POLARIZED SOURCE CHARACTERIZATION USING VECTOR-MUSIC

Sebastian Miron, Nicolas Le Bihan and Jérôme I. Mars

Laboratoire des Images et des Signaux, INPG
BP 46, 38402 Saint Martin d'Hères, Cedex, France
phone: +33 476 82 64 22, fax: +33 476 826 384
email: sebastian.miron@lis.inpg.fr

ABSTRACT

This paper proposes a version of MUSIC algorithm allowing characterization of polarized sources recorded on a vector-sensor (or multicomponent sensor) array. This method yields waves direction of arrival (DOA) and polarization estimates using multilinear algebra structures. It is based on separation of the observation space into *signal* and *noise* subspaces, using Higher-Order EigenValue Decomposition for 4^{th} order tensors. Recorded dataset is stored in tridimensional arrays rather than matrices to preserve its intrinsically trimodal organization: time, distance and components. Performances of the proposed algorithm are evaluated on simulated data.

1. INTRODUCTION

During a seismic acquisition campaign, elastic waves originating from artificial sources are generated. They propagate into the earth, are reflected or (and) refracted by the elastic discontinuities in the ground and are finally recorded on a sensor array (fig. 1). The study of the recorded data provides information on the geological features helping to discover, locate and evaluate gas concentrations or oil reservoirs. In order to analyze the seismic documents, sensor array processing techniques are used to find parameters describing the waves such as their direction of arrival, polarization and source magnitude. From the estimated parameters, it is possible to obtain information on layer structure, depth, etc [1]. Classically, a scalar-sensor array is used which gives

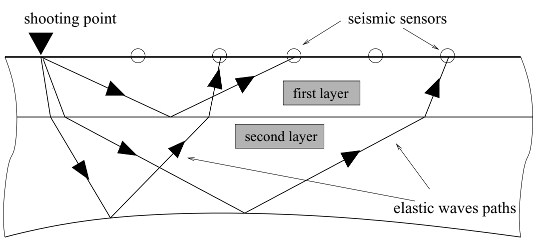


Figure 1: Seismic acquisition scheme (reflection)

a 2D signal $s(t_n, x_n)$ of size $N_x \times N_t$ (t_n is the time recording dimension and x_n is the distance dimension (array aperture)). Tendency nowadays is to replace the scalar-sensor arrays with vector-sensor ones, allowing better characterization of the layers because of the polarization dimension added to the detection process. Multicomponent datasets recorded this way demand new processing tools adapted to their trimodal intrinsic structure. Array processing techniques using vector-sensors have been developed, mainly in electromagnetics [2, 3], the majority directly derived from scalar-sensor array processing. They use long vectors formed by the concatenation of the N_c components. The originality of our method consists in keeping multidimensional structures for data organization and processing, that are more adapted to the nature of polarized signals. The proposed method is a version of MUSIC algorithm adapted to this multilinear structure. Subspace method for polarized signal separation was first proposed in [4]. Here, we propose the extension to the vectorial case of high resolution array processing technique.

In section 2 the model of a polarized wave on a vector-sensor array is presented. Section 3 illustrates the "interspectral tensor" notion and in section 4 the vector - MUSIC estimator is developed. The performances of the proposed algorithm are evaluated in section 5.

2. POLARIZED SOURCE MODEL

Some assumptions are necessary for application of the vector-MUSIC algorithm.

A1 Consider the case of a linear, uniform scalar-sensor array recording farfield seismic waves propagated in an isotropic, homogeneous medium.

A2 The signal is made of K sources (K is known). If K is not known, it can be estimated from the eigenvalues variation[5].

A3 Sources $(s_1, s_2, ..., s_K)$ are statistically decorrelated, spatially coherent and they are all confined in the array plane.

A4 Sources are considered as centered unknown deterministic processes.

A5 The noise is supposed Gaussian, spatially white and centered.

A6 We suppose that the additive noise on the sensors is not polarized (the cross-covariance matrix for noise on a vector-sensor components is diagonal).

