INFINITE LENGTH CHANNEL SHORTENING FILTERING BASED ON POLYNOMIAL APPROACH

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ABSTRACT

Maximum-likelihood estimation and multicarrier modulation techniques are effective methods to mitigate intersymbol interference (ISI) in wireless and wireline communications, but their complexity and performance are limited by the channel length. Channel shortening equalization is an efficient way to shorten the length of the ISI channel while keeping most of the channel energy inside the shortened window. In this paper, we propose an infinite impulse response (IIR) filter for channel shortening and provide polynomial based algorithms to compute the shortening filter. We also considered a very general framework allowing auto-regressive moving average (ARMA) input and noise processes together with an ARMA channel model. An illustrative numerical example shows the superior performance of our proposed scheme as compared to existing techniques.

1. INTRODUCTION

Intersymbol interference (ISI) channels are one of the main causes of performance loss in both wireless and wireline communications. It is well known that maximum likelihood (ML) estimation is the optimum method to mitigate the effect of ISI, but the complexity of the algorithm grows exponentially with the length of the channel, which makes it unfeasible to implement except for very short channels, [3]. Similarly, for systems employing discrete multicarrier modulation (such as OFDM), a cyclic prefix of length proportional to the channel length is needed between transmission blocks to prevent inter-block interference, therefore reducing the transmission rate, [5], [8].

Channel shortening is an efficient way to increase the performance for both schemes. The basic idea is to equalize the long channel into a much shorter target channel by compressing the energy in the long channel into these few taps. In the literature several algorithms can be found for channel shortening [3]-[5], [8]. The performance of finite length shortening filters is inadequate if the channel energy is spread over a large number of taps and the channel model has zeros close to the unit circle due the length of the shortening filter. In [4], an infinite length method is proposed which finds the pole-zero approximation of a very long FIR shortening filter.

In this paper, we propose a frequency domain approach to design infinite impulse response (IIR) channel shortening filters under the criterion of minimum mean square error and based on polynomial methods, [1], [2]. We consider a very general framework by allowing the channel input and the noise to be auto-regressive moving average (ARMA) processes, but various scenarios such as a finite impulse response (FIR) channel with colored noise or white noise can readily be obtained from our general solution by setting appropriate transfer functions to unity. We will show that this method outperforms FIR channel shortening filters of similar implementation complexity. In contrast to the approach in [4], our method finds the exact IIR shortening filter directly.

Throughout this paper, we will denote the backward shift operator by q^{-1}, i.e. $q^{-1}x(k) = x(k-1)$. For any polynomial with order nP, $P\left(q^{-1}\right) = p_0 + p_1 q^{-1} + \cdots + p_{nP} q^{-nP}$, the conjugate polynomial is defined as $P_*(q) = p_0^* + p_1^* q + \cdots + p_{nP}^* q^{nP}$, where $(.)^*$ denotes the scalar conjugate. A polynomial is said to be stable if all of its roots are inside the unit circle, causal if it is a function of only q^{-1}, and similarly anticausal if it is a function of only q.

Section 2 develops the motivation and framework for IIR channel shortening filters. Section 3 provides the design of the IIR channel shortening filter. In Section 4, we provide a numerical example which verifies the superior performance of our proposed method over its FIR counterpart. Finally, conclusions are drawn in Section 5.

2. PROBLEM STATEMENT

Consider the structure given in Figure 1, where channel, shortening filter and target impulse response (TIR) together with the input and noise processes are depicted. The aim is to equalize the IIR channel $B(q^{-1})/A(q^{-1})$ to a limited order (nb) FIR target impulse response, $b\left(q^{-1}\right) = b_0 + \cdots + b_{nb} q^{-nb}$, by employing a stable and causal IIR channel shortening (CS) filter $Q(q^{-1})/R(q^{-1})$ based upon the minimum mean square error (MMSE) criterion. A frequency domain approach is followed for the optimization.

