AN MMSE INTERFERENCE ESTIMATOR FOR TURBO BLAST SYSTEMS

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ABSTRACT

Turbo BLAST(Bell LAb's Layered Space Time) [1] detection and decoding has demonstrated promising performance for coded BLAST systems. In Turbo BLAST system, an iterative interference cancellation and decoding approach is adopted to simplify processing. The interference is estimated using the *extrinsic* information from the decoder, which we refer as extrinsic estimator. In this paper, we propose an optimal Minimum Mean Square Error (MMSE) interference estimator for Turbo BLAST system. We show that the mean square error of interference estimation is improved using the proposed estimator. As a result, for QPSK system, the Turbo BLAST system using the MMSE estimator achieves superior performance compared to extrinsic estimator at low and medium SNR regions. At high SNR's, performance of both systems converges to the performance lower bound. For Turbo BLAST system with 16QAM modulation, using the extrinsic estimator leads to divergence of the bit error rate (BER) performance, while using the proposed estimator, the BER still converges to the lower bound.

1. INTRODUCTION

It was shown in [2] that large capacity can be exploited in rich scattering wireless channels by employing multiple antennas at both the transmitter and the receiver. The BLAST [2] [3] structure was proposed to realize such systems. The reported spectral efficiency of the prototype Vertical-BLAST (VBLAST) system reaches 20 to 40 bps/Hz at average SNR's between 24 to 34 dB.

Error control coding (ECC) can be employed in BLAST systems to improve its robustness against fading. Iterative detection and decoding, where soft reliability information is exchanged iteratively between the detector and decoder, was proposed in [4] for multiuser detection in coded CDMA systems. The concept is extended to BLAST system and Turbo BLAST was proposed in [1]. In Turbo BLAST system, an iterative interference cancellation and decoding scheme is adopted. The interference is estimated as the statistical mean of the transmitted signal calculated from the *extrinsic* information at the output of the soft input soft output (SISO) maximum *a posteriori* probability (MAP) decoder. We, therefore, refer it as the extrinsic estimator in the rest of the paper.

In this paper, we propose a new MMSE interference estimator for Turbo BLAST systems, which minimizes the power of the residue interference in Turbo detection. We further evaluate the BER performance of Turbo BLAST system using the proposed estimator through computer simulations. We show that for system using QPSK modulation, using the proposed estimator, the performance of the Turbo BLAST system is improved at medium and low SNR regions compared to using extrinsic estimator. At high SNR region, both the proposed estimator and extrinsic estimator leads to BER convergence to the low performance bound. For system employing 16QAM modulation, the BER using the proposed estimator is significantly better than using the extrinsic estimator in all SNR regions. For a VBLAST system with 4 transmit antennas and 6 receive antennas, the proposed scheme reaches the lower BER bound at 8.5 dB. However the BER using the extrinsic estimator fails to converge.

The rest of this paper is organized as follows. In Section II, we introduce the system model of horizontal-coded VBLAST system and the conventional Turbo BLAST detector and decoder. The new MMSE interference estimator is proposed in Section III. In Section IV, we present the performance of the MMSE interference estimator for Turbo BLAST systems in terms of MSE (Mean Square Error) of interference estimation and BER. The conclusion is drawn in Section V.

2. SYSTEM DESCRIPTION

For coded BLAST systems, it was shown in [5] that independent coding on each data stream (horizontal coding) results in the best performance, compared with vertical coding and hybrid coding structures. Therefore, in this paper, we focus on the study of horizontal encoding with convolutional code. Figure 1 depicts the diagram of a horizontal-coded VBLAST system transmitter, on which the encoding, bit interleaving and constellation mapping are performed independently on each data stream. For a $m \times n$ (m denotes the number of transmit antennas and n denotes the number of receive antennas) VBLAST system, the received signal is given by

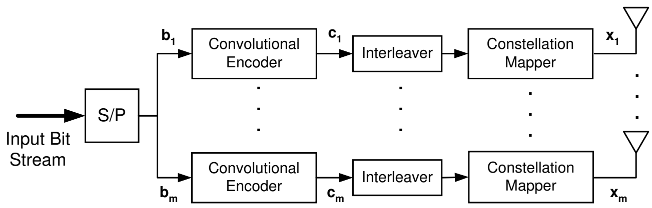

Figure 1: Horizontal-coded VBLAST system transmitter

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}$ is a $n \times m$ channel matrix with $\mathrm{H}_{i,j}$ representing the channel response between transmit antenna j and receive antenna i. $\mathbf{x}$ denotes the transmit signal from m transmit antennas and $\mathbf{n}$ is the AWGN noise vector with variance η^2.

