A VARIABLE STEP-SIZE APA ALGORITHM ROBUST UNDER IMPULSIVE NOISE INTERFERENCE

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ABSTRACT

A new Variable Step-Size Affine Projection Algorithm (VSS-APA) robust under impulsive noise interference is proposed and its performances are investigated through simulations. The proposed step size takes into account the instantaneous value of the output error and provides a trade-off between the convergence rate and the steady-state coefficient error by controling the two modes of the adaptive filter: the updating and the freezing modes. The VSS-APA algorithm is seen to robustly identify the unknown system. It presents a good behavior in terms of the convergence speed and the steady state error compared to the classical approaches based on, a nonlinear function (M-estimator of Huber) or the median filter such as: the MNLMS, the NRLS and the Median LMS algorithms.

1. INTRODUCTION

Most signal processing algorithms developed for adaptive filtering are based on the assumption that the noise is Gaussian distributed. However, in most practical environments, the noise can be generated by some natural and/or man-made electromagnetic sources and it exhibits impulsive characteristics [1]. Under this adverse condition, the performance of the conventional linear adaptive filters can be deteriorated significantly. To overcome this problem, it is desirable to build adaptive filtering algorithms that maintain appropriate functionality under a broad class of noise sources (interferences, impulsive noise, ...etc.). Many authors have noted that non-linearities, via the application of the Huber's Min-Max approach [2], can be incorporated into the product of the error signal and the tap input of the Least Mean Square (LMS) algorithm and thus the performance of the adaptive filter can be improved. The algorithm is called Non-linear LMS (NLMS) [3]. In this sense, several non-linear algorithms have also been developed, such as, the Robust Mixed Norm (RMN) [4], the mixed norm LMS (MNLMS) [5], the Order Statistic Least Mean Square (OSLMS) [6] and the Nonlinear Recursive Least Square (NRLS) [3]. Beside those solutions, a great attention has also been given to the application of the median filter in order to smooth the gradient estimate such as the Median LMS filter proposed in [7]. All the aforementioned techniques try to reduce the hostile effect of large estimation error, due to impulses, on the filter weights.

In this paper, we focus on the study of the Affine Projection Algorithm (APA). The analysis of the impact of the impulses on its convergence behavior shows that the observed instability can be controlled and reduced by the reduction of the adaptation constant denoted here by μ. However, this reduction causes the reducing of the convergence rate of the algorithm and its capacity to respond to the possibly non-stationnary input signals. To overcome the weakness, we propose, in this paper, the use of a Variable Step Size (VSS) which suppresses systematically the hostile impact of large estimation error due to impulsive noise by freezing the updating equation. Therefore, the proposed VSS-APA algorithm, which presents two modes: updating and freezing modes which are controlled by the value of the VSS, robustify, as we will see, the classical APA algorithm. This improvement in the performance is achieved at practically no increase in computational complexity.

The remainder of this paper is as follows. The next section presents the identification system and the non Gaussian noise model. Section 3 recall the classical APA algorithm and study the impact of the impulsive noise on its performances. Section 4 gives the outline of the proposed VSS. Section 5 gives some simulation results. Finally, section 6 draws our conclusion.

2. SYSTEM AND NOISE MODEL

Let us consider the system identification problem shown in Figure 1. The signal $d(n)$ is the input of the adaptive linear filter characterized by its impulse response denoted by $\mathbf{h}_{opt} = [h_{1,opt}, ..., h_{N,opt}]^T$ which represents the unknown weight vector.

$\mathbf{d}(n) = [d(n), ..., d(n-N+1)]^T$ is the input signal vector where the channel order, denoted by N, is supposed to be

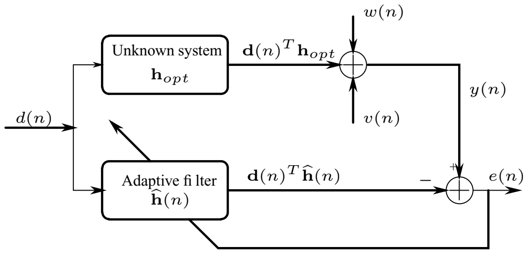

Fig. 1. System identification structure.

