UNBIASED EQUATION-ERROR APPROACH FOR EFFICIENT IIR SYSTEM IDENTIFICATION

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ABSTRACT

Based on the equation-error approach, a weighted least squares algorithm with a generalized unit-norm constraint is developed for unbiased infinite impulse response (IIR) system identification in the presence of white input and/or output noise. Through using a weighting matrix, the proposed estimator can be considered as a generalization of the Koopmans-Levin method and it produces better estimation accuracy. Computer simulations are included to evaluate the estimator performance for IIR parameters in different conditions.

1 Introduction

The problem of identifying linear systems from their input and output measurements has received considerable attention because of its important applications in signal processing, communications and control [1]-[2]. In the general case of infinite impulse response (IIR) system identification, there are two common configurations [3] to tackle the problem, namely, output-error (OE) and equation-error (EE), which correspond to minimization of multimodal and quadratic error functions, respectively. In this paper, the EE approach is considered because it has the main advantage of global convergence over the OE approach, although compensation of the parameter bias [3] induced by measurement noise is required in order to achieve unbiased estimation.

In the presence of white output noise only, Regalia [4] has proposed to minimize the EE cost function subject to a unit-norm constraint. Recently, it has been shown [5] that the constrained EE optimization can be formulated in a mixed least squares (LS)-total least squares framework. When both input and output are corrupted by white noises with known power ratio, the Koopmans-Levin (KL) method [6] can provide unbiased system parameter estimates via finding the eigenvector corresponding to the minimum eigenvalue of the sample covariance matrix determined from the noisy measurements. It is also possible to realize the KL method employing the measurements directly and the solution is solved by singular value decomposition (SVD) [7]. However, all the above schemes utilize LS optimization and thus they generally cannot produce optimum estimation performance. In this paper, we will focus on developing a constrained weighted least squares (WLS) estimator based on [4] for IIR system identification in noisy input and/or output, which expects to outperform the LS counterpart.

The rest of the paper is organized as follows. In Section 2, a EE-based impulse response estimation algorithm is devised via minimizing a WLS cost function subject to a generalized unit-norm constraint. An iterative procedure is suggested to determine the optimum weighting matrix and it can be considered as the WLS version of [7]. Simulation results are presented in Section 3 to evaluate the proposed algorithm by comparing with different benchmarks, namely, the KL, OE [2], prediction-error (PE) [2] methods as well as the asymptotic Cramér-Rao lower bound (CRLB) for infinite measurements [8]. Finally, conclusions are drawn in Section 4.

2 Algorithm Development

Let the unknown IIR system be

$$H(z) = \frac{B(z)}{A(z)} \tag{1}$$

where

$$A(z) = a_0 + \sum_{l=1}^{L} a_l z^{-l}$$

$$B(z) = -\sum_{l=0}^{L} b_l z^{-l}$$

with $a_0 = 1$. We assume that the orders of the denominator and numerator polynomials are known, and without loss of generality, they have identical values of L. The observed system input and output are

$$\begin{aligned} x_k &= s_k + m_k \\ r_k &= d_k + n_k, \quad k = 0, 1, \cdots N-1 \end{aligned} \tag{2}$$

where s_k and d_k denote the noise-free input and output, respectively, while m_k and n_k represent the input and output measurement noise which are independent of s_k. It is assumed that m_k and n_k are uncorrelated white processes with unknown variances σ_m^2 and σ_n^2, respectively, but the ratio of the noise powers, say, $\alpha^2 = \sigma_n^2/\sigma_m^2$, is available. Given N samples of x_k and r_k, the task is to find a_l and b_l, $l = 0, 1, \cdots, L$.

In the EE formulation, the error function e_k is computed from

$$e_k = \sum_{l=0}^{L} \hat{a}_l r_{k-l} + \sum_{l=0}^{L} \hat{b}_l x_{k-l} \qquad (3)$$

where $\{\hat{a}_l\}$ and $\{\hat{b}_l\}$ are the estimates of $\{a_l\}$ and $\{b_l\}$ up to a scalar since $\hat{a}_0$ is not fixed to be unity. The corresponding LS cost function is then

$$J(\hat{\boldsymbol{\rho}}) = \mathbf{e}^T \mathbf{e} = \hat{\boldsymbol{\rho}}^T \mathbf{Y}^T \mathbf{Y} \hat{\boldsymbol{\rho}} \qquad (4)$$

where

$$\mathbf{e} = [e_{N-1}, e_{N-2}, \cdots, e_L]^T$$

$$\mathbf{Y} = \begin{bmatrix} r_{N-1} & \cdots & r_{N-L-1} & x_{N-1} & \cdots & x_{N-L-1} \\ r_{N-2} & \cdots & r_{N-L-2} & x_{N-2} & \cdots & x_{N-L-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ r_L & \cdots & r_0 & x_L & \vdots & x_0 \end{bmatrix}$$

and

$$\hat{\boldsymbol{\rho}} = [\hat{a}_0, \hat{a}_1, \cdots \hat{a}_L, \hat{b}_0, \hat{b}_1, \cdots, \hat{b}_L]^T$$