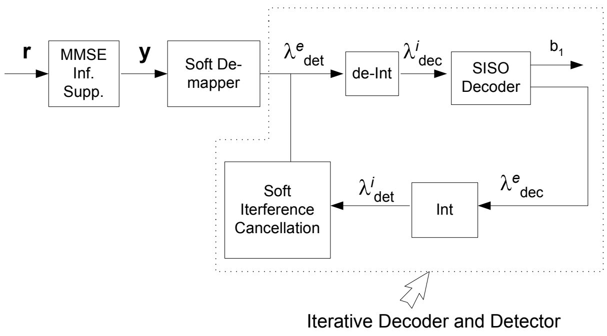

Figure 2: Conventional Turbo BLAST detector and decoder using extrinsic information for interference cancellation

Figure 2 depicts the conventional Turbo BLAST iterative detector and decoder [1]. The received signal is first passed through an MMSE interference suppression filter given by $\mathbf{w}_i^H = \mathbf{h}_i^H \left[\mathbf{H}\mathbf{H}^H + \eta^2 \mathbf{I}\right]^{-1}$ with $\mathbf{h}_i$ indicating the ith column of $\mathbf{H}$. To detect x_1, we calculate

$$\begin{aligned} y_1 &= \mathbf{w}_1^H(\mathbf{H}\mathbf{x}+\mathbf{n}) \\ &= \underbrace{\mathbf{w}_1^H \mathbf{h}_1 x_1}_{desired\ signal} + \underbrace{\mathbf{w}_1^H(\mathbf{h}_2 x_2 + \cdots + \mathbf{h}_m x_m)}_{Interference} \\ &\quad + \underbrace{\mathbf{w}_1^H \mathbf{n}}_{AWGN\ noise}, \end{aligned} \tag{2}$$

which consists the desired signal, the interference from the other transmit antennas and the AWGN noise.

From [6], the distribution of the interference plus the AWGN noise can be well approximated by Gaussian. Therefore, we can write y_1 using an equivalent AWGN channel model $y_1 = a_1 x_1 + \tilde{n}$, where $a_1 = \mathbf{w}_1^H \mathbf{h}_1$ is the equivalent channel gain and variance of $\tilde{n}$, $\sigma_1^2 = a_1 - |a_1|^2$ [4]. To explain the bit metric calculation of the transmitted bits, we use λ with subscript *det* and *dec* to denote the a posteriori Log Likelihood Ratio (LLR) obtained from the *detector* and *decoder* respectively. We also use λ, λ^a and λ^e to indicate the *a posteriori*, *a priori* and *extrinsic* LLR.

Let c_k^i be the kth bit of x_i, the LLR of coded bits c_k^i is defined as

$$\lambda_{det}(c_k^i) = \ln \frac{P(c_k^i = 1|y_i)}{P(c_k^i = 0|y_i)} = \lambda_{det}^e(c_k^i) + \lambda_{det}^a(c_k^i), \tag{3}$$

where the *a posteriori* LLR is the sum of the *extrinsic* LLR and the *a priori* LLR.