A7 Sources polarization is constant in time and along the antenna (temporal and spatial stationarity).

A8 Sources have different polarizations.

We focus on the case of a two components ($N_c = 2$) vector-sensor array (the formalism remains the same in the case of an arbitrary N_c). The energy repartition of a polarized source s_k on the antenna is given by the *outer product*[1] of a vector containing information on the source behavior along the antenna $\mathbf{a}_k(\theta_k) = [1,\ e^{-j\theta_k}, \dots, e^{-j(N_x-1)\theta_k}]$ and a polarization vector $\mathbf{p}_k(\rho_k, \varphi_k) = \frac{1}{\sqrt{\rho_k^2+1}}\left[1,\ \rho_k e^{j\varphi_k}\right]$ yielding the energy repartition between the components of the vector-

[1]The *outer product* $\mathscr{A} \circ \mathscr{B}$ of a tensor $\mathscr{A} \in \mathbb{C}^{I_1 \times I_2}$ and a tensor $\mathscr{B} \in \mathbb{C}^{J_1 \times J_2}$, is defined by: $(\mathscr{A} \circ \mathscr{B})_{i_1 i_2 j_1 j_2} = a_{i_1 i_2} b_{j_1 j_2}$, for all values of indices.

sensor:

$$\mathbf{A}_k(\theta_k,\rho_k,\varphi_k) = \mathbf{a}_k(\theta_k)\circ\mathbf{p}_k(\rho_k,\varphi_k)$$

In this relation, θ_k is the inter-sensor phase shift corresponding to source k and ρ_k, φ_k are the amplitude ratio and the phase shift respectively, between the second and the first sensor component. We have to estimate θ_k, ρ_k and φ_k to characterize a source on a $N_c = 2$ components vector-sensor array. Inter-sensor phase-shift θ_k will be used to characterize the direction of arrival (DOA) of the source on the array rather than the incidence angle α_k (because it is independent of physical quantities such as inter-sensors distance Δx and wave propagation velocity v_k). The incidence angle α_k can then be found using the following relation: $\theta_k = 2\pi\nu\frac{\Delta x \sin\alpha_k}{v_k}$.

In frequency domain, output of the vector-sensor array is given by a $N_x \times N_c$ matrix in which every column corresponds to a component of the vector-sensor array:

$$\mathbf{X}(\nu) = \begin{pmatrix} x_{11}(\nu) & x_{12}(\nu) \\ \vdots & \vdots \\ x_{N_x 1}(\nu) & x_{N_x 2}(\nu) \end{pmatrix}$$

When assumptions **A7** and **A8** are considered, at a given frequency ν_0, $\mathbf{X}(\nu_0)$ can be written as a linear combination of K unknown deterministic signals s_k with additive white noise. For simplification, argument ν_0 will be omitted, so that $\mathbf{X}(\nu_0) = \mathbf{X}$.

$$\begin{pmatrix} x_{11} & x_{12} \\ \vdots & \vdots \\ x_{N_x 1} & x_{N_x 2} \end{pmatrix} = \mathscr{A} \bullet_2 \begin{pmatrix} s_1 \\ \vdots \\ s_K \end{pmatrix}^T + \begin{pmatrix} n_{11} & n_{12} \\ \vdots & \vdots \\ n_{N_x 1} & n_{N_x 2} \end{pmatrix} \quad (1)$$

The operator $\bullet_2$ represents the 2^{nd} *mode product*[2] of a tensor by a matrix. $\mathscr{A}$ is a 3^{rd} order tensor regrouping all the information on the sources behavior on the vector-sensor array. Another way of writing (1) is:

$$\mathbf{X} = \sum_{k=1}^{K} \mathbf{A}_k s_k + \mathbf{N} \quad (2)$$

with $\mathbf{N}$ the additive noise matrix.