It is easy to show that the expected value of $J(\hat{\boldsymbol{\rho}})$ is

$$\begin{aligned} E\{J(\hat{\boldsymbol{\rho}})\} &= (\mathbf{S}\hat{\boldsymbol{\rho}})^T \mathbf{S}\hat{\boldsymbol{\rho}} + \hat{\boldsymbol{\rho}}^T E\{\mathbf{Q}^T \mathbf{Q}\} \hat{\boldsymbol{\rho}} \\ &= \hat{\boldsymbol{\rho}}^T \mathbf{S}^T \mathbf{S} \hat{\boldsymbol{\rho}} + (N-L) \hat{\boldsymbol{\rho}}^T \boldsymbol{\Sigma} \hat{\boldsymbol{\rho}} \sigma_m^2 \qquad (5) \end{aligned}$$

where

$$\boldsymbol{\Sigma} = \mathrm{diag}(\underbrace{\alpha^2, \cdots, \alpha^2}_{L+1}, \underbrace{1, \cdots, 1}_{L+1})$$

and

$$\mathbf{S} = \begin{bmatrix} d_{N-1} & \cdots & d_{N-L-1} & s_{N-1} & \cdots & s_{N-L-1} \\ d_{N-2} & \cdots & d_{N-L-2} & s_{N-2} & \cdots & s_{N-L-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ d_L & \cdots & d_0 & s_L & \vdots & s_0 \end{bmatrix}$$

is the signal component of $\mathbf{Y}$ and $\mathbf{Q}$ represents the noise component such that $\mathbf{Y} = \mathbf{S} + \mathbf{Q}$. We observe that when $\{\hat{a}_l\}$ and $\{\hat{b}_l\}$ equal the true system parameters, then $\mathbf{S}\hat{\boldsymbol{\rho}}$ is a zero vector. However, the minimum of $E\{J(\hat{\boldsymbol{\rho}})\}$ will not correspond to the desired $\{a_l\}$ and $\{b_l\}$ in the presence of input and/or output noise because the second term of (5) also contains the parameter estimates. Extending the idea of the unit-norm constraint approach [4] to noisy input-output systems, unbiased IIR system identification can be achieved via minimizing $E\{J(\hat{\boldsymbol{\rho}})\}$ subject to

$$\hat{\boldsymbol{\rho}}^T \boldsymbol{\Sigma} \hat{\boldsymbol{\rho}} = 1 \qquad (6)$$

which is a generalized unit-norm constraint and we refer this scheme as the unit-norm LS method. Using the technique of Lagrange multipliers, the constrained optimization problem can be solved by computing the generalized eigenvector corresponding to the smallest generalized eigenvalue of the pair $(\mathbf{Y}^T\mathbf{Y}, \boldsymbol{\Sigma})$. In fact, the unit-norm LS algorithm is identical to the KL method [7] and the only difference is that the former uses eigenvalue decomposition while the latter employs SVD.

Nevertheless, we do not expect that the unit-norm LS method will produce optimum parameter estimates because of its LS realization. A straightforward improvement to it is to add a symmetric positive definite matrix, say, $\mathbf{W}$, to the LS cost function. An optimal choice of $\mathbf{W}$ is the Markov estimate [2]:

$$\mathbf{W} = (E\{\boldsymbol{\epsilon} \cdot \boldsymbol{\epsilon}^T\})^{-1} \qquad (7)$$

where

$$\boldsymbol{\epsilon} = [\epsilon_{N-1}, \epsilon_{N-2}, \cdots, \epsilon_L]^T$$

with

$$\epsilon_k = \sum_{l=0}^{L} a_l n_{k-l} + \sum_{l=0}^{L} b_l m_{k-l}$$

Since $\{a_l\}$ and $\{b_l\}$ as well as the noise powers are unknown, practically we should substitute the system parameters with $\hat{\boldsymbol{\rho}}$ in $\mathbf{W}$ and then multiplying it by σ_m^2, so that the resultant matrix is characterized by $\hat{\boldsymbol{\rho}}$ and α^2 only. An iterative procedure for the weighting matrix computation will be presented shortly. Now we first review the constraint of (6) as the cost function is modified as

$$J_w(\hat{\boldsymbol{\rho}}) = \mathbf{e}^T \mathbf{W} \mathbf{e} = \hat{\boldsymbol{\rho}}^T \mathbf{Y}^T \mathbf{W} \mathbf{Y} \hat{\boldsymbol{\rho}} \qquad (8)$$

