INFLUENCE OF SPECKLE AND POISSON NOISE ON EXOPLANET DETECTION WITH A CORONAGRAPH

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ABSTRACT

We study in this communication the statistical spatial properties of the residual speckles in the Point Spread Function (PSF) of a large telescope with Adaptive Optics. These speckles form a noisy background outside the PSF central core, which makes it very difficult to detect an exoplanet. It can be shown that these speckles are due to small defaults of the wavefront, amplified by the coherent part of the wave. Using reasonable physical assumptions, their statistics is described by a modified Rice distribution. A simple analytic form can be derived for the Moment Generating Function (MGF). Analytical expressions can also be obtained for the statistics at photon counting levels. Using properties of the MGF, simple expressions are obtained for the variances of the noise. We discuss the relative importance of speckle and photon noise and present conclusions on the limits of coronagraphy for the detection of an exoplanet.

1. INTRODUCTION

Direct imaging of an extrasolar planet, orbiting a nearby star is an ambitious project with strong astrophysical drivers, including for example the study of planet formation and the search for life. At the present time, 119 exoplanets have been detected using radial velocities or transits [19]. The required angular resolution is already achievable with today's telescopes. The main problem comes from the extremely large intensity ratio between the star and the planet, typically between a million and a billion, depending on wavelengths and particular science goals.

Several nulling or coronagraphic techniques have been proposed to cancel the star light and achieve such a direct detection [3]. Several Instrumental projects are under construction or study for ground or space based observations. The problem is to detect a very faint pattern (the planet) over a bright background produced by the star diffraction wings. In the case of ground based observations with Adaptive Optics (AO) the uncorrected aberrations of the wavefront produce random intensity fluctuations of this background (residual speckles). Even at very high AO corrections, those speckles still exist, and are "pinned" on the first diffraction rings for short exposure images [6, 20, 5, 18].

We study the statistics of these pinned speckles, and evaluate the usefulness of a coronagraph. A very simple modelling of the wavefront amplitude permits to obtain the Probability Density Function (PDF) of residual speckles at high light level of AO correction. The photon counting statistics is then derived by Poisson-Mandel transform. Similar results can be obtained using Moment Generating Functions (MGF). From a practical point of view, the relevant information comes from the comparison of the variances of the noises coming either from the coherent part of the wave (that can be suppressed by a coronagraph) or by the residual speckles (that cannot). We emphasize in this study that speckles are amplified by the coherent part of the wave, if they are not suppressed by a coronagraph. The relative weights of these noises are represented by the expressions for their variances, and the usefulness of coronagraphs in the different regimes is discussed.

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2. STATISTICAL MODEL FOR THE WAVE AMPLITUDE AND LIGHT INTENSITY

In this section, we introduce a model to describe the statistical properties of the light intensity at high AO corrections. This model was proposed by Goodman [12] for laser speckles (in the context of holography) and applied to AO images by Cagigal and Canales [7, 10, 9, 8, 11] mainly for the study of the statistics of AO images, as a function of the degree of correction.

2.1 Statistics of the wave amplitude

In the general case, the wavefront amplitude at the entrance pupil can be written as the coherent sum of two terms, a deterministic term A corresponding to a perfect plane wave and a random term $a(x,y)$ corresponding to the uncorrected part of the wavefront (either phase or amplitude errors):

$$\Psi_1(x,y) = [A + a(x,y)]\ P(x,y), \tag{1}$$

where the function $P(x,y)$ describes the aperture transmission. The complex amplitude of the wave in the focal plane is given by a scaled Fourier Transform of this pupil amplitude [14]:

$$\Psi_2(x,y) = \mathscr{F}[\Psi_1(x,y)]_{\frac{x}{\lambda f},\frac{y}{\lambda f}} \tag{2}$$

where f denotes the telescope focal length, λ the monochromatic wavelength and the symbol $\mathscr{F}$ the Fourier Transform (FT). The focal complex amplitude can be written as the sum of two complex amplitudes:

$$\begin{aligned}\Psi_2(x,y) &= A \times \mathscr{F}[P(x,y)]_{\frac{x}{\lambda f},\frac{y}{\lambda f}} \\ &\quad + \mathscr{F}[a(x,y) \times P(x,y)]_{\frac{x}{\lambda f},\frac{y}{\lambda f}} && (3)\\ &= C(x,y) + S(x,y) && (4)\end{aligned}$$

The deterministic term $C(x,y) = A \ \mathscr{F}\ [P(x,y)]_{x/(\lambda f),y/(\lambda f)}$ is proportional to the wave amplitude without atmospheric turbulence. The second component is a random term, associated to the speckles:
$S(x,y) = \mathscr{F}[a(x,y) \times P(x,y)]_{x/(\lambda f),y/(\lambda f)}$.