3. INTERSPECTRAL TENSOR

In order to characterize the incident field, second order statistics are considered through a "interspectral tensor". Interspectral tensor is a 4^{th}-order complex tensor of size $N_x \times N_c \times N_x \times N_c$ defined as the second order auto-moments and cross-moments between all the components of all sensors:

$$\mathscr{T} = \xi\{\mathbf{X}\circ\mathbf{X}^*\} \quad (3)$$

in which $\xi\{.\}$ is the mathematical expectation operator. If we replace (2) in (3) and using **A3**, **A4**, **A7** and **A8**, $\mathscr{T}$ can be written as:

$$\mathscr{T} = \sum_{k=1}^{K} \sigma_k^2 \mathbf{A_k}\circ\mathbf{A_k}^* + \mathscr{N} \quad (4)$$

in which σ_k^2 are the different powers of the K sources and $\mathscr{N} = \xi\{\mathbf{N}\circ\mathbf{N}^*\}$ is a 4^{th} order tensor containing the second order noise statistics. As interspectral tensor presents a higher-order hermitian symmetry ($t_{i_1 i_2 i_3 i_4} = t^*_{i_3 i_4 i_1 i_2}$), [6] it can be decomposed in eigenelements by means of Higher-Order tensors EigenValue Decomposition (HOEVD) [6]. Interspectral tensor can thus be written as:

$$\mathscr{T} = \sum_{p=1}^{P} \lambda_p \mathbf{U_p}\circ\mathbf{U_p}^* \quad (5)$$

where $P = N_x N_c$, λ_p are real eigenvalues and $\mathbf{U_p} \in \mathbb{C}^{N_x \times N_c}$ are P mutually orthonormal eigentensors. Two tensors are *mutually orthonormal* if their *scalar product*[3] equals 0, and their *Frobenius norm*[4] is unity. To obtain the HOEVD of interspectral tensor, we apply the EVD (EigenValue Decomposition) on standard matrix unfoldings [6]. For a fourth order tensor $\mathscr{T} \in \mathbb{C}^{N_x \times N_c \times N_x \times N_c}$, there are four different ways to unfold it in order to obtain a square matrix. In our particular case $t_{i_1 i_2 i_3 i_4} = t^*_{i_3 i_4 i_1 i_2}$, only two of these unfolding techniques yield Hermitian symmetric matrices. So, two linear mappings between the vectorial space of $\mathscr{T}$-like tensors and the vectorial space of Hermitian matrices $\mathbf{G}$ of size $N_x N_c \times N_x N_c$ can be defined:

$$g1_{((i_2-1)Nx+i_1,\,(i_4-1)Nx+i_3)} = t_{(i_1,i_2,i_3,i_4)}$$

$$g2_{((i_1-1)Nc+i_2,\,(i_3-1)Nc+i_4)} = t_{(i_1,i_2,i_3,i_4)}$$

for all indices i_1, i_2, i_3, i_4. In these expressions $g_{(i,j)}$ are the entries of $\mathbf{G}$. The two decompositions $g1$ and $g2$ are equivalent, yielding the same eigentensors with HOEVD. This is a natural result, the HOEVD of a pair-wise symmetric tensor being unique. The 2^{nd} order eigentensors are obtained by reintroduction of the tensor notation for the resulting matrices in the decomposition (inverse operation of matrix unfolding).

4. VECTOR-MUSIC ESTIMATOR

By identification of (4) with (5), we associate the first K eigenvalues to the signal part of the observation and the other $P-K$ eigenvalues to the noise part. We build the noise subspace projector using the last $P-K$ eigentensors:

$$\mathscr{P}_{\mathscr{N}} = \sum_{p=K+1}^{P} \mathbf{U_p}\circ\mathbf{U_p}^*$$

The steering-tensor $\mathbf{M}(\theta,\rho,\varphi)$ is generated:

$$\mathbf{M}(\theta,\rho,\varphi) = \mathbf{a}(\theta)\circ\mathbf{p}(\rho,\varphi) \quad (6)$$