known. The received signal, $y(n)$, can be written as follows,

$$y\left(n\right)=\mathbf{d}(n)^T\mathbf{h}_{opt}+w(n)+v(n) \tag{1}$$

where $w(n)$ is the Gaussian background noise with zero mean and variance σ_w^2 and $v(n)$ is the impulsive component. In the following, the impulse noise is modeled as following:

$$v(n)=\gamma(n)g(n) \tag{2}$$

where $\{\gamma(n)\}$ stands for a Bernoulli process, a sequence of zeroes and ones with $p(\gamma=1)=\epsilon$, where ϵ is the contamination constant. It expresses the probability that an impulse occurs. $g(n)$ is a white Gaussian noise with zero mean and variance σ_v^2 such as $\sigma_w^2 \ll \sigma_v^2$. In this paper, we take a linear variation of the two variances,

$$\sigma_v^2=\kappa\sigma_w^2 \ \text{ with } \ \kappa \gg 1 \tag{3}$$

Under this model, the probability density of the channel noise $b(n)=w(n)+v(n)$ can be expressed as

$$p(b(n))=(1-\epsilon)\mathcal{N}(0,\sigma_w^2)+\epsilon\mathcal{N}(0,(\kappa+1)\sigma_w^2) \tag{4}$$

where $\mathcal{N}(m_x,\sigma_x^2)$ is the Gaussian density function with mean m_x and variance σ_x^2. $\{b(n)\}$ is called an ""$\varepsilon--$ *contaminated*"" noise sequence. This model serves as an approximation to the more fundamental Middleton $Class\ A$ noise model [1].

In the next section, we recall the well known APA algorithm and we study the impact of impulsive noise on its performances.

3. INSTABILITY OF APA ALGORITHM IN IMPULSIVE ENVIRONMENT

The LMS algorithm is well known in adaptive filtering, however, its convergence speed may be far from acceptable for many applications, such as echo cancellation, when the input sequence is correlated. To overcome this problem, many modifications that aim to decorrelate the input signal are done. In particular, this lead to the affine projection algorithm [8]. The APA algorithm is given by the following equations,

$$\begin{cases} \widehat{\mathbf{h}}(n)=\widehat{\mathbf{h}}(n-1)+\mu\frac{\mathbf{u}(n)}{\mathbf{u}(n)^T\mathbf{u}(n)+\eta}e(n) \\ e(n)=y(n)-\mathbf{d}(n)^T\widehat{\mathbf{h}}(n-1) \end{cases} \tag{5}$$

Where the estimation error at time n is given by

$$e(n)=y\left(n\right)-\widehat{y}\left(n\right)=y(n)-\mathbf{d}(n)^T\widehat{\mathbf{h}}(n)$$

where $\widehat{\mathbf{h}}(n)=\left[\widehat{h}_1(n),...,\widehat{h}_N(n)\right]^T$ is the estimated coefficients vector delivered by the identification algorithm. μ is a positive step size. η is a positive regularization constant which is added in order to prevent undesired behavior when $\mathbf{u}(n)=\mathbf{0}$.

The vector $\mathbf{u}(n)$ can be defined as a direction vector since it determines the direction of the update. It is defined by [8],

$$\mathbf{u}(n)=\mathbf{d}(n)-\mathbf{M}_n\mathbf{v}(n) \tag{6}$$

where $\mathbf{v}(n)=\left[\mathbf{M}_n^T\mathbf{M}_n\right]^{-1}\mathbf{M}_n^T\mathbf{d}(n)$ and the matrix $\mathbf{M}_n=[\mathbf{d}(n-1),...,\mathbf{d}(n-m)]$ is the collection of the last m observation of $\mathbf{d}(n)$ with $m<N$. In this paper, we choose $m=2$.

The APA algorithm suffers from serious performance degradation and may fail entirely when the signal $e(n)$ is corrupted by impulsive noise. In Figure 2, we plot the evolution of the squared error estimation, $\left\|\widehat{\mathbf{h}}(n)-\mathbf{h}_{opt}\right\|^2$, in the presence of impulsive noise (characterized by $\epsilon=10^{-2}$ and $\kappa=1000$). The channel weights are chosen $\mathbf{h}_{opt}=[1,0.2,0.5]^T$. The step size parameter is chosen $\mu=0.02$ and $\mu=0.002$. We remark that the APA algorithm with $\mu=0.02$ exhibits instability. This problem can be controlled, as is shown in Figure 2, by the reduction of the adaptation constant μ, (in our case $\mu=0.002$). This reduction causes the reduction of the convergence rate and the capacity of the algorithm to respond to a non stationnary input signals.