To calculate the extrinsic LLR of c_k^i, we denote χ_k^1 and χ_k^0 as the subsets of all the constellation points χ, with elements having the kth bit equal to 1 and 0, respectively. Subsequently, $\lambda_{det}^e(c_k^i)$ after the MMSE detection can be obtained as

$$\begin{aligned} \lambda_{det}^e(c_k^i) &= \ln \frac{P(y_i|c_k^i = 1)}{P(y_i|c_k^i = 0)} \\ &= \ln \frac{\sum_{x_i \in \chi_k^1} \left[\frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{|y_i - a_i x_i|^2}{2\sigma_i^2}\right) P(x_i)\right]}{\sum_{x_i \in \chi_k^0} \left[\frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{|y_i - a_i x_i|^2}{2\sigma_i^2}\right) P(x_i)\right]} \end{aligned} \tag{4}$$

Assuming all x_i's are equal likely, (4) can be further simplified to

$$\lambda_{det}^e(c_k^i) = \sum_{x_i \in \chi_k^1} \left[-\frac{|y_i - a_i x_i|^2}{2\sigma_i^2}\right] - \sum_{x_i \in \chi_k^0} \left[-\frac{|y_i - a_i x_i|^2}{2\sigma_i^2}\right] \tag{5}$$

The extrinsic LLR $\lambda_{det}^e(c_k^i)$ is passed through the de-interleaver, the output of which forms the *a priori* LLR to be used by the SISO decoder

$$\lambda_{dec}^a(c_k^i) = \Pi^{-1}(\lambda_{det}^e(c_k^i)), \tag{6}$$

where Π^{-1} denotes de-interleaving.

The diversity achieved through MMSE interference suppression is only 1. To fully explore the diversity of BLAST system, the soft reliability information from the output of the SISO decoder is feedback to perform soft interference cancellation and maximal ratio combining (MRC) as following,

$$\tilde{y}_k = \mathbf{h}_k^H \left(\mathbf{r} - \sum_{i=1, i\neq k}^{m} \mathbf{h}_i \hat{x}_i\right) = \mathbf{h}_k^H \left(\mathbf{h}_k x_k + \sum_{i=1, i\neq k}^{m} \mathbf{h}_i (x_i - \hat{x}_i) + \mathbf{n}\right), \tag{7}$$

where $\hat{x}_i$ indicates the soft estimate of the transmitted signal x_i from the SISO decoder output. For extrinsic estimator, $\hat{x}_i = \mathrm{E}(x_i)$, where E denotes statistical expectation. For an M-ary modulation system, it is the function of the extrinsic LLR $\lambda_{dec}^e(c_k^i)$ for $k = 1, \cdots, M$ of all the coded bits c_k^i of x_i from the SISO decoder. For example, the extrinsic estimate for QPSK modulated signals is given by

$$\mathrm{E}(x_i) = \frac{1}{\sqrt{2}} \left\{ \tanh\left(\frac{\lambda_{dec}^e(c_1^i)}{2}\right) + j \cdot \tanh\left(\frac{\lambda_{dec}^e(c_2^i)}{2}\right) \right\}. \tag{8}$$

if the mapping rule is $x_i = \frac{1}{\sqrt{2}}\left[(2c_1^i - 1) + j(2c_2^i - 1)\right]$.

3. MMSE INTERFERENCE ESTIMATOR

From (7), we can see the total noise after MRC is the sum of the residue interference $x_i - \hat{x}_i$ and the Gaussian noise. In this paper, we propose a new estimator which minimizes the power of the residue interference. The cost function can be formulated as:

$$J(\hat{x}_i) = \mathrm{E}\left[|x_i - \hat{x}_i|^2\right]. \tag{9}$$

The expectation is taken over all possible values of x_i, which belongs to the constellation set χ, and y_i.

Expanding (9), we get

$$\begin{aligned} J(\hat{x}_i) &= \int_{y_i} \sum_{x_i \in \chi} |x_i - \hat{x}_i|^2 P(x_i, y_i) dy_i \\ &= \int_{y_i} \left[\sum_{x_i \in \chi} |x_i - \hat{x}_i|^2 P(x_i|y_i)\right] P(y_i) dy_i \end{aligned} \tag{10}$$

where $P(\bullet)$ denotes probability.

Since $P(y_i) \geq 0$ for all the values of y_i, the cost function can be minimized by minimizing the terms in the square bracket for each value of y_i. The optimal MMSE solution can be found as

$$\hat{x}_i = \sum_{x_i \in \chi} x_i P(x_i|y_i) = \mathrm{E}(x_i|y_i), \tag{11}$$

which is same as the Bayesian's estimator given in [7].