Whatever the statistics of $a(x,y)$, the complex amplitude $S(x,y)$ follows a circular gaussian distribution, thanks to the central limit theorem, assuming a large enough number of independent phasors on the pupil. Therefore, the wave complex amplitude in the focal plane $\Psi_2(x,y)$ follows gaussian law, decentered by the mean of the amplitude $< \Psi_2(x,y) > \ = \ C(x,y)$.

This problem is equivalent the study of speckles over a coherent background and the statistics of $\Psi_2(x,y)$, given by Goodman [13] is:

$$P(\Psi_2^{(r)}, \Psi_2^{(i)}) = \frac{1}{\pi I_s} \exp\left(\frac{-(\Psi_2^{(r)} - C)^2 + (\Psi_2^{(i)})^2}{I_s} \right), \quad (5)$$

where $\Psi_2^{(r)}$ and $\Psi_2^{(i)}$ denote the real and imaginary part of $\Psi_2(x,y)$. We omit the variables (x,y) everywhere, for clarity. We also use the notation $I_s =< |S(x,y)|^2 >$ for the mean intensity of the speckle term.

The deterministic term $C(x,y)$ is a complex quantity. However, for an aperture with symmetries, $C(x,y)$ can be real. For example, in the case of a circular aperture of diameter D, we obtain the Airy amplitude $C(r) = D\ \frac{J_1(\pi D r)}{2r}$, with $r = \sqrt{x^2+y^2}$

2.2 Statistics of the PSF light intensity in the residual speckle zone

The instantaneous intensity in the focal plane is the modulus squared of the amplitude:

$$\begin{aligned} |\Psi_2(x,y)|^2 &= |C(x,y) + S(x,y)|^2 & (6)\\ &= |C(x,y)|^2 + |S(x,y)|^2 + 2\mathrm{Re}[C^*(x,y)\, S(x,y)]. & (7)\end{aligned}$$

The term coupling the two deterministic and random parts (C and S) corresponds to the so-called "speckle pinning", discussed by several authors [6, 20, 5, 18].

The mean intensity (long exposure image), is simply the sum of the deterministic diffraction pattern with a halo produced by the average of the speckles.

$$\begin{aligned} < |\Psi_2(x,y)|^2 > &= |C(x,y)|^2 + < |S(x,y)|^2 > \\ &= I_c + I_s, \qquad (8)\end{aligned}$$

with $< S(x,y)^* > = < S(x,y) >^* = 0$ (circular gaussian distribution). $I_c = |C(x,y)|^2$ is the intensity of the deterministic part of the wave, and $I_s =< |S(x,y)|^2 >$ as previously. For a totally developed speckle structure, the intensity I_s is a constant, but a function of the radial distance r for a AO halo. The long exposure image is therefore the sum a term proportional to the perfect impulse response (Airy pattern) and a halo.

The statistics of the light intensity can be derived from that of the complex amplitude. The marginal PDF for the intensity, known as a *modified Rician density*, was given by Goodman [13] and also used by [7]:

$$P_I(I) = \frac{1}{I_s} \exp\left(-\frac{I+I_c}{I_s}\right) \mathrm{I}_0\left(\frac{2\sqrt{I I_c}}{I_s}\right), \quad (9)$$

This Rician distribution is illustrated in Fig.1, for a same speckle intensity I_s and several continuous intensities I_c.

The corresponding MGF (Laplace transform of the PDF), finds an analytical expression of the form :

$$\begin{aligned} M(u) &=< \exp[uI] > \\ &= \exp\left(\frac{I_c u}{1 - I_s u}\right) \frac{1}{1 - I_s u} \qquad (10)\end{aligned}$$

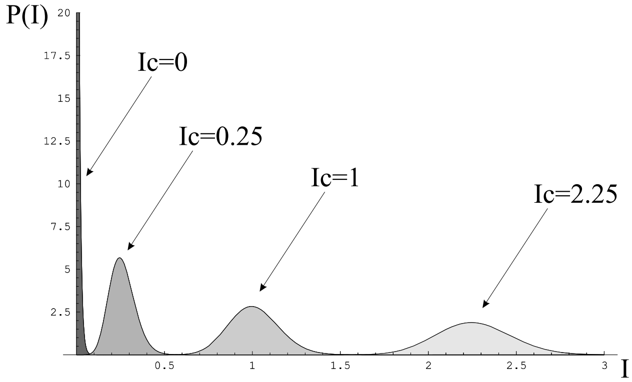

Figure 1: Superimposition of intensity PDFs (grey shades) for four continuous background level I_c and same value of $I_s = 0.1$. The width of the distribution strongly increases with the level of continuous from $Ic = 0$ to $Ic = 2.25$. This result shows the amplification of the speckle by the coherent part of the wave (speckle pinning).