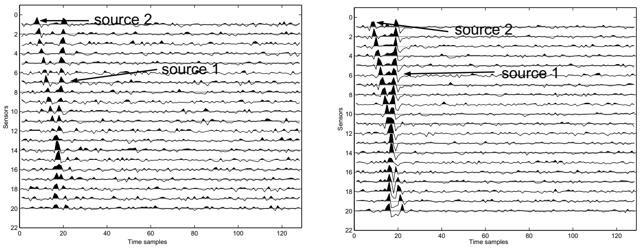
Expanding (6), we get:

$$\mathbf{M}(\theta,\rho,\varphi) = \frac{1}{\sqrt{N_x(1+\rho^2)}} \begin{bmatrix} 1 & \rho e^{j\varphi} \\ e^{-j\theta} & \rho e^{j(\varphi-\theta)} \\ \vdots & \vdots \\ e^{-j(N_x-1)\theta} & \rho e^{j(\varphi-(N_x-1)\theta)} \end{bmatrix}$$

[2] The *2-mode product* of a tensor $\mathscr{A} \in \mathbb{C}^{I_1 \times I_2 \times I_3}$ and a matrix $\mathbf{U} \in \mathbb{C}^{J_2 \times I_2}$, denoted by $\mathscr{A} \bullet_2 \mathbf{U}$, is an $(I_1 \times J_2 \times I_3)$ tensor given by: $(\mathscr{A} \bullet_n \mathbf{U})_{i_1 j_2 i_3} = \sum_{i_2} a_{i_1 i_2 i_3} u_{j_2 i_2}$.

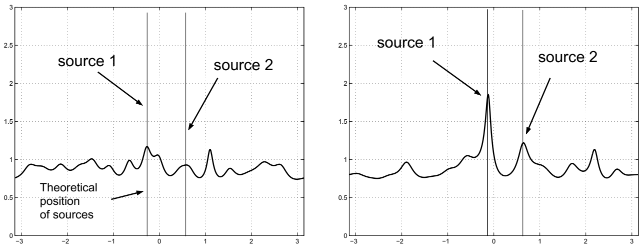
[3] The *scalar product* $<\mathscr{A},\mathscr{B}>$ of two tensors $\mathscr{A},\mathscr{B} \in \mathbb{C}^{I_1 \times I_2}$ is defined as: $<\mathscr{A},\mathscr{B}> = \sum_{i_1,i_2} b^*_{i_1 i_2} a_{i_1 i_2}$.

[4] The *Frobenius norm* of a tensor $\mathscr{A}$ is given by $\|\mathscr{A}\| = \sqrt{<A,A>}$.



a. First component   b. Second component

Figure 2: Two component seismic section



θ (rad) θ (rad)

a. Scalar-MUSIC (1stC)   b. Scalar-MUSIC (2ndC)

Figure 3: Scalar-MUSIC on each component

Vector-MUSIC estimator VM is then computed by projection of the steering tensor $\mathbf{M}$ on the noise subspace:

$$VM(\theta,\rho,\varphi) = \frac{1}{\| < \mathscr{P}_{\mathscr{N}}, \mathbf{M}(\theta,\rho,\varphi) >_{i_1 i_2} \|} \quad (7)$$

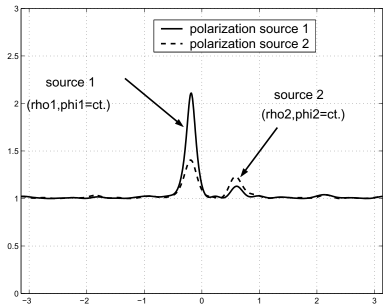
in which $< \mathscr{P}_{\mathscr{N}}, \mathbf{M} >_{i_1 i_2}$ is the inner product over the first two indices and $\|.\|$ is the Frobenius norm.