Both the channel input $s(k)$ and the noise $n(k)$ are assumed to be ARMA processes as respectively with the transfer functions $C(q^{-1})/D(q^{-1})$ and $M(q^{-1})/N(q^{-1})$. The stationary processes $d(k)$ and $v(k)$ are assumed to be zero-mean and white with variances λ_d and λ_v respectively. Except B, which has an arbitrary but nonzero leading coefficient, all polynomials are monic. The polynomials A, D, and N are assumed also to be stable. The delay parameter m is adjusted to maximize the performance. The polynomials Q and R are coprime, otherwise the common terms cancel each other. Consider the CS filter input (for simplicity the parentheses of polynomials will be dropped where convenient)

$$y(k) = \frac{B}{A}\frac{C}{D} d(k) + \frac{M}{N} v(k). \tag{1}$$

This signal can be written in the innovations form

$$y(k) = \frac{\beta}{ADN} \sqrt{\lambda_\eta}\, \eta(k) \tag{2}$$

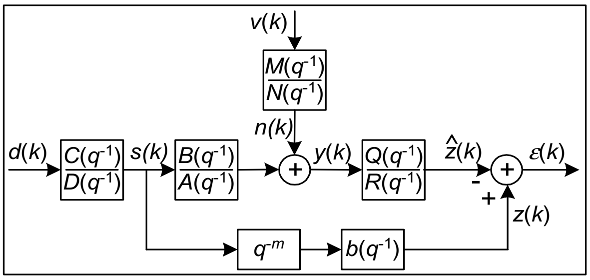

Figure 1: Channel shortening model employing an IIR shortening filter to approximate an IIR channel to a finite length target impulse response. The channel input $s(k)$ and the noise $n(k)$ are ARMA processes.

where $\eta(k)$ is the innovation sequence (white with unity variance), the scalar η is a gain term, and the polynomial $\beta(q^{-1}) = 1+\beta_1 q^{-1}+\cdots+\beta_{n\beta}q^{-n\beta}$ is the monic and stable spectral factor of $y(k)$. Equating the spectral densities of $y(k)$ from (1) and (2), we obtain

$$\frac{r\beta\beta_*}{AA_*DD_*NN_*} = \frac{BB_*CC_*}{AA_*DD_*} + \rho\frac{MM_*}{NN_*}$$

$$r\beta\beta_* = BB_*CC_*NN_* + \rho AA_*DD_*MM_* \tag{3}$$

where $r = \lambda_\eta/\lambda_d$ and $\rho = \lambda_v/\lambda_d$. The degree of β is $n\beta = \max\{nB+nC+nN, nA+nD+nM\}$.

3. IIR CHANNEL SHORTENING EQUALIZER

The signals $z(k)$ and $\hat{z}(k)$ can be written as

$$\begin{aligned} z(k) &= q^{-m} b \frac{C}{D} d(k) \\ \hat{z}(k) &= \frac{Q}{R}\left(\frac{B}{A}\frac{C}{D}d(k) + \frac{M}{N}v(k)\right) \end{aligned}$$

The error signal $\varepsilon(k)$ is defined as the difference between these signals

$$\begin{aligned} \varepsilon(k) &= z(k) - \hat{z}(k) \\ &= \left[q^{-m}b - \frac{Q}{R}\frac{B}{A}\right]\frac{C}{D}d(k) - \frac{Q}{R}\frac{M}{N}v(k) \end{aligned} \tag{4}$$

Using the orthogonality principle of MSE estimation which states that the error signal and filter input are orthogonal, we obtain (5) given at the top of the next page, where Parseval's formula is employed.

We conclude that to make this integral equal to zero, all stable poles in the denominator must be cancelled by the numerator (Cauchy's integral theorem, [6]) (poles outside the unit circle do not contribute to the integral since the integration contour is the unit circle)

$$\frac{z^{-m}RbAB_*CC_*NN_* - r\beta\beta_*Q}{zRADN} = L_*$$

where L_* is an anticausal polynomial to satisfy the equality. Rearranging we obtain

$$z^{-m}RbAB_*CC_*NN_* = r\beta\beta_*Q + zRADNL_* \tag{6}$$

We observe that R is common to both the left-hand-side (lhs) and the second term in the right-hand-side (rhs). Since R and Q are coprime and the only stable factor of the first term in the rhs is β, R must be equal to β. Similarly, since N and A are common to both the lhs and the second term in the rhs, they must be factors of Q, therefore $Q = Q_1AN$. Therefore the optimum shortening filter is found to be

$$\frac{Q}{R} = \frac{Q_1AN}{\beta} \tag{7}$$

where Q_1 is a function of b which will be calculated in the next step. Substituting Q and R into (6) we get