To obtain $\mathrm{E}(x_i|y_i)$, we make use of the LLR of coded bits from the output of the SISO decoder. The expected value of c_k^i given y_i can be calculated as

$$\begin{aligned}\mathrm{E}(c_k^i|y_i) &= (+1)P(c_k^i=+1|y_i)+(-1)P(c_k^i=-1|y_i)\\ &= (+1)\frac{\exp\left(\lambda_{dec}(c_k^i)\right)}{1+\exp\left(\lambda_{dec}(c_k^i)\right)}+(-1)\frac{\exp\left(-\lambda_{dec}(c_k^i)\right)}{1+\exp\left(-\lambda_{dec}(c_k^i)\right)}\\ &= \tanh\left(\frac{\lambda_{dec}(c_k^i)}{2}\right). \qquad (12)\end{aligned}$$

For QPSK modulated signal, $\hat{x}_i$ can be calculated as

$$\begin{aligned}\hat{x}_i &= \mathrm{E}(x_i|y_i)=\frac{1}{\sqrt{2}}\left[\mathrm{E}\left(c_1^i|y_i\right)+\mathrm{E}\left(c_2^i|y_i\right)\right]\\ &= \frac{1}{\sqrt{2}}\left[\tanh\left(\frac{\lambda_{dec}(c_1^i)}{2}\right)+j\tanh\left(\frac{\lambda_{dec}(c_2^i)}{2}\right)\right]. \qquad (13)\end{aligned}$$

For 16QAM modulated signal, $\hat{x}_i$ is given by

$$\begin{aligned}\hat{x}_i &= \frac{-3\exp\left[-\lambda(c_1^i)-\lambda(c_2^i)\right]-\exp\left[-\lambda(c_1^i)\right]+3\exp\left[-\lambda(c_2^i)\right]+1}{\sqrt{10}\left(1+\exp[-\lambda(c_1^i)]\right)\left(1+\exp[-\lambda(c_2^i)]\right)}\\ &+j\frac{-3\exp\left[-\lambda(c_3^i)-\lambda(c_4^i)\right]-\exp\left[-\lambda(c_3^i)\right]+3\exp\left[-\lambda(c_4^i)\right]+1}{\sqrt{10}\left(1+\exp[-\lambda(c_3^i)]\right)\left(1+\exp[-\lambda(c_4^i)]\right)} \qquad (14)\end{aligned}$$

Here, all the λ's denotes LLR information from the SISO decoder. We omit the subscript *dec* to avoid long equations.

Compare this with the extrinsic estimator, we are now using the *a posteriori* LLR from the SISO decoder rather than the extrinsic LLR to estimate the interference in (2). As the calculation of the new estimate is the same as the conventional approach, no extra complexity is involved using the new estimator. Using the new estimate, we are able to obtain a lower MSE of interference estimation. As a result, the proposed estimator leads to better BER performance in Turbo BLAST systems.

4. SIMULATION RESULTS

In this section, we present the simulation results of the Turbo BLAST system using the proposed MMSE estimator. The system is a bit-interleaved convolutional-coded BLAST system with horizontal encoding. The channel is narrow band fast and flat Rayleigh fading. We assume the channel response between different pairs of antennas are uncorrelated. We also assume perfect channel state information at the receiver. The error control coding used in the simulations is a half-rate convolutional code with constraint length of 7 and generator polynomials $g_0 = 133_8$ and $g_1 = 171_8$. Interleaving is done in two stages. The first stage maps adjacent coded bits onto nonadjacent constellation symbols, while the second guarantees that adjacent coded bits are mapped alternately onto less and more significant bits of the constellation [8].

Figure 3 shows the comparison of the MSE for the proposed estimator using *a posteriori* LLR and the extrinsic estimator using extrinsic LLR for three different $\frac{E_b}{N_0}$ values. There are 8 transmit and 8 receive antennas in the system and QPSK modulation is used. We can see for all the three cases, the MSE using the proposed estimator is significantly better. As a result, a better estimate is provided for interference cancellation, which leads to less residue interference and better BER performance.