MUSIC estimator values are stored in a multidimensional table of 3^{rd} order. So, we can detect $N_x N_c$ sources, N_c times more than in monocomponent-sensor case. In the general case of an arbitrary number N_c of components, the number of parameters to estimate is $2N_c - 1$. In the following section, this algorithm will be evaluated on synthetic data in order to characterize and separate seismic sources in DOA-polarization domain.

5. SIMULATION RESULTS

To illustrate performances of vector-MUSIC algorithm, we have considered a scenario with two seismic sources recorded on a two-component vector-sensor array. The polarization parameters simulated are: $\rho_1 = 2$, $\varphi_1 = -80°$ $(-1.4$ rad) for the first source and $\rho_2 = 3$, $\varphi_2 = 60°$ (1.04 rad) for the second. The intersensor phase-shift corresponding to the DOAs of the sources are $\theta_1 = -0.18$ and $\theta_2 = 0.58$. The simulations have been performed with an array of $N_x = 20$ vector-sensors recording $N_t = 128$ time samples each. The two components of the original data are presented in fig. 2 (a. and b.). Gaussian noise has been added to a signal to noise ratio $SNR_1 = -7$ dB for the first component and $SNR_2 = 12$ dB for the second one. In order to decorrelate the two sources in the interspectral tensor, a frequency smoothing technique [7] has been used with averaging over 5 frequency channels. This technique induces bias in θ and φ estimates because of the dependency on frequency of these quantities. At the same time, it reinforces the detection of sources having an intersensor phase-shift close to zero, as it is the case for source 1. Vector-MUSIC estimator has been calculated as well as the scalar-MUSIC [8] estimate on each of the components independently. For θ we have used a computation step of 0.01 in the interval $[-\pi\ \pi]$, $\rho \in [0.1 : 0.1 : 10]$ and $\varphi \in [-\pi : 0.1 : \pi]$. In fig.3 (a and b) the results of the scalar version of MUSIC estimator applied to each one of the components separately are presented. This method is commonly used in seismic vector-sensor array data processing.

We see that on the first component the two sources are not clearly visible (fig . 3.a). On the second component (fig. 3.b), the detection is more accurate but there are still some false alarms that could mask the real detection. Furthermore, the estimates for θ_1 and θ_2 are slightly different on the two components, mainly because of frequency smoothing technique. Fig. 4 shows vector-MUSIC estimator for two sets of polarization parameters. The solid line corresponds to $\rho_1 = 2$, $\varphi_1 = -80°$ (the polarization parameters of the first source) and the dashed line corresponds to the second source polarization parameters ($\rho_2 = 3$, $\varphi_2 = 60°$). We can observe (fig.4) that both detections are a lot smoother than in scalar case (the secondary lobs are much more attenuated). It means that this estimator is more robust to source correlation than the scalar one, allowing better results for the same averaging technique. This improvement can be explained by the fact that vector-MUSIC algorithm takes into account the coherent relationship existing between the components of vector-sensors.



θ (rad)

Figure 4: Vector-MUSIC

Polarization parameters ρ and φ can also be estimated with this method. For two values $\theta_1 = -0.18$ and $\theta_2 = 0.58$ corresponding to sources DOAs, we represent the estimator values in the polarization $(\rho\ ;\varphi)$ plane (fig. 5 and fig. 6). Simulated parameters found are slightly biased due to the frequency averaging. Next, the separation powers of these two algorithms are compared on a synthetic example. We consider the same seismic sources with the same polarization parameters as before $\rho_1 = 2$, $\varphi_1 = -80°$ $\rho_2 = 3$, $\varphi_2 = 60°$ but with very close DOAs (fig. 7 a. and b.) ($\theta_1 = 0.3$ rad and $\theta_2 = 0.5$). In this case, averaging over five frequency channels is not an accurate choice because of the important value of the resulting bias. This is why a mix smoothing technique over three channels in frequency and three sensors along the antenna (spatial smoothing) is preferred. The results of MUSIC algorithm applied to each component independently is shown in fig. 8 (a and b). Theoretical position of the detec-