It is clear that the major cause of the observed instability is the linear dependency between the output error, $e(n)$, and the impulsive component. So, in the literature, it is often proposed to use the M-estimate proposed initially by Huber or the median filter in order to reduce the impact of the large estimation error on the weight adaptation, through the term $e(n)\mathbf{d}(n)$, while maintaining performance comparable to a non-impulsive environment. In this paper, instead of reducing the step size or incorporating a nonlinear function which performances are dependant of the choice of the clipping threshold, we here propose to freeze the updating equation when an impulse occurs in the received signal. For this aim, we must modify the fixed step size to a VSS which will be able to freeze automatically the updating, when an impulse occurs in the received signal, and will keep an optimal behavior in the absence of impulses. In the next section, we present the proposed VSS.

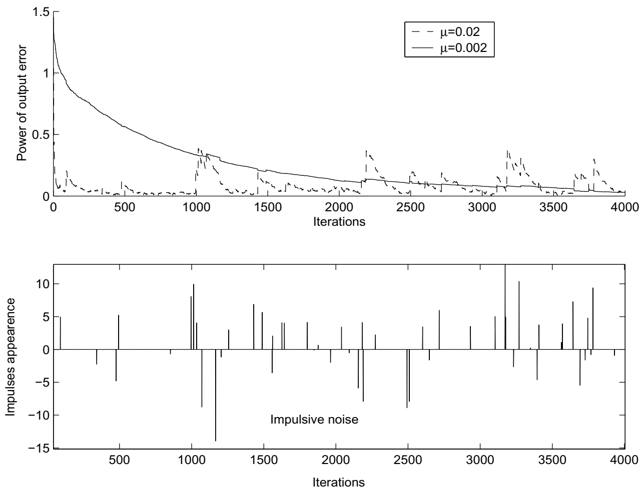

Fig. 2. Instability of the APA2 algorithm in the presence of impulsive noise.

4. PROPOSED APPROACH

In [9], it is shown that the detection of impulses in the received signal given by equation (1) can be cast as a binary hypothesis-testing problem as follows: H_1 presence of impulsive noise and H_0 absence of impulsive noise. When, H_1 hypothesis is decided, we freeze the updating. However, in this case, we should update the Likelihood Ratio Test at the sampling rate which would increase the complexity of the algorithm.
For this purpopse, we propose to build a step size which is able to decrease very rapidly to zero when an impulse occurs in order to stop the updating step. For this purpose, we propose,

$$\mu(n) = \frac{a}{b + ce^2(n)} \tag{7}$$

where a, b et c are positive constants. We notice that the proposed step size takes into account the instantaneous power of the output error $e(n)$ in order to robustify the detection of the presence of the impulses in the received signal denoted by $y(n)$. The behavior of the proposed VSS replaces the hypothesis test proposed in [9] without any complexity increase.
So, the proposed VSS denoted by $\mu(n)$ is bounded by $\mu_{\max}$ from above and $\mu_{min} = 0$ from below. Typically, the value of $\mu_{\max}$ is selected to provide the maximum possible rate of convergence. $\mu_{\min} = 0$ implies the freeze of adaptation of the algorithm especially when an impulse occurs. In fact:

$$\begin{cases} \text{Appearence of impulses} \Rightarrow & e(n) \nearrow \quad \Rightarrow \mu(n) = 0 \\ \text{absence of impulses} \Rightarrow & e(n) \to 0 \quad \Rightarrow \mu(n) = \frac{a}{b} \end{cases}$$