From (8), it can be deduced that for WLS minimization, the constraint will become:

$$\hat{\boldsymbol{\rho}}^T \boldsymbol{\Upsilon} \hat{\boldsymbol{\rho}} = 1 \qquad (9)$$

where $\boldsymbol{\Upsilon} = E\{\mathbf{Q}^T \mathbf{W} \mathbf{Q}\}$. The matrix $\boldsymbol{\Upsilon}$ can be expressed as

$$\begin{bmatrix} \alpha^2 D_0 & \cdots & \alpha^2 D_L & 0 & \cdots & 0 \\ \alpha^2 D_1 & \cdots & \alpha^2 D_{L-1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \alpha^2 D_L & \cdots & \alpha^2 D_0 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & D_0 & \cdots & D_L \\ 0 & \cdots & 0 & D_1 & \cdots & D_{L-1} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & D_L & \cdots & D_0 \end{bmatrix}$$

where $D_j = \sum_{i=1}^{N-L-j} [\mathbf{W}]_{i,i+j}$, $j = 0, 1, \cdots, L$, with $[\mathbf{W}]_{i,j}$ represents the (i,j)-entry of $\mathbf{W}$. It is observed that (4) and (6) are only a special case of (8) and (9) when $\mathbf{W}$ is the identity matrix and we refer the generalized one as the unit-norm WLS estimator. In practice, we suggest the following procedure for the constrained WLS-based system identification:

(i) Find initial estimates of the system parameters by computing the generalized eigenvector corresponding to the minimum generalized eigenvalue of the pair $(\mathbf{Y}^T\mathbf{Y}, \mathbf{\Sigma})$.

(ii) Use the estimated $\hat{\boldsymbol{\rho}}$ to construct $\mathbf{W}$ as well as $\boldsymbol{\Upsilon}$.

(iii) Compute the generalized eigenvector corresponding to the minimum generalized eigenvalue of the pair $(\mathbf{Y}^T\mathbf{W}\mathbf{Y}, \boldsymbol{\Upsilon})$.

(iv) Repeat Steps (ii)-(iii) until parameter convergence.

It is noteworthy that when there is either input or output noise only, $\boldsymbol{\Upsilon}$ will not be of full rank. However, the proposed algorithm can be easily modified to operate in these scenarios via some matrix partitioning work.

3 Numerical Examples

Simulation results had been performed to evaluate the performance of the developed approach for estimating IIR system parameters using noisy measurements. The scenarios of noisy output only and noisy input-output were considered. In the former case, comparison was made with the OE and PE methods [2] while for the latter, we contrasted the estimator performance with the KL method [7] and asymptotic CRLB for white Gaussian disturbance [8]. The noise-free input s_k was a white Gaussian process of unity power and the unknown system was of second order with parameters as follows: $a_1 = -1$, $a_2 = 0.5$ and $b_0 = -1$. The measurement noise m_k and n_k were also independent white Gaussian random variables. The sample size was $N = 200$ and three iterations were used in the estimation procedure. All results provided were averages of 100 independent runs.

Figures 1 to 3 show the mean square errors (MSEs) for a_1, a_2 and b_0, respectively, of the unit-norm LS, unit-norm WLS, OE and PE methods in estimating the output-noise-only system at noise power ranged from -30dB to 20dB. As expected, the unit-norm LS was inferior to the other three estimators which had very similar performance before the threshold region. Since for IIR system identification with only output noise, it is well known that [2] the PE method is optimum for general noise models while the OE method is optimum for white noise, we concluded that the proposed WLS estimator also produced optimum system parameter estimates for sufficiently small noise levels.

The above test was repeated for the noisy input-output system with $\alpha^2 = 20$ and the corresponding results are shown in Figures 4 to 6. It is seen that the unit-norm LS and KL methods performed very similar and were suboptimum. While the unit-norm WLS estimator was again optimum as it agreed with the asymptotic CRLB for sufficiently high signal-to-noise ratios, although the MSEs could below the bound because N was not chosen large enough.

4 Conclusions

The relationship between the unit-norm constraint approach and Koopmans-Levin method for impulse response estimation has been illustrated via the development of a unit-norm least squares algorithm. This algorithm is then improved by using the technique of weighted least squares (WLS), and the resultant parameter estimates are obtained via minimization of a WLS cost function subject to a generalized unit-norm constraint where the weighting matrix is the Markov estimate. The optimality of the proposed WLS estimator for output-noise-only and noisy input-output systems is demonstrated by contrasting with the corresponding benchmarks via computer simulations.