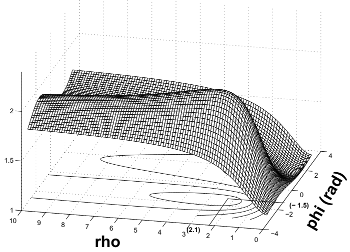


Figure 5: Vector-MUSIC for $\theta = \theta_1(-0.18\, rad)$

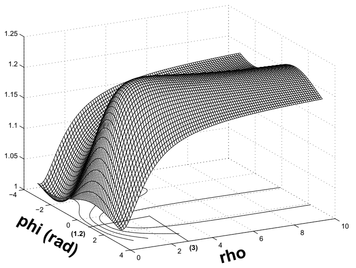


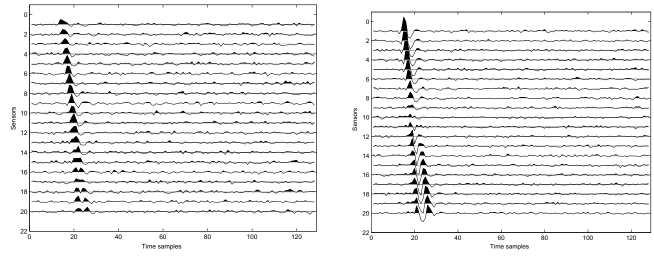
Figure 6: Vector-MUSIC for $\theta = \theta_2(0.58\, rad)$

tion peaks has been marked by a vertical line for each source. One can see that on the first component, scalar-MUSIC estimator presents only one detection peak corresponding to an average direction of arrival of the two sources (8.a). On the second one the algorithm detects only the first source (the one that is closer to normal incidence). The multicomponent version still performs a correct detection (fig. 9) of both sources.

The explanation is on one side the use of coherent information between the two components, and on the other side the isolation of each source on one detection curve corresponding to its polarization parameters. This way, the superposition of detection lobes is avoided and detection is possible.

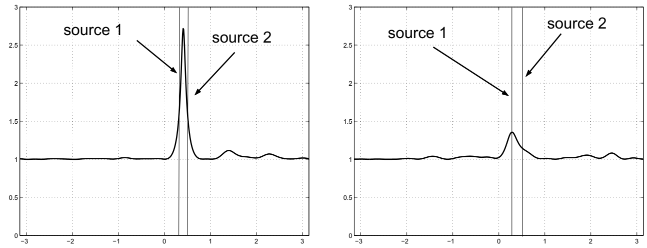
6. CONCLUSIONS

Polarization provides an additional dimension to source parameters space. Taking it into account, we have proposed a new model and an eigenstructured-based algorithm using the intrinsic data structure and enabled characterization of N_c times more sources than in monocomponent array case. Simulations were carried out to evaluate the performance of the proposed method. We have shown that polarization diversity enhances the performance of the direction finding system provided that a tensor-model is used. Both accuracy and resolving power are improved. Vector-MUSIC algorithm is less sensible to the correlation of the sources than its scalar version, providing at the same time a better separation power. This algorithm performs an averaging between the N_c components, so, it should be used only when the *SNR* is comparable on all of them. Using a very noisy component could strongly deteriorate the estimation performance of the algorithm.



a. First component b. Second component

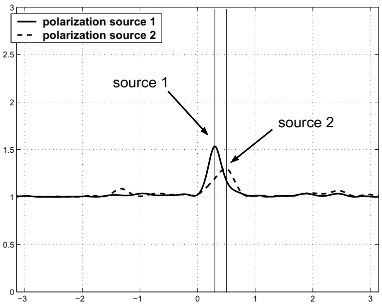
Figure 7: Two component seismic section



θ (rad) θ (rad)

a. Scalar-MUSIC (1stC) b. Scalar-MUSIC (2ndC)

Figure 8: Scalar-MUSIC on each component



θ (rad)

Figure 9: Vector-MUSIC for very close DOAs

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