k \sim \lambda \mathcal{N}(0, \sigma_1^2) + (1-\lambda)\mathcal{N}(0, \sigma_0^2), \tag{2}$$

where λ is the probability of having a reflector at a given position $(p(q_k = 1) = 1 - p(q_k = 0) = \lambda)$ and $\sigma_1^2 \gg \sigma_0^2$.

The finite impulse response of the wavelet is modeled as the ARMA systems. The observed trace for (p,q) order ARMA (ARMA(p,q)) process is given by

$$y_k + \sum_{i=1}^{p} a_i r_{k-i} = r_k + \sum_{j=0}^{q} b_j r_{k-j} \tag{3}$$

3. PARAMETER ESTIMATION

We address the blind deconvolution problem through the classical Maximum Likelihood criterion [9, 10] which leads to calculate

$$\hat{\theta}_{\mathbf{MV}} = \arg\max_{\theta} ln(p(\mathbf{y}|\theta)), \quad (4)$$

where θ is the parameter vector of interest.

The vector parameter $\theta = (\mathbf{h}, \lambda, \sigma_0^2, \sigma_1^2, \sigma_w^2)$ estimation is obtained using a standard maximum likelihood criterion, maximized thanks to an Stochastic Expactation Maximization (SEM) algorithm [6].

4. IMPROVED WAVELET ESTIMATION

In some seismic experiments the wavelet impulse response $\mathbf{h}$ is quite long. In such cases, the mean square error of the estimator is quite large. In particular, the last coefficients of $\mathbf{h}$, which have small values, are poorly estimated. For this reason, searching for a vector $\mathbf{h}$ with reduced length generally enables a good compromise between bias and variance properties of the estimator.

However, performing the deconvolution with a truncated wavelet will generate degradated performance for the reflectivity sequence.

In order to improve the deconvolution performance, we assume that the MA(L) wavelet model that has been estimated by means of the SEM procedure described in the previous section is in fact a truncated version of the true wavelet, of length $L' > L$. The value of L is not much critical. Simply, the envelope of the MA(L) impulse response should not decay too much. Since L' can be quite large in practice and, often, the wavelet has an oscillatory shape, it can be modeled efficiently as an ARMA(p,q) impulse response. In order to estimate it from the initial MA(L) wavelet, we propose to use the Prony method [11].

Initialization wavelet: It is well known that the non-minimum phase structure of the wavelet $\mathbf{h}$ makes its estimation complicated. In particular the wavelet estimation is not robust to initialization. A simulated annealing version of the SAEM algorithm [12] could be used to overcome this problem [7]. Here, we propose a deterministic procedure for initializing the MA(L) wavelet estimate. First, we initialize $\mathbf{h}$ with the vector $\mathbf{h}^{(i)}$ and we perform the deconvolution for each initialization $\mathbf{h}^{(i)}$. Now, let us note $C_i = 1$ if $\hat{\mathbf{h}}_i$ has a positive derivative at the origin, and $C_i = -1$ if it is negative. It can be checked that $\mathbf{C} = (C_i)_{i=1,k_1}$ changes of sign for the values of i corresponding to local optima of the true wavelet. A justification of this result is presented in the Appendix. Simulations on several examples show the very good practical behavior of this technique. The retained solution for the maximum position is chosen among the values i for which $\mathbf{C}$ changes of sign, by selecting the one for which the Kurtosis of the estimated reflectivity $\hat{\mathbf{r}}$ is maximum.

5. DECONVOLUTION

When $\mathbf{h}$ has been estimated, the last step consists in a deconvolution via an MPM approach that yields the final estimate of the reflectivity sequence [6].

Note that, in situations where several traces are available for the same reflectivity sequence but significantly distinct wavelets, a multi-channel version of the SEM deconvolution can be applied [13]. In this case both the parameters and the reflectivity sequence are well estimated from SEM algorithm and no additional MPM procedure needs to be employed.

6. RESULTS

In this section, we present simulation examples . Figure 1 and 2 represent the simulated reflectivity and observation ($\sigma_w^2 = 1.0^{-3}$, $\lambda = 0.07$, $\sigma_0^2 = 10^{-4}$, $\sigma_1^2 = 0.1$). The true wavelet and the function $\mathbf{C}$ introduced in the previous section are given in Figure 3, 4 and 5 for distinct wavelets. Table 1 and Figure 5 show that the maximum position is correctly recovered with the proposed procedure. Figure 6 shows the improvement brought by the MA truncated wavelet + ARMA extension modelization compared to a direct estimation of the full length wavelet. Figure 7, shows the deconvolved reflectivity estimated to gether with the original reflectivity.

