ROBUST TIME-FREQUENCY ANALYSIS: DEFINITIONS AND REALIZATIONS

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ABSTRACT

In signal analysis the standard linear signal transforms follow as the maximum likelihood solutions for the Gaussian additive noise environment. In some applications signals are corrupted by a heavy-tailed kind of noise. The resulting noise in quadratic and higher order time-frequency distributions (TFD) is inherently a mixture of the Gaussian input noise and an impulse noise component, due to the higher order functions of a signal. For these forms of input or resulting noise the robust signal transforms and TFDs can outperform standard ones. Several forms of the robust TFDs are considered: iterative based, median based, and $L-$estimation based representations. In addition to the overview of these recently introduced forms, in this paper we propose a recursive on-line realization of the robust TFDs based on the robust STFT. It is shown that higher order TFDs, developed by using the robust STFT in the initial stage, and calculated in recursive manner, produce significantly better results than their counterparts based on the local autocorrelation function.

1. INTRODUCTION

Standard signal transforms and time-frequency distributions (TFD) are very sensitive to an impulse noise influence. They can be considered as the maximum likelihood estimates, resulting from corresponding minimization problems for Gaussian noise environment. In the quadratic and higher order TFDs resulting noise can be consider as a mixture of the Gaussian and impulse noise, even for pure Gaussian input noise. These facts motivated rewriting signal transforms and TFDs within Huber robust statistics concept resulting in the robust M-, median filter and L-filter forms of the corresponding transforms. Details on these transforms realization can be found in [1, 2, 3]. Two applications of these transforms are highlighted: instantaneous frequency (IF) estimation and design of robust filters in frequency domain [4, 5]. Note that all these transforms are calculationally demanding with iterative or sorting procedures performed for each point of the TF plane. Another drawback of the mentioned higher order TFDs is the fact that they are more sensitive to the impulse noise influence than the robust STFT. At the same time the robust STFT has a weak TF resolution.

In order to overcome the mentioned problems, higher order TFDs, calculated in recursive manner from the robust STFT, are presented. Also, a recursive on-line realization of the robust STFT is proposed. In this way we get highly concentrated TFDs of non-linear FM signals, calculated in an efficient manner and robust to the impulse noise influence.

The paper is organized as follows. Brief review of the robust TFDs is given in Section II. Recursive realization of the robust STFT is given in Section III. The new forms of the robust TFDs are presented in Section IV. Comparison of the presented transforms in the IF estimation is done in Section V.

2. ROBUST TRANSFORMS

General unitary signal transforms can be obtained as solution of

$$S(k) = \arg\min_{m} \sum_{n=0}^{N-1} F(s(n)\varphi_k(n) - m), \qquad (1)$$

where $F(.)$ is the loss function while $\varphi_k(n)$, $k \in [0, N)$ are the basis functions [2]. In the same way, the generalized TFDs can be written as

$$TF(n,k) = \arg\min_{m} \sum_{l=0}^{N-1} F(\gamma(n,l,k) - m), \qquad (2)$$

where

$$\gamma(n,l,k) = x(n+l)\exp(-j2\pi lk/N),$$
$$\gamma(n,l,k) = x(n+l)x^*(n-l)\exp(-j4\pi lk/N) \qquad (3)$$

produce the STFT and the Wigner distribution (WD), respectively. For other forms of $\gamma(n,l,k)$ various versions of distributions from the Cohen class (CD), L-Wigner distribution (LWD), and Polynomial-Wigner Ville distribution (PWVD) [6, 7, 8] can be defined. Standard signal transforms and TFDs are produced from (1) and (2) for the loss function $F(e) = |e|^2$. However, the standard transforms are very sensitive to an impulse noise environment. In the robust statistics other loss functions are used to overcome this drawback [9]. The commonly used loss functions are given in Table I. For example, the standard and the marginal median form of the robust STFT produced by the loss function given in the third column of Table I are:

$$STFT_S(n,k) = \text{mean}\{x(n+l)\exp(-j2\pi lk/N) | l \in [0,N)\}$$

Standard	$F(e) = \|e\|^2$
Robust 1	$F(e) = \|e\|$
Robust 2	$F(e) = \|\mathrm{Re}(e)\| + \|\mathrm{Im}(e)\|$
Myriad form	$F(e) = \log(\|e\|^2 + K^2)$

Table 1. Some common loss functions.

$$STFT_R(n,k) = \text{median}\{\mathrm{Re}\{x(n+l)\exp(-j2\pi lk/N) | l \in [0,N)\}\} + +j\text{median}\{\mathrm{Im}\{x(n+l)\exp(-j2\pi lk/N) | l \in [0,N)\}\} \quad (4)$$

For impulse noise environment $\gamma(n,l,k)$ (3) has twice as many impulses as the input signal, meaning that these representations would be more sensitive to the impulse noise than the robust STFT. Situations becomes worse in the case of the higher order robust TFDs (for example for the robust PWVD) where number of impulses can be even larger. These facts motivated development of the new robust TFDs that will be presented in Section IV.