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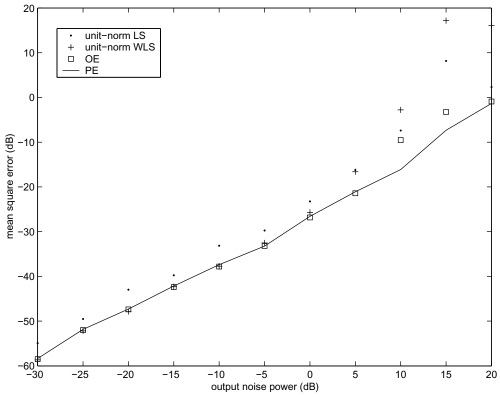

Figure 1: Mean square errors of a_1 for output-noise-only system

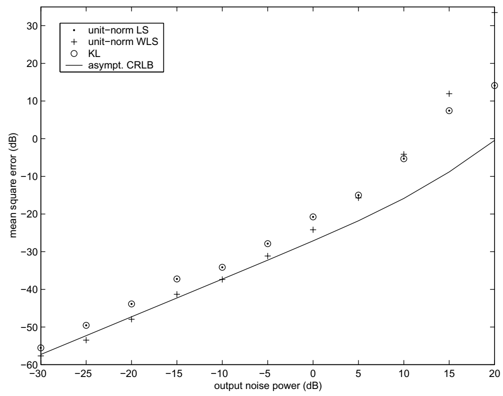

Figure 4: Mean square errors of a_1 for noisy input-output system

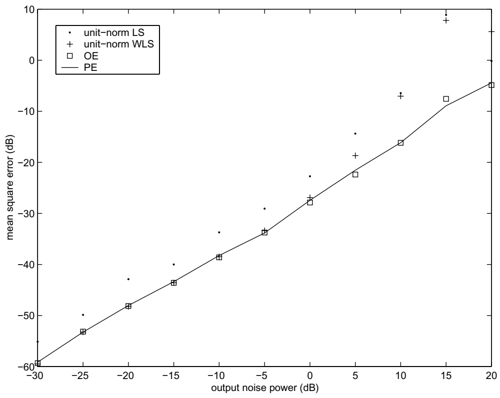

Figure 2: Mean square errors of a_2 for output-noise-only system

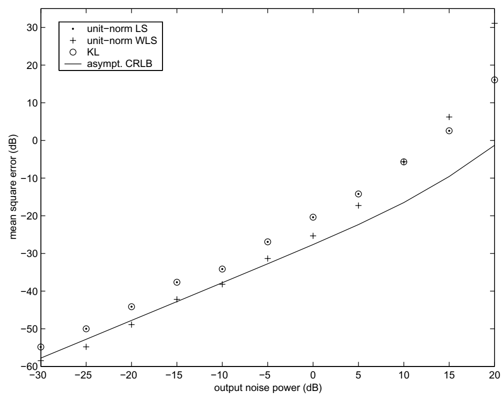

Figure 5: Mean square errors of a_2 for noisy input-output system

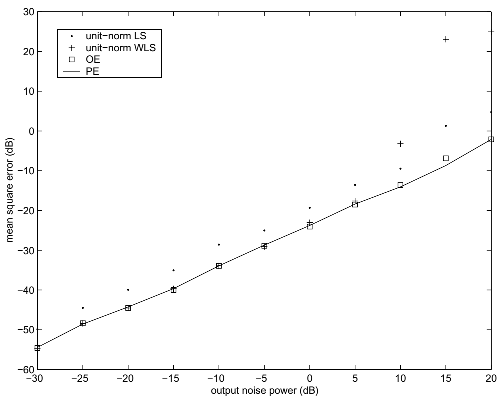

Figure 3: Mean square errors of b_0 for output-noise-only system

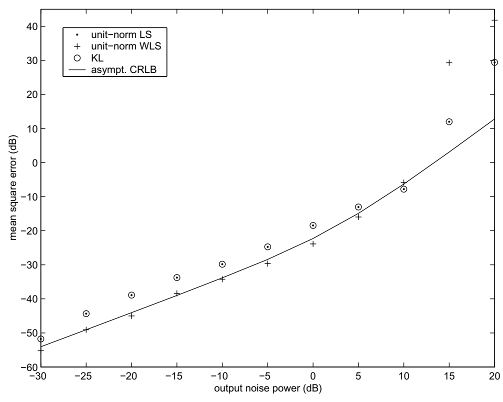

Figure 6: Mean square errors of b_0 for noisy input-output system