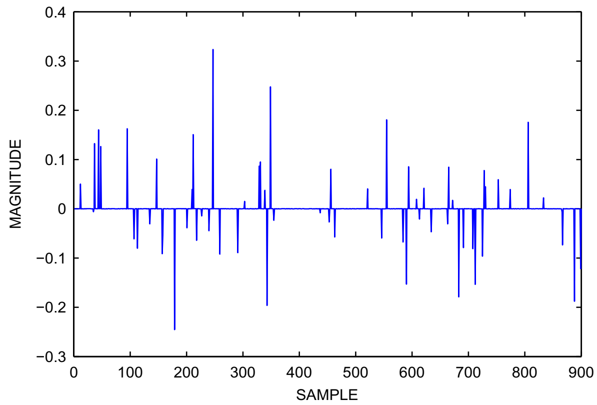

Figure 1: simulated reflectivity sequence.

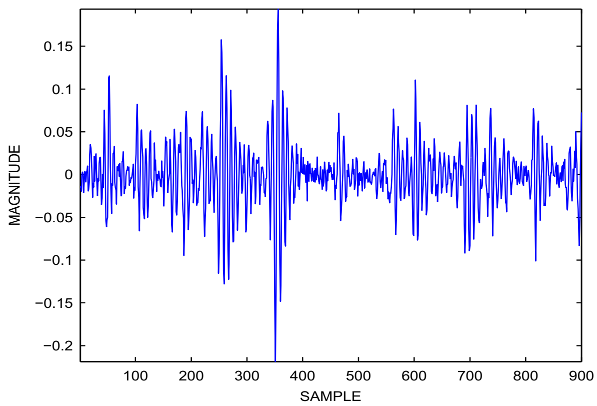

Figure 2: simulated noisy seismic data. SNR=17dB.

maximum position candidates (Fig. 4)	4	9	13	17
kurtosis	22.37	25.54	23.43	22.2

Table 1: estimated kurtosis at changes of $\mathbf{C}$.

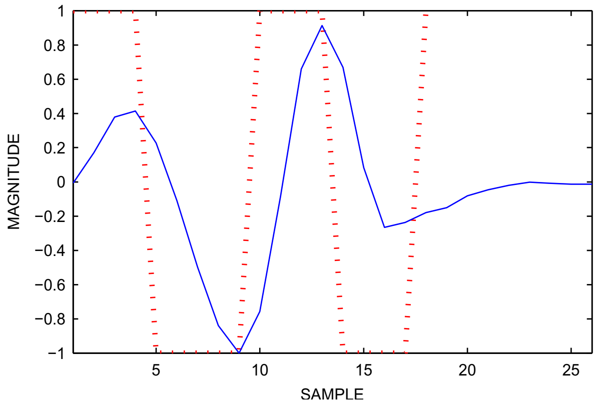

Figure 3: '-': Marmousi wavelet '...': **C**.

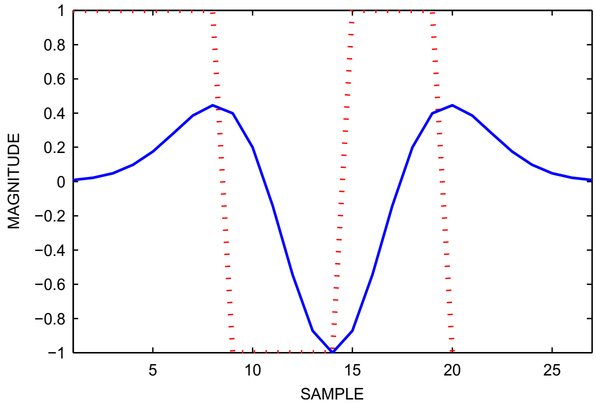

Figure 4: '-': Ricker's wavelet'...': **C**.