The L-filter forms of the signal transforms and TFDs are introduced as an extension of the robust forms based on the minimization problem (1) and (2). The L-filter form of the robust STFT can be written as:

$$STFT_L(n,k) = \sum\nolimits_{l=0}^{N-1} a_l[r_l(n,k) + ji_l(n,k)],$$

where $r_l(n,k)$ and $i_l(n,k)$ are elements from the sets $R(n,k) = \{\mathrm{Re}\{x(n+m)\exp(-j2\pi km/N)\}, m \in [0,N)\}$ and $I(n,k) = \{\mathrm{Im}\{x(n+m)\exp(-j2\pi km/N)\}, m \in [0,N)\}$, respectively, sorted into a non-decreasing order

$$r_l(n,k) \le r_{l+1}(n,k), \;\; i_l(n,k) \le i_{l+1}(n,k). \quad (5)$$

The L-filters are usually generated in such a way to produce an unbiased estimate of the input signal [10]. This condition holds for: $\sum_{l=0}^{N-1} a_l = 1$ and $a_l = a_{N-1-l} \quad l \in [0,N)$. The α-trimmed version of the L-filters will be used in this paper:

$$a_l = \begin{cases} \frac{1}{N-2\alpha(N-2)} & l \in [(N-2)\alpha, (2-N)\alpha + N - 1] \\ 0 & \text{elsewhere,} \end{cases} \quad (6)$$

where $\alpha \in [0, 0.5]$. Note that the α-trimmed form for $\alpha = 0$ reduces to the standard STFT, while for $\alpha = 0.5$ it reduces to the robust STFT (4). All presented robust forms are computationally demanding, since they require that corresponding procedures (iterative or sorting) are performed for each point in the TF plane.

3. RECURSIVE REALIZATION OF THE ROBUST STFT

The standard STFT can be efficiently realized in a recursive manner as:

$$STFT(n+1,k) = \left[\frac{1}{N}x(n+N) - \frac{1}{N}x(n) + STFT(n,k)\right] e^{j2\pi k/N}. \quad (7)$$

Robust STFTs are accurate estimates of the standard STFT of non-noise signal. This fact motivated introduction of the recursive realization based on the widely known recursive realization of the standard STFT.

Step 1: Calculation of the robust STFT $STFT_\Delta(n,k)$ in the initial instant, by using iterative or sorting procedures. Set $R = 0$, $P = 0$.

Step 2: Calculation of the inverse DFT based on the robust STFT:

$$\hat{f}(n+m) = N \sum_{k=0}^{N-1} STFT_\Delta(n,k) e^{j2\pi km/N}, \; m \in [0,N). \quad (8)$$

Step 3: Calculation of the maximal value of $\hat{f}(m)$ within the interval:

$$f_{\max} = \max\{|\hat{f}(n+m)|, m \in [0,N)\}. \quad (9)$$

Step 4: If the following condition holds:

$$|x(n+N)| \le (1+\eta) f_{\max} \quad (10)$$

then $\hat{f}(n+N) = x(n+N)$ and $P = 0$; otherwise $\hat{f}(n+N) = \hat{f}(n+N-1)$ and $P = P+1$.

Step 5: Recursive calculation of the robust STFT in the next instant:

$$STFT_\Delta(n+1,k) = \left[\frac{1}{N}\hat{f}(n+N) - \frac{1}{N}\hat{f}(n) + STFT_\Delta(n,k)\right] e^{j2\pi k/N} \quad (11)$$

and $R = R+1$, $n = n+1$.

Step 6: If $R \le R_{\max}$ and $P \le P_{\max}$, go to Step 3, otherwise go to Step 1 and recalculate the robust STFT by using iterative or sorting procedures.