So, it is clear that the choice of the parameters a and b can be done using the main result of the paper [10], which consider a Gaussian noise case. In fact, the appearance of impulses is controlled by Bernoulli process. So, when $\gamma = 0$ the main component of the observation noise, $b(n)$, is Gaussian (first term in the equation (4)), the result of the paper [10] can be used. We note also that the proposed VSS can be used in order to robustify the LMS algorithm and in this case the parameters a and b are chosen using this condition [11]

$$\frac{a}{b} <= \frac{1}{N\sigma_d^2} \tag{8}$$

where σ_d^2 is the variance of input signal.
The parameter c accelerates the convergence of the VSS to zero especially when the impulses occur. The parameter c is fixed by simulation.
In order to reduce the complexity of the VSS-APA2 algorithm, the product $\mathbf{u}(n)^T\mathbf{u}(n)$, $\mathbf{d}(n)^T\mathbf{d}(n-1)$ and $\mathbf{d}(n-1)^T\mathbf{d}(n-1)$ are computed recursively where we incorporate an exponential windowing via a positive constant λ, $(0 < \lambda < 1)$ which governs the averaging time constant. So the VSS-APA2 can be described as following:

$$\begin{aligned}
e(n) &= y(n) - \mathbf{d}(n)^T\widehat{\mathbf{h}}(n-1) && (9)\\
p(n) &= \lambda p(n-1) + (1-\lambda)\left([\mathbf{d}(n-1)]_1\right)^2 && (10)\\
q(n) &= \lambda q(n-1) + (1-\lambda)\,[\mathbf{d}(n-1)]_1\,[\mathbf{d}(n)]_1 && (11)\\
\mathbf{u}(n) &= \mathbf{d}(n) - \frac{p(n)}{q(n)}\mathbf{d}(n-1) && (12)\\
g(n) &= \lambda g(n-1) + (1-\lambda)\left([\mathbf{u}(n)]_1\right)^2 && (13)\\
\widehat{\mathbf{h}}(n) &= \widehat{\mathbf{h}}(n-1) + \mu(n)\frac{e(n)}{g(n)+\eta}\mathbf{u}(n) && (14)
\end{aligned}$$

where $[\mathbf{x}]_1$ represents the first component of the vector $\mathbf{x}$.

5. SIMULATION RESULTS

In order to evaluate the performance of the proposed robustification approach, simulation is carried out on the system identification problem as shown in Figure 1. The unknown system is modelled as a FIR filter with impulse response, $\mathbf{h}_{opt} = [1, 0.2, 0.5]^T$. The adaptive filter is assumed to have the same length as the unknown system, i.e. $N = 3$. The parameters of the impulsive noise are $\epsilon = 10^{-2}$ and $\kappa = 1000$. The Signal to Noise Ratio is chosen equal to 15 dB. The initial weights of the adaptive filter are set to zeroes. We choose $a = 1$, $b = 40$ and $c = 20$. So $\mu(0) = \frac{1}{40}$. The forgetting factor λ is chosen equal to 0.98.
First, we plot on Figure 3 the variation of the VSS $\mu(n)$ versus iterations. We remark the good behavior of the step size which switch the APA algorithm on two modes: updating and freezing modes. The value of $\mu(n)$ is around $\frac{1}{40}$ in an

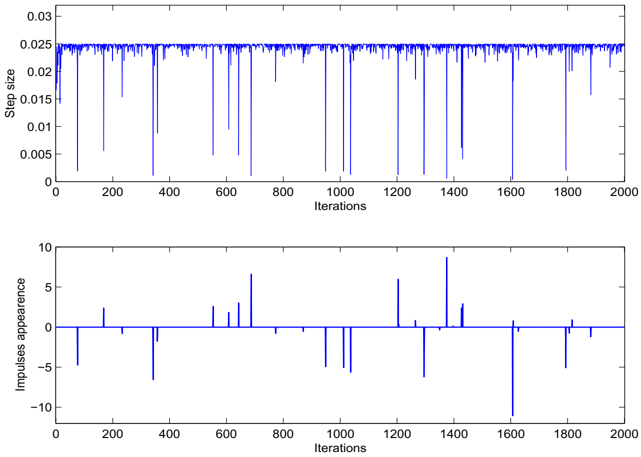

Fig. 3. Variation of the value of the VSS, $\mu(n)$, versus iterations

impulse free signal (i.e. $\gamma = 0$) and around 0 in an impulse signal (i.e. $\gamma = 1$).
Second, we plot the normalized square norm of the weight error vector. We have compared the convergence behavior of the proposed approach, the MNLMS proposed in [5, 12] and the Median LMS proposed in [7].