$$z^{-m}bB_*CC_*N_* = r\beta_*Q_1 + zDL_* \tag{8}$$

rewriting

$$\frac{z^{-m}bB_*CC_*N_*}{rD\beta_*} = \frac{Q_1}{D} + \frac{zL_*}{r\beta_*} \tag{9}$$

the rhs is the causal-anticausal factorization of the lhs. Since Q_1 is purely causal and zL_* is purely anticausal they are independent, which means L_* can be written in terms of only b. It can easily be verified that the degrees of Q_1 and L_* are respectively $nQ_1 = max\{nb+nC+m, nD-1\}$ and $nL = max\{nB+nC+nN-m, n\beta\}-1$. In fact, (8) is a diophantine equation and several efficient methods exist to solve this equation [1], [7].

The mean square error can be written as in $(10)-(11)$ which can be found at the top of the next page. On the second line, we used the completing to the squares method after substituting (4) into (10). A closer investigation of the first bracket in (11) reveals that Q/R, when chosen as in (7), cancels the causal part of the second term in that bracket (which is Q_1/D in (9)). This optimizes the MSE with respect to the strictly causal and stable polynomials Q and R. Therefore, for this choice of Q and R, we could write the MSE in (11) as

$$J = \frac{\lambda_d}{2\pi j}\oint_{|z|=1}\left\{\frac{LL_* + \rho bb_*AA_*CC_*MM_*}{r\beta\beta_*}\right\}\frac{dz}{z} \tag{12}$$

The polynomials A, C and M are known, and β and L can be determined using equations (3) and (8) respectively. The only unknown b is chosen to minimize the MSE in (12). To calculate the integral we can employ Cauchy's integral formula, [6], by first expanding the integrand using partial fractions. We should note that after partial fractions expansion, all of the numerators are constant with respect to z, it suffices to add the numerators corresponding to non-zero poles located in the unit circle (which are actually the roots of $\beta(q^{-1})$) and the numerator of the $1/z$ fraction, therefore there is no need to solve the integral explicitly. Upon summation of the numerators contributing to the integration, we obtain a quadratic function of $b\left(q^{-1}\right)$,

$$J = \mathbf{b}^H\mathbf{R}\mathbf{b}$$

where $\mathbf{b} = [b_0 \quad b_1 \quad \cdots \quad b_{nb}]^T$. The matrix $\mathbf{R}$ is positive definite, therefore the eigenvector corresponding to the smallest eigenvalue is the coefficients of the optimum TIR in polynomial, $b_{opt}(q^{-1})$. The optimum channel shortening filter can be obtained by first substituting $b_{opt}\left(q^{-1}\right)$ back into (9) to calculate Q_1 which is then substituted into (7).

$$E\{\varepsilon(k)y^*(k)\} = \frac{\lambda_d}{2\pi j}\oint_{|z|=1}\left\{\frac{z^{-m}RbAB_*CC_*NN_* - r\beta\beta_*Q}{RAA_*DD_*NN_*}\right\}\frac{dz}{z} = 0 \quad (5)$$

$$\begin{aligned} J &= E\{\varepsilon(k)\varepsilon^*(k)\} & (10)\\ &= \frac{\lambda_d}{2\pi j}\oint_{|z|=1}\left\{r\left[\frac{Q\beta}{RADN} - \frac{z^{-m}bB_*CC_*N_*}{rD\beta_*}\right]\left[\frac{Q_*\beta_*}{R_*A_*D_*N_*} - \frac{z^m b_*BCC_*N}{rD_*\beta}\right] + \frac{\rho bb_*AA_*CC_*MM_*}{r\beta\beta_*}\right\}\frac{dz}{z} & (11)\end{aligned}$$

The equivalent channel which is the convolution of the channel and the shortening filter is

$$C_{eq} = \frac{B}{A}\frac{Q}{R} = \frac{Q_1BN}{\beta}. \quad (13)$$

Remarks:

1. If the channel shortening is performed only with a numerator polynomial, $Q(q^{-1})$, it can be verified that this filter, $Q' = Q_1AN\beta^{-1}$, would be of infinite length. The FIR shortening filter is the truncation of this polynomial into a finite length. Therefore, if the stronger taps of the impulse response of the IIR channel are accumulated in a relatively short window, the performance of FIR and IIR filters can be very close. However, if the channel has many zeros close to the unit circle which results in very long Q', the truncation would yield performance loss as demonstrated in the next section.
2. When we set the TIR to $b = 1$ the channel shortening filter reduces to the Wiener equalizer [1], [2]. This provides complete equalization, but at the expense of degradation of performance as compared to a full complexity ML detector. In terms of complexity, the channel shortening filter, when applied as a preprocessor, lies between a linear equalizer and a ML detector. Channel shortening techniques also have potential application in multicarrier modulation schemes to reduce the required prefix length, hence to increase the bandwidth efficiency.

