An Approach to Transform Domain Variable Step-size LMS Adaptive Filter

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ABSTRACT

This paper introduces a new transform domain variable step-size LMS algorithm, in order to deal with highly correlated inputs. In the algorithm the step-size rule is based on the weighting coefficients bias to variance trade-off. Performance of this method is promising, especially in nonstationary environment with abrupt change of unknown system. Computer simulation results are provided to support the proposed implementation of the new TDVSS LMS.

1 Introduction

The least mean square (LMS) algorithm [1, 2, 3] is one of the most popular algorithms in adaptive signal processing. Due to its simplicity and robustness this algorithm has been the focus of much study, leading to its implementation in many applications. Many different modifications were proposed to improve performance of the LMS, and a large number of results on its steady state misadjustments and its tracking ability has been obtained [1, 2, 3].

Unfortunately, its convergence rate is highly dependent on the conditioning of autocorrelation matrix of its inputs. When inputs are highly correlated, convergence rate degrades radically. In order to cope with this problem, transform domain algorithms have been developed (TDLMS)[4, 5]. In the case of the TDLMS, the input signal is transformed by the use of an orthogonal transform and the filter coefficients are updated independently.

Variable step-size LMS (VSS LMS) algorithms are applied [6], with the intention of decreasing misadjustment and to maximize convergence rate, . To design an appropriate variable step-size method, one should intuitively use a larger step-size when the estimate is far from the optimum and a smaller step-size as it approaches the optimum.

In this paper, we will combine the transform domain technique with a new variable step-size LMS (NVSS LMS) algorithm, that is based on the weighting coefficients bias/variance trade-off [7, 8].

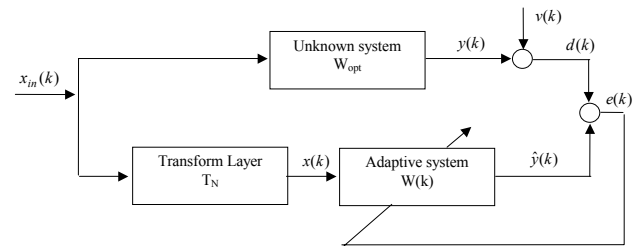In Section 2, we explain basic characteristics of existing transform domain LMS algorithms, that we used to evaluate our results. Analytical form of our approach to transform domain variable step-size LMS algorithm is presented in Section 3. Then we demonstrate performances of the proposed approach by using the appropriate simulations.

Figure 1: Block diagram of system identification using the transform domain adaptive filtering.

2 TDLMS Adaptive Filters With Variable Step-Size

The transform domain concept will be presented by using the adaptive system identification problem. The block diagram is shown in figure 1, where the block denoted by T_k represents the transform matrices applied to the block of the input signal $\mathbf{X}_{in}(k)$. In literature, many different transformations are proposed: the discrete cosine transform (DCT), discrete Fourier transform (DFT) or the Harley transform (DHT). Here we used the DCT. $\mathbf{X}(k)$ is the transformed signal, $v(k)$ is output noise, $d(k)$ and $e(k)$ are reference signal and the error signal, respectively. The output signals of the adaptive filter and the unknown system are $\hat{y}(k)$ and $y(k)$, respectively. These signals are computed by:

$$y(k) = \mathbf{W}_{opt}^T(k)\mathbf{X}(k), \quad \hat{y}(k) = \mathbf{W}^T(k)\mathbf{X}(k),$$

$$(1)$$

$$d(k) = y(k) + v(k), \quad (2)$$

$$e(k) = d(k) - \hat{y}(k), \quad (3)$$

Updating of each TDLMS adaptive filter coefficient is described by:

$$W_i(k+1) = W_i(k) + \frac{\mu}{\epsilon + \sigma_i^2}e(k)x(k-i), \qquad (4)$$

where W_i is the ith coefficient of the adaptive filter, $\sigma_i^2(k)$ is the power estimate of the ith transform coefficient $x_i(k)$, and ϵ is a small constant that eliminates the overflow when the values of $\sigma_i^2(k)$ are small [4]. The value of $\sigma_i^2(k)$ is usually computed by:

$$\sigma_i^2(k) = \beta\sigma_i^2(k-1) + (1-\beta)|x_i(k)|^2, \qquad (5)$$

where $\beta \in [0,1]$ is forgetting factor.