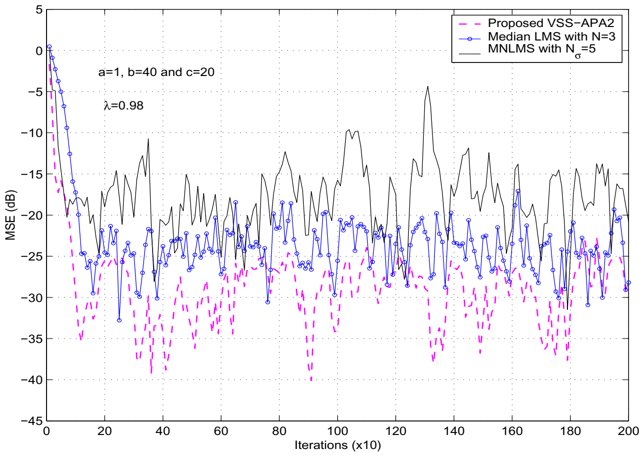

Fig. 4. MSE performance of the proposed VSS-APA2 algorithm compared to the MNLMS and the Median LMS algorithms under impulsive noise.

We remark that the proposed approach robustly identify the unknown system. It presents a good behavior in term of the convergence speed and the steady state error, (for example, about $-32dB$ in the MSE for the VSS-APA2 when MNLMS presents a MSE of $-20dB$) without any complexity increase.

6. CONCLUSION

In this paper, a Variable Step Size for the robustification of the APA algorithm is proposed. It controls the two modes of the adaptive filter: updating mode and freezing mode. Simulation results show that the performance of the proposed approach is better than the MNLMS, Median LMS algorithms based on the classical approach (nonlinear function, median filter). Work is now in progress to generalize these results to the multiuser CDMA context.

7. REFERENCES

[1] D. Middleton, “Non-gaussian models in signal processing for telecommunications: New methods and results for class A and class B models,” *IEEE Trans. on Information Theory*, vol. 45, No 4, pp. 1129–1149, May 1999.

[2] P.J. Huber, *Robust Statistics*, vol. vol 43, New York Wiley, 1981.

[3] J.F. Weng and S.H. Leung, “Adaptive non linear RLS algorithm for robust fi ltering in impulse noise,” *IEEE Int. Symp. on Circuits and Systems*, pp. 2337–2340, 1997.

[4] J.A. Chambers and A. Avlonitis, “A Robust Mixed-Norm (RMN) adaptive fi lter algorithm,” *IEEE Signal Processing Letter*, vol. 4, pp. 46–48, Feb 1997.

[5] D. P. Mandic, E.V. Papoulis, and C.G. Boukis, “A normalized mixed-norm adaptive fi ltering algorithm robust under impulsive noise interference,” *IEEE ICASSP conf.*, pp. 333–336, 2003.

[6] T.I. Haweel and P.M. Clarkson, “A class of order statistic LMS algorithms,” *IEEE Trans. on Signal Processing*, vol. 40, pp. 44–53, 1992.

[7] P.M. Clarkson and T.I. Haweel, “Median LMS algorithm,” *Electronics Letters*, vol. 25, No 8, pp. 520–522, April 1989.

[8] M. Rupp, “A family of adaptive fi lter algorithms with decorrelating properties,” *IEEE Trans. on Signal Processing*, vol. 46, pp. 771–775, March 1998.

[9] B. Sayadi and S. Marcos, “A robustifi cation method of the adaptive fi ltering algorithms in impulsive noise environments based on the likelihood ratio test,” *accepted in First International Symposium on Control, Communications and Signal Processing, ISCCSP04*, March 2004.

[10] H. Besbes, Y. Ben-Jemaa, and M. Jaidane, “Exact convergence analysis of affi ne projection algorithm: The fi nite alphabet inputs case,” *ICASSP*, pp. 1669–1672, 1999.

[11] E. Soria-Olivas and al., “An easy demonstration of the optimum value of the adaptation constant in the LMS algorithm,” *IEEE Trans on Education*, vol. 41, No 1, pp. 81, Feb 1998.

[12] J.A. Chambers, O. Tanrikulu, and A.G. Constantinides, “Least mean mixed-norm adaptive fi ltering,” *Electronics Letters*, vol. 30, No 19, pp. 1574–1575, Sept 1994.