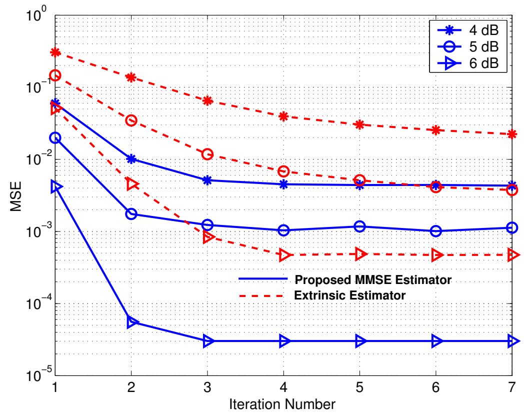

Figure 3: MSE of interference estimation using different estimators (8×8 coded BLAST system QPSK modulation)

Figure 4 compares the MSE using the proposed MMSE estimator and the extrinsic estimator for system using 16QAM modulation for 3 different $\frac{E_b}{N_0}$ values. Here, we use a VBLAST system with 4 transmit and 6 receive antennas. Again we can see that the MSE produced by the MMSE estimator is significantly lower compared to extrinsic estimator. An interesting observation is that using the extrinsic estimator, the MSE fails to converge through iterations. This will lead to worse BER performance with larger number of iterations using the extrinsic estimator for 16 QAM systems, as we are going to show later.

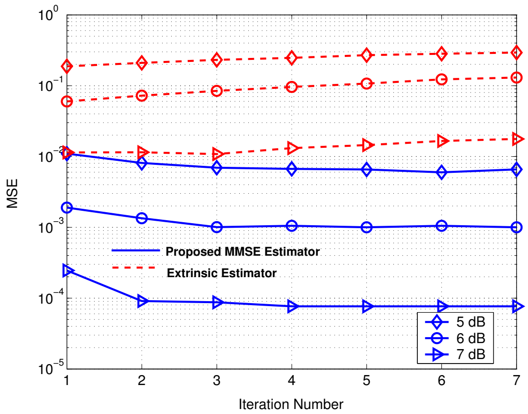

Figure 4: MSE of interference estimation using different estimators (4×6 coded BLAST system, 16QAM modulation)

Figure 5 shows the BER performance of an 8×8 Turbo BLAST system using the proposed MMSE estimator. In comparison, we include the BER using the extrinsic estimator as well. Also shown in the figure is the low bound for Turbo BLAST system, which is obtained by assuming perfect estimation of interference so that the interference can be removed completely. We can see that with both estimators, the BER of the Turbo BLAST is improved through iterations. In practise, we only need to perform 3 iterations as the improvement from the 3rd to the 4th iteration, for both cases, is marginal. We can see the Turbo BLAST system with the MMSE estimator has clearly better performance over extrinsic estimator. We can achieve a performance gain of 0.5dB at BER of 10^{-3} and 0.45dB at BER of 10^{-4}. For higher SNR values, as shown in Figure 3, the MSE for both estimators is small. The difference between the MSE's of two estimators becomes less important as the Gaussian noise becomes dominant in the total noise contribution in (7). Hence, the performance difference of Turbo BLAST system using two estimators becomes smaller. The BER of both system merges with the lower BER bound at high SNR region.

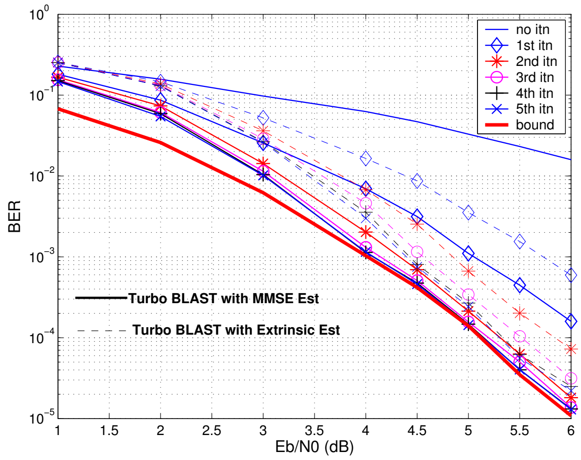

Figure 5: Comparison of BER performance for Turbo BLAST system using the proposed estimator and extrinsic estimator (8×8 coded BLAST system, QPSK modulation).