Although many different algorithms deal with variable μ, we have decided to compare our algorithm with DCT-LMS [5] and TDVSS, because those algorithms have shown best performance so far [6].

In DCT-LMS [5], the following relations are used:

$$W_i(k+1) = W_i(k) + \mu_i(k)e(k)x(k-i), \qquad (6)$$

$$\mu_i(k+1) = \beta\mu_i(k) + \gamma(1-\beta)\left(\frac{1}{\epsilon + \frac{1}{M}\mathbf{X}_i(k)^T\mathbf{X}_i(k)}\right), \qquad (7)$$

where $\mathbf{X}_i(k) = [x_i(k), x_i(k-1), ..., x_i(k-M+1)]^T$ is the vector of the past M values of the ith transform coefficient, and $\beta \in [0,1], \gamma \in [0,1]$ and $0 < \epsilon \leq 1$ are some constant parameters.

As for TDVSS [6], the step-size is obtained from:

$$A(k) = \alpha\mu(k) + \frac{\gamma}{L}\sum_{i=k-L+1}^{k} e(i)^2, \qquad (8)$$

$$\mu(k+1) = \begin{cases} A(k), & \text{if } k = nL, \text{and} \\ & \quad A(k) \in (\mu_{min}, \mu_{max}) \\ \mu_{max}, & \text{if } k = nL, \text{and } A(k) \geq \mu_{max} \\ \mu_{min}, & \text{if } k = nL, \text{and } A(k) \leq \mu_{min} \\ \mu(k), & \text{if } k \neq nL \end{cases} \qquad (9)$$

where $n = 1, 2, 3, ...,$ and μ_{min}, μ_{max} are the minimal and maximal values of step-size, respectively.

$$\mu_i(k+1) = \frac{\mu(k)}{\epsilon + \sigma_i^2(k)}, \qquad (10)$$

The weighting coefficients of TDVSS are computed by using relation (6). This algorithm has outstanding performance, high convergence rate and comparable complexity, although with many adjustable parameters [6].

3   New TDVSS LMS Adaptive Algorithm

In order to derive our VSS LMS algorithm, let us apply the presented combining method to two standard LMS adaptive algorithms with different step sizes. Let the first one have the maximal step size value μ_{max} which does not violate the algorithm convergence condition [1, 3], while the second one is characterized in each iteration by the variable step size $\mu_i(k)$.

The analysis from [7, 8] may now be applied to these two algorithms. After choosing the better algorithm, based on the proposed criterion [7, 8], both algorithms will, in each iteration, take the set of better values of the weighting coefficients as a starting point for the next iteration.

Thus, according to [1, 2, 3], denote the ith weighting coefficient at an instant k by $W_i^p(k)$, $W_i^q(k)$, for the first and the second LMS algorithm, respectively. Weighting coefficients for these algorithms would be calculated, in each iteration, according to the relations

$$W_i^p(k+1) = W_b + 2\mu_{max}e(k)x(k-i), \qquad (11)$$

$$W_i^q(k+1) = W_b + 2\mu_i(k)e(k)x(k-i), \qquad (12)$$

where W_b is the coefficient value selected as the best choice from the previous iteration.

As the criterion for choosing better weighting coefficient and the step-size value, at an instant $k+1$, we get the inequality:

$$a_i(k)(\sqrt{\mu_{max}} - \sqrt{\mu_i(k)}) < \frac{(\kappa+1)}{2}\sigma_n, \qquad (13)$$

where $a_i(k) = |e(k)x(k-i)|$ represents half of the estimated ith coordinate of the performance criterion gradient for the LMS algorithm, [7, 8, 10].