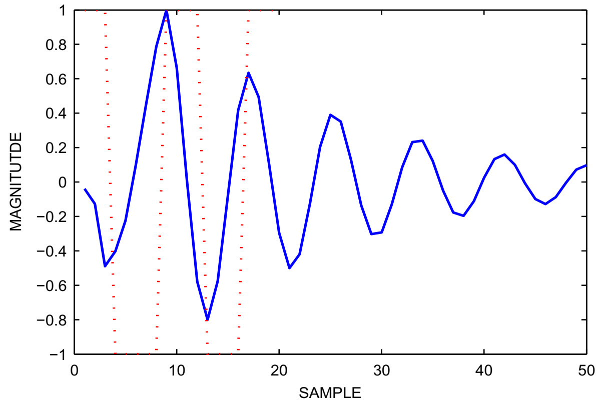

Figure 5: '-' true wavelet: '...': **C**.

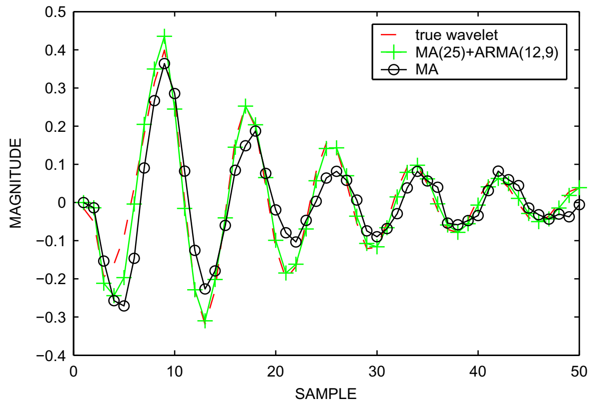

Figure 6: estimated wavelet for SNR=17dB.

7. CONCLUSION

In this paper, we have proposed a new method to solve the wavelet initialization problem in the SEM identification procedure. We have applied it to a recent method for blind identification of long ARMA models impulse response.

Appendix: SEM algorithm Initialization

In order to justify why the proposed criterion for maximum position selection works efficiently, as shown in the simulation part, let us recall, as discussed in section 4, that when the impulse response $\mathbf{h}$ is initialized with one at the k^{th} entry and zeros at other entries, then the SEM algorithm converges to a solution where the estimated $\mathbf{h}$ has its maximum at position k. In other words, we can say that the SEM algorithm looks for a solution that minimizes the norm error $\| \mathbf{y} - \mathbf{R}\mathbf{h} \|$ with a maximum constraint at position k.
Let us rephrase this idea in the time continuous domain: we are led to search for a solution h_t that achieves minimum error norm under the null derivative constraint $h'_{t_0} = 0$. In mathematical terms, we are considering the following problem:

$$\begin{cases} \min \| y_t - (r * h)_t \| \\ h'_{t_0} = 0. \end{cases} \tag{5}$$

Equivalently, problem (5) can be rewritten in the Fourier transform domain:

$$\begin{cases} \min \int_B |\hat{y}(f) - \hat{r}(f)\hat{h}(f)|^2 df \\ h'_{t_0} = \int_B (2i\pi f) e^{2i\pi f t_0} \hat{h}_{t_0}(f) df = 0, \end{cases} \tag{6}$$

where $\hat{g}(f)$ denotes the Fourier transform of $g(t)$ and B is the signals bandwidth. Using Lagrange multipliers (see for instance [14]) and introducing real and complex variations of $\hat{h}$ yields the following conditions upon the solution of problem (6), denoted $\hat{h}^{(t_0)}(f)$:

$$\begin{cases} 2\mathcal{R}e\{[\hat{r}(f)\hat{h}^{(t_0)}(f) - \hat{y}(f)]^* \hat{r}(f)\} + \lambda 2i\pi f e^{2i\pi f t_0} = 0 \\ 2i\mathcal{I}m\{[\hat{r}(f)\hat{h}^{(t_0)}(f) - \hat{y}(f)]^* \hat{r}(f)\} + i\lambda 2i\pi f e^{2i\pi f t_0} = 0. \end{cases} \tag{7}$$

Indeed, considering the functional

$$\begin{aligned} J(\hat{h}, \lambda) &= \int_B |\hat{y}(f) - \hat{r}(f)\hat{h}(f)|^2 df \\ &+ \lambda \int_B (2i\pi f) e^{2i\pi f t_0} \hat{h}_{t_0}(f) df, \end{aligned} \tag{8}$$

and denoting by $\delta \hat{h}_r(f)$ any real valued small variation of $\hat{h}(f)$, the optimality constraint $J(\hat{h}^{(t_0)} + \delta\hat{h}_r, \lambda) -$