Comments on the algorithm. It can happen that the true value of new sample $f(n+N)$ is larger than the maximal sample of the noise-free signal within the previous window $f_{\max}$. In order to avoid situation that this sample is recognized as an impulse, we set larger value than $f_{\max}$ in (10), $\eta \ge 0$. In our numerical calculation $\eta = 0.25$ is used. In order to avoid accumulation of the discretization error in the registers, recursive realization of the standard STFT needs recalculation by using the FFT techniques after relatively large number of samples. It should be done in the recursive realization of the robust STFT, as well. However, since the recursive robust STFT is not equal to any of the directly calculated form, we set number of samples for recursive calculation of the robust STFT relatively small, $R_{\max} = 32$. Furthermore, if there is no signal within a wide interval, and $f_{\max}$ is close to zero,then the new sample of signal can be considered as an impulse, and it can be neglected in our algorithm. In order to avoid this case, we set that maximal number of the consecutive samples, where the previous sample is taken as the signal estimate, in the current sample, $P_{\max}$, is relatively small. In numerical examples $P_{\max} = 3$ is used.

4. REALIZATION OF THE ROBUST TFDS

Calculation of the higher order TFDs from the STFT is a common approach in the TF analysis. The robust STFT can be used as the initial stage for calculation of robust higher order TFDs, in the same manner as in the case of the standard transforms. Here, property that the robust STFT reduces impulse noise is combined with the property that higher order TFDs are highly concentrated. Several forms of the robust higher order TFDs, based on the robust STFT, are itemized below.

- The windowed robust STFT can be calculated by $rSTFT(n,k) = STFT_{\Delta}(n,k) *_k W(k)$, where $W(k)$ is the FT of the desired window function and $*_k$ represents the convolution in frequency. For instance, for a Hanning window (discrete domain), this convolution reduces to

$$rSTFT(n,k) = \sum_{i=-1}^{1} c_i STFT_{\Delta}(n,k+i),$$

where $c_0 = 1/2$, $c_{\pm 1} = 1/4$. Similar expressions can be derived for other window types.

- Robust forms of the WD based on the robust STFT are determined as [11]
$rWD(n,k) =$

$$\sum\nolimits_{l=-N/2}^{N/2} STFT_{\Delta}(n,k+l) STFT^*_{\Delta}(n,k-l). \quad (12)$$

Pseudo form of the rWD can be determined in the same manner as the windowed form of the robust STFT.

- The S-method (SM) [11] combines favorable properties of the STFT (non-aliased for the Nyquist rate sampled signal, along with cross-terms reduction) and the WD (high concentration). We define its robust counterpart as
$rSM(n,k) =$

$$\sum\nolimits_{l=-N/2}^{N/2} P(l) STFT_{\Delta}(n,k+l) STFT^*_{\Delta}(n,k-l). \quad (13)$$

where $P(l)$ is the window function in the frequency domain. Wider window means a better TF concentration while a narrower window means a better cross-terms interference suppression. Details on the window length determination can be found in [11].

- Realization of the quadratic CD class by using the standard STFT was introduced in [12]. We define the robust CDs as
$rCD'(n,k) =$

$$\sum\nolimits_{i} \lambda_i |STFT_{\Delta}(n,k) *_{\omega} Q_i(k)|^2. \quad (14)$$

In the above expression, λ_i represents the eigenvalues of the rotated kernel function in the time-lag domain of the particular member of the CD class, $Q_i(k)$ represents the FT of the corresponding eigenvectors. Details about this decomposition and its associated quantities can be found in [12].

- The LWD, proposed in [7], is an appropriate tool for nonlinear FM signals. This distribution can be evaluated recursively by using the WD or the SM. In a similar way, we propose to evaluate its robust counterpart based on the rWD or the rSM defined in (12) and (13). We define the robust LWD by using the recursive expression
$rLWD_L(n,k) = \sum_{l=-N/2}^{N/2} P(l) \times$

$$rLWD_{L/2}(n,k+l) rLWD_{L/2}(n,k-l), \quad (15)$$

where $rLWD_1(n,k) = rWD(n,k)$ or $rLWD_1(n,k) = rSM(n,k)$.

- The PWVD is a TF distribution proposed to deal with polynomial FM signals. For multicomponent signals, this distribution suffers from the presence of cross-terms. To mitigate this problem, an implementation procedure for the fourth order PWVD was proposed in [13]. Its robust counterpart can be defined as
$rPWVD(n,k) = \sum_l P(l) \times$

$$rLWD_2(n,k+l) rWD(n,k+[l/A]), \quad (16)$$

where $A = 0.85/1.35$ and $[\cdot]$ denotes rounding to the closest integer value.