The best bias-to-variance ratio is obtained for the particular step-size that turns (13) into an equality, [7, 8, 10]. By solving the inequality (13), we arrive at the relation for the step size calculation for the $i-th$ weighting coefficient in the $k-th$ iteration:

$$\sqrt{\mu_i(k)} = \begin{cases} \sqrt{\mu_{max}} - \frac{(\kappa+1)\sigma_n}{2a_i(k)} & ,\mu_i(k) > \mu_{min} \\ \sqrt{\mu_{min}} & ,\mu_i(k) \leq \mu_{min} \end{cases}. \qquad (14)$$

It may be shown that for practical applications of (14) one may use:

$$\sqrt{\mu_i(k)} = \begin{cases} \sqrt{\mu_{max}} - \frac{(\kappa+1)\sigma_n}{2a_i(k)} & ,a_i(k) > C \\ \sqrt{\mu_{min}} & ,a_i(k) \leq C \end{cases}, \qquad (15)$$

where $C = \frac{(\kappa+1)\sigma_n}{2(\sqrt{\mu_{max}} - \sqrt{\mu_{min}})}$.

Analysis of (15) leads to the idea of avoiding advance calculation of the weighting coefficients, i.e. the parallel LMS algorithms. Instead, based on the calculation of the parameter $a_i(k)$ and taking into account (15), we determine the value $\mu_i(k)$, i.e. a more appropriate step size of the LMS algorithm for each weighting coefficient at each instant of time. Obviously, this is just the idea of a VSS LMS algorithm, [7].

Note that the only unknown value in (15) is the noise variance σ_n^2. It should either be apriori known or estimated at the beginning of the adaptation procedure

if the input noise is stationary. In our simulations we have estimated the value of σ_n^2 by the weighting coefficient variance σ_i^2, where it is obtained by, [1, 3, 7, 8, 10]:

$$\sigma_i = \frac{median(|W_i(k) - W_i(k-1)|)}{0.6745\sqrt{2}}, \qquad (16)$$

for $k = 1, 2, .., L$. The above relation produces good estimates for all stationary cases, as well as for the indicated nonstationary ones, including abrupt coefficient changes.

Note that any other estimation of σ_n^2 is valid for the algorithm. Namely, although it somewhat affects the precision of step size choice, this error does not significantly affect the overall algorithm performance, as shown in [11].

Adjustable step-size (15) and the relation for updating weighting coefficients:

$$W_i(k+1) = W_i(k) + \frac{\mu_i(k)}{\epsilon + \sigma_i^2} e(k)x(k-i), \qquad (17)$$

completes new TDVSS (NTDVSS) adaptive algorithm.

4   Simulation Results

For these simulations the input signal was

$x_{in}(k) = 1.79x_{in}(k-1) - 1.85x_{in}(k-2) + 1.27x_{in}(k-3) - 0.41x_{in}(k-4) + z(k),$

where $z(k)$ is white Gaussian random signal with zero mean and variance $\sigma_z^2 = 0.14817$. In the presented simulations, the reference signal $d(k)$ is also corrupted white Gaussian random noise $v(k)$ with zero mean and variance $\sigma_v^2 = 0.0001$. The eigenvalue spread ratio of the input signal is 944.67. The signal to noise ratio at the output of the unknown system was 50 dB. The new algorithm is compared with the DCT-LMS and TDVSS [5, 6] algorithms. In order to make the algorithms comparison more transparent, unknown system is the same as in [6]. Presented results were obtained by averaging 250 Monte-Carlo simulations of the algorithms. The values of parameters for DCT-LMS, TDVSS (recommended in [6]) and NTDVSS algorithm are given in Table 1.