At photon counting levels, the PDF suffers a Poisson-Mandel transformation. An analytical expression of the light intensity probability can be obtained:

$$\begin{aligned} P(n) &= \frac{1}{n!} \int_0^\infty P_I(I) I^n \exp(-I) \mathrm{d}I & (11)\\ &= \frac{1}{I_s+1} (1 + \frac{1}{I_s})^{-n} \exp\left(-\frac{I_c}{I_s}\right)\ {}_1F_1\left(n+1; 1; \frac{I_c}{I_s^2 + I_s}\right), \end{aligned}$$

where ${}_1F_1$ is the Kummer confluent hypergeometric function. An equivalent expression was used by Cagigal and Canales [9] using the Laguerre polynomial, related to the Hypergeometric function by the relation: ${}_1F_1(n+1, 1, x) = L_n(-x)\exp(x)$. This distribution is illustrated in Fig.2.

The MGF at low light levels is easily obtained from the high light level MGF, making the simple change of variable $u \rightarrow exp(u) - 1$. We obtain :

$$M(u) = \frac{1}{1 - (\exp(u) - 1) I_s} \exp \frac{(\exp(u) - 1) I_c}{(1 - (\exp(u) - 1) I_s)} \quad (12)$$

2.3 Statistics of the intensity at the planet position

In the previous section we have studied the statistical properties of a speckle pattern when a coherent amplitude was added to it.

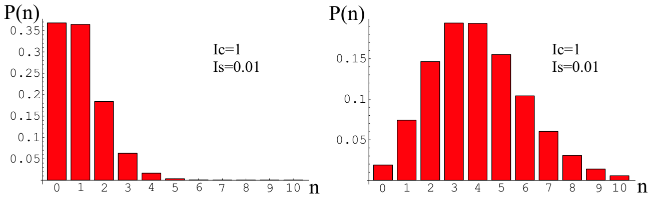

Figure 2: Illustration of the low flux statistics of AO speckles (Eq.11), for an arbitrary speckle intensity $I_s = 0.01$ and two different values of I_c.

We also have to study the problem of incoherent intensity added to a speckle pattern, corresponding to the planet light added to the star speckle background. We shall assume that the planet adds a constant value m in one pixel only, and use the approach developed by Aime [2]. The PDF is simply shifted of the quantity m, so that:

$$P_{Ip}(I) = P_I(I - m) \tag{13}$$

and the MGF is therefore multiplied by a term $exp(mu)$:

$$M_p(u) = M(u)exp(mu) \tag{14}$$

Using the same change of variable as above, the photon counting MGF for a pixel at the planet position becomes :

$$M(u) = \frac{1}{1-(\exp(u)-1)I_s} \exp \frac{(\exp(u)-1)I_c}{(1-(\exp(u)-1)I_s)} \times \exp(m(\exp(u)-1)) \tag{15}$$

These distributions can be used for a deeper understanding of several speckle techniques proposed for the detection of exoplanets, like the dark-speckles [16]. However, in the present communication, we focus the presentation on noise fluctuations alone, and study their possible reduction, using coronagraphic techniques.

3. VARIANCE OF THE INTENSITY

In this note, our goal is to determine under what conditions the use of a coronagraph will substantially reduce this variance (suppressing the coherent term that amplifies the speckles), when is it is worth constructing such a device (a very elaborated and expensive interferometric device). The variance of the intensity at and around the planet location in the image can be used as a simple criterion for that. The variance of the speckles alone can be obtained by several ways: one is to express the second order moment $< I^2(x,y) >$ with $C(x,y)$ and $S(x,y)$ and to use the properties of gaussian distributions [12, 13]. The moments of Eq.9 also find an analytical expression [12]:

$$< I^n >= I_s^n \exp\left(\frac{-I_c}{I_s}\right) n!\,{}_1F_1\left(n+1, 1, \frac{I_c}{I_s}\right) \tag{16}$$

The easiest way is probably to compute them from the MGF, using the relation $< I^n >= M^{(n)}(0)$. Whatever the mean used, we obtain:

$$< I >= I_s + I_c \tag{17}$$
$$< I^2 >= I_c^2 + 4I_cI_s + 2I_s^2 \tag{18}$$

and the variance:

$$\sigma_I^2 = I_s^2 + 2I_sI_c \tag{19}$$

This result was discussed by Goodman for the addition of a laser speckle pattern with a continuous background, and by Cagigal and Canales (reference herein) for the study of corrected PSF intensity statistics as a function of the degree of correction.