4. A NUMERICAL EXAMPLE

Consider a second order IIR channel with parameters $B = (1-0.95q^{-1}+0.9025q^{-2})$ and $A = (1-1.1314q^{-1}+0.64q^{-2})$, which correspond to two zeros at $0.95e^{\pm j\pi/3}$, and two poles at $0.8e^{\pm j\pi/3}$. The ARMA models for the input and noise processes are chosen such that $C = D = 1$, $M = (1-0.3q^{-1})$, $N = (1+0.7q^{-1})$, with $\lambda_d = 1$, $\lambda_v = 0.05$, $\rho = 0.05$. Therefore the SNR at the shortening filter input is calculated as 10 dB. It is desired to shorten the channel to two taps ($nb = 2$). The optimum delay $m = 0$ is considered. Therefore, the filter input is

$$y(k) = \frac{1-0.95q^{-1}+0.9025q^{-2}}{1-1.1314q^{-1}+0.64q^{-2}}d(k) + \frac{1-0.3q^{-1}}{1+0.7q^{-1}}v(k)$$

The spectral factor β of this signal and the scalar r in (3) are found to be

$$\begin{aligned}\beta &= 1-0.3334q^{-1}+0.3041q^{-2}+0.5412q^{-3}\\ r &= 1.1495\end{aligned}$$

The degrees of polynomials Q_1 and L are calculated as $nQ_1 = 1$, $nL = 2$. To manipulate the causal-anticausal factorization (9), we write the equation (8) in matrix form

$$\begin{bmatrix} 1.1495 & 0 & 0 & 0 & 0\\ -0.3831 & 1.1495 & 0 & 0 & 0\\ 0.3496 & -0.3831 & 1 & 0 & 0\\ 0.6221 & 0.3496 & 0 & 1 & 0\\ 0 & 0.6221 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix} q_1\\ q_0\\ l_0^*\\ l_1^*\\ l_2^*\end{bmatrix} = \begin{bmatrix} b_1\\ b_0-0.25b_1\\ -0.25b_0+0.2375b_1\\ 0.2375b_0+0.6318b_1\\ 0.6318b_0\end{bmatrix}$$

where we obtain the polynomials Q_1 and L_* as

$$Q_1 = (0.8699b_0+0.0724b_1)+(0.8699b_1)q^{-1} \quad (14)$$

$$\begin{aligned}L_* = {} & (0.0833b_0-0.0389b_1)+(-0.0666b_0\\ & +0.0652b_1)q+(0.0905b_0-0.0451b_1)q^2\end{aligned}$$

Substituting L_* into (12) and performing partial fractions expansion, the MSE expression becomes

$$\begin{aligned} J &= 0.1301b_0^*b_0 - 0.0724b_0^*b_1 - 0.0724b_1^*b_0 + 0.1241b_1^*b_1\\ &= \begin{bmatrix} b_0^* & b_1^*\end{bmatrix}\begin{bmatrix} 0.1301 & -0.0724\\ -0.0724 & 0.1241\end{bmatrix}\begin{bmatrix} b_0\\ b_1\end{bmatrix}\end{aligned}$$

The minimum eigenvalue, which is also the MSE, is 5.4566×10^{-2}. The corresponding eigenvector, hence the TIR, in polynomial form is

$$b(q^{-1}) = 0.6922+0.7217q^{-1}$$

Substituting b into (14), the polynomial Q_1 is found to be

$$Q_1(q^{-1}) = 0.6545+0.6278q^{-1}$$

Therefore the optimum shortening filter given in (15) at the top of the next page is found by substituting Q_1, β, A and N into (7). The equivalent channel impulse response given in (16) at the top of the next page is calculated by substituting the polynomials Q_1, B, N and β into (13).