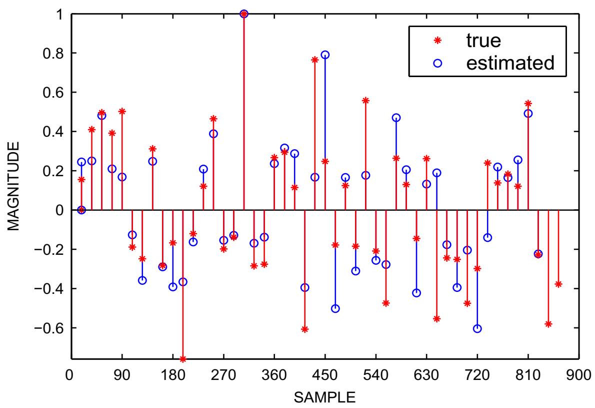

Figure 7: estimated reflectivity squence for SNR=13dB.

$J(\hat{h}^{(t_0)}, \lambda) = 0$ leads to

$$\int_B [\hat{r}(f)\hat{h}^{(t_0)}(f) - \hat{y}(f)]\hat{r}^*(f) + [\hat{r}(f)\hat{h}^{(t_0)}(f) - \hat{y}(f)]^*\hat{r}(f) + \lambda \int_B (2i\pi f)e^{2i\pi f t_0}\delta\hat{h}_r(f)df = 0, \tag{9}$$

leading thus to the first equation of (7). The second equation of (7) is derived in a similar way by considering imaginary small variation of $\hat{h}(f)$. Then, summing both equations of (7) yields

$$[\hat{r}(f)\hat{h}^{(t_0)}(f) - \hat{y}(f)]\hat{r}(f)^* = -N(2i\pi f)e^{-2i\pi f t_0}, \tag{10}$$

with $N = (1+i)\lambda/2$. Then, the solution of (6) is of the form

$$\hat{h}^{(t_0)}(f) = \frac{\hat{y}(f)}{\hat{r}(f)} - N\frac{(2i\pi f)e^{-2i\pi f t_0}}{|\hat{r}(f)|^2}. \tag{11}$$

Now, let us denote $\tilde{h}(f) = \hat{y}(f)\hat{r}^{-1}(f)$, which corresponds to the true wavelet in the noise free case. Then, inserting solution (11) in the constraint equation $h'_{t_0} = \int_B (2i\pi f)e^{2i\pi f t_0}\hat{h}_{t_0}(f)df = 0$, it comes that

$$N = \tilde{h}'_{t_0}\left(\int_B (2i\pi f)^2|\hat{r}(f)|^{-2}df\right)^{-1}. \tag{12}$$

Then, from equations (11) and (12), we get the time domain solution:

$$h_t^{(t_0)} = \tilde{h}_t - \tilde{h}'_{t_0}\left(\frac{\displaystyle\int_B (2i\pi f)e^{2i\pi f(t-t_0)}|\hat{r}(f)|^{-2}df}{\displaystyle\int_B (2i\pi f)^2|\hat{r}(f)|^{-2}df}\right). \tag{13}$$

Now, let us remark that

$$h_0^{(t_0)\prime} = \tilde{h}'_0 - \tilde{h}'_{t_0}\left(\frac{\displaystyle\int_B (2i\pi f)^2 e^{-2i\pi f t_0}|\hat{r}(f)|^{-2}df}{\displaystyle\int_B (2i\pi f)^2|\hat{r}(f)|^{-2}df}\right). \tag{14}$$

Clearly, if $\tilde{h}'_0 = 0$, then $h_0^{(t_0)\prime}$ changes of sign at $t = t_0$ if $\tilde{h}$ has a local optimum at point t_0. This shows that, provided the true wavelet has an horizontal tangent at point 0, the derivative of the estimated wavelet $h^{(t)}$ changes of sign around point $t = t_0$ (note that the term in the parenthesis is always positive). In practice, we have shown that this result remains true for wavelets $\tilde{h}$ with small tangent slopes at point 0. We explain this good practical behaviour by the fact that, in practice we are working with a discretized version of the wavelet.

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