Figure 6 shows the comparison of BER performance for a 16-QAM modulated Turbo BLAST system with 4 transmit and 6 receive antennas using the MMSE estimator and the extrinsic estimator. Again we can see using the proposed estimator, the BER of the system is improved by performing more iterations. Only 3 iterations are necessary since the improvement by performing more than 3 iterations is marginal. The BER using the proposed estimator converges to the lower BER bound at 8.5 dB. However, using the extrinsic estimator, the BER of the Turbo BLAST system is getting worse performing more iterations. This is expected because the extrinsic interference estimator produces large MSE for larger number of iterations as shown in Figure 4.

5. CONCLUSION

In this paper, we proposed a new MMSE estimator for Turbo BLAST systems. We show that this proposed estimator minimizes the power of the residue interference in the iterative interference cancellation stage. It results in much lower MSE of interference estimation, hence less residue interference, compared to the conventional extrinsic estimator. From the simulation result, we observe for QPSK modulated Turbo BLAST system, the BER using both estimators can converge to the lower performance bound, while the proposed estimator gives superior performance at medium to low SNR. For higher order modulations, such as 16 QAM, using the pro-

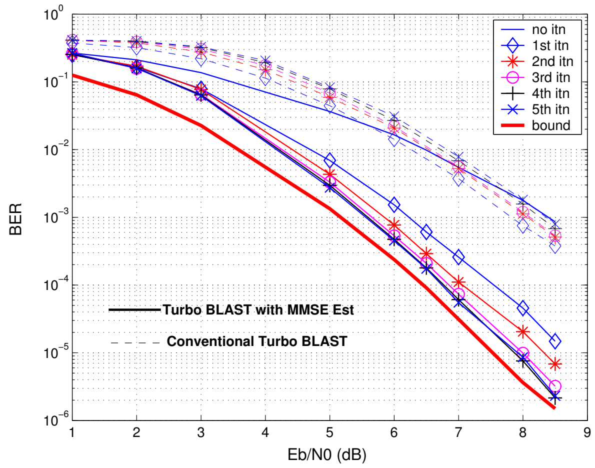

Figure 6: Comparison of BER performance for Turbo BLAST system using the proposed estimator and extrinsic estimator (4×6 coded BLAST system, 16QAM modulation).

posed estimator, the BER of Turbo BLAST system can still converge to lower BER bound, however, the BER fails to converge using the conventional extrinsic estimator.

REFERENCES

[1] M. Sellathurai and S. Haykin, "TURBO-BLAST for wireless communications: theory and experiments ," *IEEE Transactions on Communications*, vol. 50, Issue: 10, pp. 2538-2546, Oct. 2002.

[2] G. J. Foschini, "Layered space-time architecture for wireless communications in a fading environment when using multi-element antennas," *Bell Labs Technical Journal*, pp.41-59, 1996.

[3] G. J. Foschini, G. D. Golden, R. A. Valenzuela, and P. W. Wolnianski, "Simplified processing for high spectral efficiency wireless communication employing multi-element arrays," *IEEE Journal on Selected Areas of Communications*, vol. 17, No.11, pp. 1841-1852, Nov. 1999.

[4] X. Wang and H. V. Poor, "Iterative (Turbo) soft interference cancellation and decoding for coded CDMA," *IEEE Transactions on Communications*, vol. 47, no.7, pp. 1046-1061, Jul. 1999.

[5] X. Li, H. Huang, G. J. Foschini, and R. A. Valenzuela, "Effects of iterative detection and decoding on the performance of BLAST," *IEEE Global Telecommunications Conference, 2000*, vol. 2, pp. 1061-1066, 2000

[6] H. V. Poor and S. Verdú, "Probability of error in MMSE multiuser detection," *IEEE Transactions on Infomation Technology*, vol. 43, no.3, pp. 858 - 871, May 1997.

[7] S. M. Kay, "Fundamentals of statistical signal processing: estimation theory," *Prentice Hall*, 1998.

[8] "Part11: wireless LAN medium access control (MAC) and physical layer (PHY) specifications: High-speed physical layer in the 5GHz band," *IEEE std 802.11a-1999:Supplement to IEEE std 802.11-1999*, Sept, 1999.