At low light levels, we must take into account the variance associated with photodetection (Poisson process) and the total variance becomes:

$$\sigma^2 = \sigma_I^2 + \sigma_P^2 \tag{20}$$

where, σ_P^2 is the variance associated to the poisson statistics. With our notations, $\sigma_P^2 = I_c + I_s$. The total variance is:

$$\sigma^2 = I_s^2 + 2I_sI_c + I_c + I_s \tag{21}$$

This result can be also directly obtained from the photon-counting MGF. Now we will see that the variance at the planet position is the same. The variance of the high light level signal is unaffected by the planet, which is a simple intensity shift (this is verified using the MGF for the planet). The planet contributes to the overall variance by the quantity m, its own photon variance. In all cases of interest this term can be neglected compared to the others.

4. DISCUSSION

The goal of stellar coronagraphy is to remove, as best possible the star diffracted light, using optical filtering. Coronagraphy basically consists of using two masks (opaque or phase shifting), in the focal plane and in a relay pupil plane. Some coronagraphs can reach a total star extinction in the perfect case [4, 21, 1], rejecting all the star energy outside the telescope aperture, in a relay pupil plane. However, in presence of residual aberrations, those coronagraphs will only work on perfect part $C(x,y)$ of the wave, whilst the speckle part $S(x,y)$ will remain mostly unaffected.

The model we presented in this communication provides a tool to understand how a coronagraph can reduce the noise variance and improve the detection limit of an exoplanet. Let us partition for that the total variance of Eq.21 into two contributions, one coming from the continuous background I_c and the other one from the speckle term I_s:

$$\sigma^2 = \sigma_c^2 + \sigma_s^2 \tag{22}$$
$$= (2I_sI_c + I_c) + (I_s^2 + I_s), \tag{23}$$

with $\sigma_c^2 = 2I_sI_c + I_c$ and $\sigma_2^2 = I_s^2 + I_s$. A coronagraph will only have an effect on the term σ_c.

- If $I_c << I_s$, far from the optical axis or with a perfect coronagraph, then the variance reduces to $\sigma_s^2 = I_s^2 + I_s$ and $I_s = 1$ photon per pixel is a limit of regime (the variance is dominated either by the speckle noise, or by the photon noise).
- If $I_c >> I_s$, close to the optical axis (on the first diffraction rings) or without any coronagraph, the variance becomes $\sigma_c^2 = 2I_cI_s + I_c$. One photon per pixel is also a limit of regime.
- The transition domain for $I_c = I_s$ leads to the variance $\sigma_I^2 = 3I_s^2 + 2I_s$

A coronagraph will be efficient in the part of the focal field where $\sigma_c^2 > \sigma_s^2$. We can deduce the following criterion for coronagraphic efficiency:

$$\begin{aligned} \sigma_c^2 &> \sigma_s^2 \\ Ic &> \frac{I_s(I_s+1)}{2I_s+1}. \end{aligned} \qquad (24)$$

At high flux, $I_s >> 1$ and the condition of Eq.24 is equivalent to $I_c^{(1)} > \frac{I_s}{2}$. At photon counting rates $I_s << 1$, this limit is equivalent to $I_c^{(2)} > I_s$. In both cases, the order of magnitude is similar, and does not depend on the number of photons. We can conclude that for either low or high flux, a coronagraph is efficient as long as $I_c > I_s$, in terms of S/N.

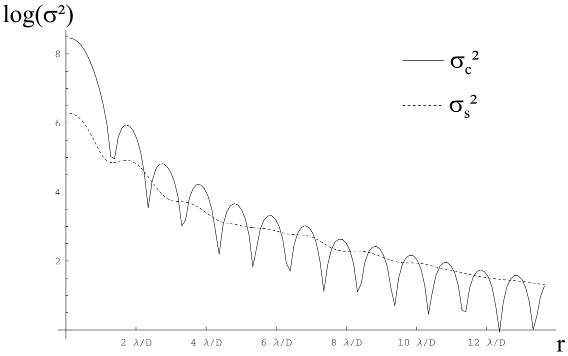

Figure 3: Illustration of σ_c^2 and σ_s^2, as a function of the radial position in the focal plane, without coronagraph. The simulation is made for a 3.6m telescope in the H Band with a Strehl Ratio of 90%. This simple simulation corresponds to the Lyot project coronagraph on the AEOS telescope in Hawaii [17]. A coronagraph will be efficient everywhere the variance σ_c^2 (hard line) is greater than σ_s^2 (dashed line). This simulation correspond to a high flux regime and is independent of the exposure time.

As a conclusion, we have presented a statistical model that can provide a valuable tool for the analysis of a coronagraph, at high flux or photon counting rates. This work will be extended to actual S/R computations, based on this formalism. A coronagraph for a given telescope and AO, should reduce the contribution σ_c^2 lower than σ_s^2 everywhere in the field. This is illustrated in Fig.3, using the PAOLA software of the AO simulation [15]

Several other noise terms are involved and have not been taken into account in this study. In particular, a coronagraph will provide also a gain on the read out noise, allowing increased exposure time.

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