Figure 2 depicts the impulse responses of the equalized channel obtained using the IIR and the FIR methods together with the original channel impulse response. The TIR window has been chosen as the first two taps, and the remaining

$$\frac{Q}{R} = \frac{Q_1 A N}{\beta} = \frac{0.6545 + 0.3455q^{-1} - 0.3703q^{-2} + 0.1978q^{-3} + 0.2813q^{-4}}{1 - 0.3334q^{-1} + 0.3041q^{-2} + 0.5412q^{-3}} \tag{15}$$

$$C_{eq} = \frac{Q_1 B N}{\beta} = \frac{0.6545 + 0.4642q^{-1} - 0.0015q^{-2} + 0.5626q^{-3} + 0.3966q^{-4}}{1 - 0.3334q^{-1} + 0.3041q^{-2} + 0.5412q^{-3}} \tag{16}$$

taps correspond to the interference terms.It can be seen that a good compression is achieved with the IIR filter as compared to the FIR filter for the same complexity.

We compare the performance of the proposed method to that of an FIR channel shortening filter of length $nQ + nR - 1 = 8$ (which has the same complexity as the ARMA filter) designed with the MMSE criterion as given in [5]. We consider two types of performance measure. The first one is the compression ratio which is defined as the ratio of the energy inside the TIR window to that of the whole channel impulse response window after equalization. This quantity for the original channel is 67%. We obtained 85% compression ratio with an FIR channel shortening filter, whereas our proposed ARMA based channel shortening filter achieved 99.7% compression. We also studied the performance based on the shortening signal-to-interference plus noise ratio (SINR), where the signal component is considered as the energy inside the TIR window and the interference component is considered as to the energy outside the window. The method based on the FIR filter results in 6.5 dB SINR, whereas our proposed method results in 12.4 dB SINR.

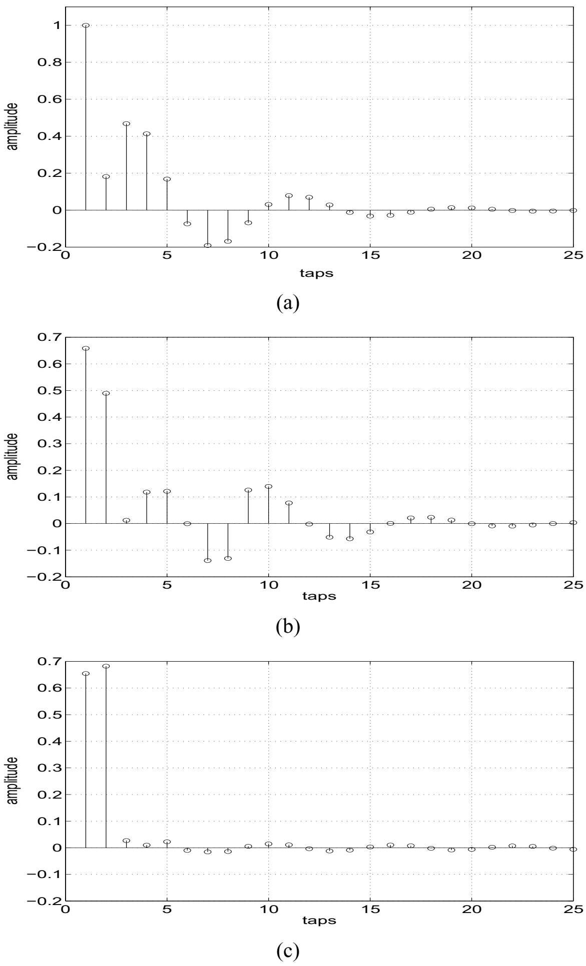

Figure 2: Impulse responses of (a) the original channel, (b) the equalized channel using an FIR filter of length eight, (c) the equalized channel using the proposed IIR filter with four zeros and three poles.

5. CONCLUSIONS

We proposed the design of a channel shortening filter based upon an IIR structure using a polynomial approach. We considered a very general framework allowing the signal and the noise to be ARMA processes and the channel to be an ARMA model. Various simple scenarios such as an FIR channel and white or colored noise can readily be obtained from the proposed method. Using the simulation results, we have demonstrated superior performance of our proposed IIR based channel shortening filter as compared to its FIR counterpart. Extreme compression ratio achieved by this method encourages application of this technique in wireless and wireline communications as a preprocessor at the receiver for reducing the complexity of a ML detector or for reducing the cyclic prefix length in multicarrier modulation schemes such as OFDM.

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