5. IF ESTIMATION

We consider the signal $f(t) = \exp(j256\pi t^3 - j192\pi t)$, $t \in [-1,1]$. Sampling rate is $\Delta t = 1/512$. Signal is embedded in an impulse noise, a cube of the Gaussian noise, $\nu(t) = (\sigma\nu_1(t))^3 + j(\sigma\nu_2(t))^3$, where $\nu_i(t)$, $i = 1,2$ are mutually independent white Gaussian noises $E\{\nu_i(t)\} = 0$ and $E\{\nu_i(t)\nu_j(t)\} = \delta(i-j)$, while σ is a noise amount. Various STFT forms, the robust WD calculated by using minimization problem (2) and the robust SM evaluated by using (13), are compared in the IF estimation. The IF estimation has been formed as

$$\hat{\omega}(n) = \hat{\omega}_m(n) + \delta(n)$$

where $\hat{\omega}_m(n)$ is estimation based on the maxima of the TF representation:

$$\hat{\omega}_m(n) = \Delta\omega \hat{k}_m(n), \;\; \hat{k}_m(n) = \arg\max_k TF(n,k)$$

where $\Delta\omega$ is the frequency step considered, while the so-called displacement is:

$$\delta(n) = \frac{[Q_1(n) - Q_{-1}(n)]}{2[2Q_0(n) - Q_1(n) - Q_{-1}(n)]} \Delta\omega$$

where $Q_i(n) = TF(t, \hat{k}_m(n) + i)$. Note that this simple displacement is based on quadratic interpolation of the TFD around its maxima. The displacement is introduced in order to reduce effects of frequency discretization. Other displacement forms can be found in [14].

Various STFT forms are compared for different noise amounts in Fig1.a. It can be seen that the robust STFT forms, even for a light amount of noise, outperform the standard STFT form. Also, it can be seen that corresponding robust forms produce very similar results (i.e., the robust STFT evaluated by using the L-filter in each point of the TF plane, produces almost the same results as the robust STFT evaluated recursively with the L-filter form of the robust STFT in the initial stage). The robust STFT is

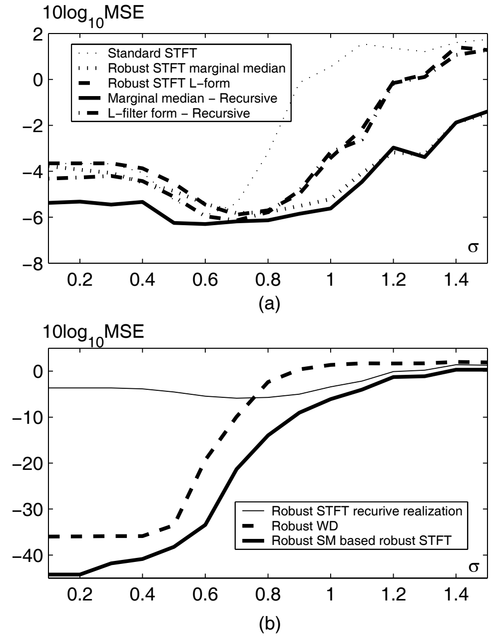

Fig. 1. MSE in the IF estimation by using: (a) STFT forms; (b) L-filter forms of the robust STFT, WD and SM.

compared with the robust WD calculated by using minimization problem (2) and the SM (13), Fig.1b. It can be seen that the robust WD outperforms the robust STFT for a weak noise but it is worse than the robust STFT for a higher noise. At the same time the robust SM outperforms both the robust STFT and the robust WD for all noise amounts.

6. CONCLUSION

Overview of the robust TFDs is presented in this paper. In order to reduce significant calculation complexity, inherited in sorting and iterative procedures used to calculate of the robust STFT, we proposed a recursive realization of the robust STFT. It is slightly more demanding than the recursive realization of the standard STFT. Furthermore, the robust higher-order TFDs can be calculated by using the robust STFT. In this paper we presented a brief comparison of the STFT forms with the robust WD and the robust SM in the IF estimation. A detailed calculation study will be reported later. Recently, several other TFDs based approaches have been used for the FM signals parameter estimation in the case of impulse noise environment (see [15]). These methods are based on the fractional lower order moments [16]. However, calculation savings, as compared to the signal parameters estimation based on the proposed recursive realization of the robust TFDs, are not significant. Since the robust STFTs are accurate estimates of the standard STFT, they are suitable for analysis of multicomponent signals, while the fractional lower order moments could be inaccurate in this case.

7. REFERENCES

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