In Figure 2 we can see the MSE behavior of all three algorithms for time-invariante unknown system. As it can be seen form figure 1, TDVSS and NTDVSS have similar performance, but better than DCT-LMS.

Performance of considered algorithms in nonstationary environment with abrupt change of unknown system is presented on figure 3. Abrupt change is made by multiplying impulse response coefficients of unknown system with –1 at the middle of iteration process. As it may be observed, after abrupt change, new TDVSS LMS algorithm has better MSE speed of convergence than DCT-LMS and TDVSS. The reason for that is more adaptive step-size behavior of new TDVSS LMS algorithm.
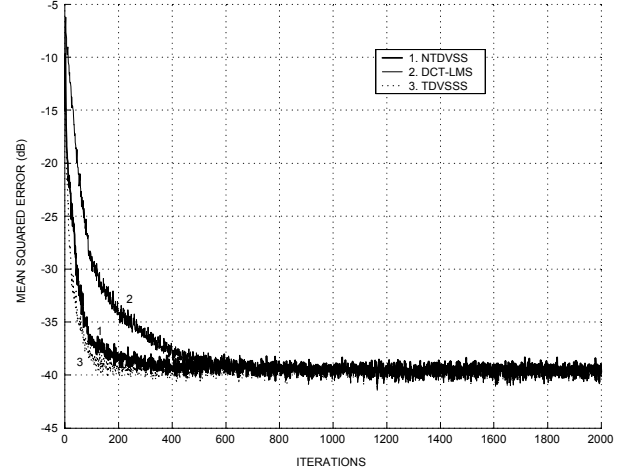

Figure 2: Comparison of MSE for considered algorithms.

DCT-LMS		
$M = 10$	$\gamma = 2 \cdot 10^{-3}$	
$\beta = 0.9985$	$\varepsilon = 8 \cdot 10^{-4}$	
TDVSS		
$\mu_{max} = 5 \cdot 10^{-2}$	$\alpha = 0.99$	
$\mu_{min} = 4.7 \cdot 10^{-3}$	$\gamma = 10^{-3}$	
$L = 10$ $\beta = 0.9985$	$\varepsilon = 2.5 \cdot 10^{-2}$	
NTDVSS		
$\mu_{max} = 0.1$	$\mu_{min} = \mu_{max}/10$	$\kappa = 1.8$

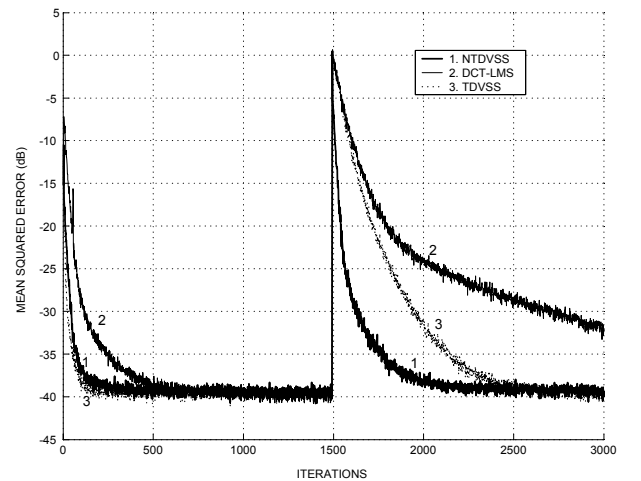Table 1: Parameters of the compared algorithms

Figure 3: Comparison of MSE for considered algorithms, with abrupt change of unknown system.

5 Conclusion

A new transform domain variable step-size LMS algorithm is introduced here. Using transform domain concept and appropriate step-size adjustment, we have shown that this algorithm can successfully deal with highly correlated inputs, without considerable increase in the computational complexity. Also, this algorithm has less adjustable parameters than the compared algorithms. Due to more adaptive step-size behavior of our method, the proposed algorithm has improved convergence rate after abrupt change of unknown system.

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