APPLICATION OF 1-D FILTERS TO IMAGE TRANSMULTIPLEXING

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ABSTRACT

Several images are combined into one image for transmission by a single channel and recovery by a receiver. The distortion or mixture of the received images is invisible. Transmultiplexer consists of upsamplers and 1-D FIR filters. Detransmultiplexer consists of 1-D FIR filters and downsamplers. Bilinear constraints on the synthesis and analysis FIR filters are imposed to achieve the perfect reconstruction. The optimization method for filter design was applied.

1. INTRODUCTION

Transmultiplexing [1,2,5] is a structure that combines a collection of suitably filtered signals for the transmission by a single channel. The transmultiplexers were originally studied for 1-D signals in the context of converting Time Division Multiplexing (TDM) into Frequency Division Multiplexing (FDM) with a goal of converting back to TDM at some later point. Transmultiplexers have some important applications, in particular in telecommunications, to provide many signals over a single transmission line. The separation of signals should be perfect and the recovery of each signal should be performed without leakage of signal from one channel to another [3,4]. This goal can be achieved by a choice of filters that ensure perfect reconstruction.

The optimization method of filter bank design reducing the above-mentioned effects is presented below. Using this approach, the obtained 1-D filters were applied to image transmultiplexing and preserving the perfect reconstruction.

Transmultiplexing is usually associated with the 1-D signals. In this paper we attempt to increase the variety of signals that can be transmultiplexed. Methods usually used for the acoustic signals were successfully applied to images.

2. TRANSMULTIPLEXING

A transmultiplexer combines several images into a single image. Its application is for simultaneous transmission of several images through a single channel.

Fig.1 shows the structure of the four-channel image transmultiplexer. The input images are upsampled horizontally, filtered vertically and summed to obtain two composite images. These composite images are then upsampled vertically and filtered horizontally and summed to obtain the single image. This image can be transmitted over a single transmission channel. At the receiver end, the signal is relayed first to the two channels of the detransmultiplexation part, where the signals are filtered horizontally and downsampled vertically. Then these signals are relayed to the four channels where images are filtered vertically and downsampled horizontally to recover the original images. Although an example presented in Fig.1 consists of four channels, only two different filters (H_1^t and H_1^t) were used for transmultiplexing and other two (H_1^d and H_1^d) for the detransmultiplexing. The basic idea is the reversibility of all procedures of transmultiplexation in such a way that all input images could be recovered as precisely as possible.

3. PERFECT RECONSTRUCTION CONDITIONS

For the 1-D signal, the dependence of output s_i^{out} from inputs s_k^{in}, where $i,k \in \{1,2,\cdots,M\}$ is described [2,5] in the z-transformation domain by

$$\bar{s}_i^{\,out}(z) = \frac{1}{M}\sum_{k=1}^{M}\bar{s}_k^{\,in}(z)\left[\sum_{m=0}^{M-1} H_i^{\,d}(w_M^m z^{1/M}) H_k^{\,t}(w_M^m z^{1/M})\right], \quad (1)$$

where $w_M = e^{-2\pi\cdot\sqrt{-1}/M}$, $\bar{s}_i(z)$, $H^t(z)$, $H^d(z)$ stand for the z-spectrum of discrete signal $\{s_i(n)\}$, transmultiplexer and detransmultiplexer transfer functions, respectively. This means that each output signal depends on all inputs signals. A key point is that the constituent signals should be recoverable from the combined signal. To fulfill the perfect reconstruction condition, we obtain from (1) a set of M^2 equations

$$\frac{1}{M}\sum_{m=0}^{M-1} H_i^d\left(w_M^m z^{1/M}\right) H_k^t\left(w_M^m z^{1/M}\right) = c\, z^{-\tau}\delta_{i,k}\,, \quad (2)$$

where $\delta_{i,j}$ is a Kronecker function and the natural number τ is some shifting inserted by the causal filters.

Under assumption that all filters are FIR type of order *I*, conditions (2) are equivalent to

$$\sum_{n=0}^{[2I/M]} z^{-n} \sum_{j=\max\{0,Mn-I\}}^{\min\{Mn,I\}} h_i^d(j)\, h_k^t(Mn-j) = c\, z^{-\tau}\delta_{i,k}\,, \quad (3)$$

where $h_i^d(j)$ means the j-th coefficient of the filter

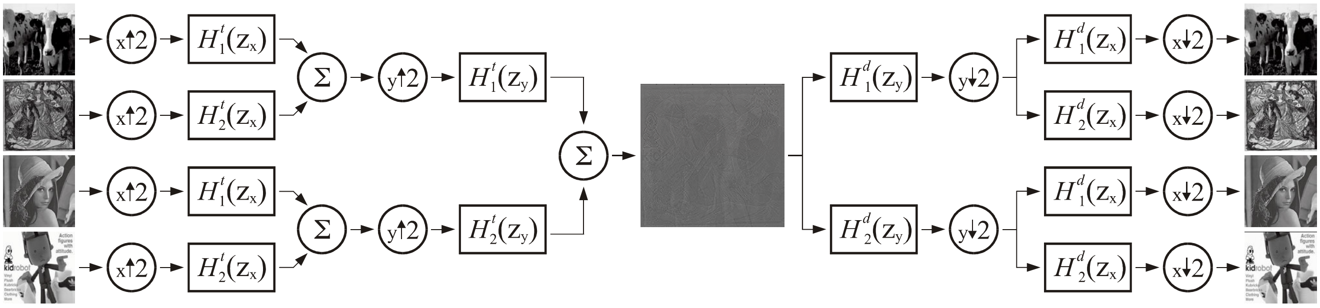

Figure 1: The 4-channel image transmultiplexer system.

$H_i^d(z), i \in \{1,2,\ldots,M\}$. Similarly $h_k^t(Mn-j)$ stands for the coefficients of the $H_k^t(z)$ filter. Equalities (3) are fulfilled for all $z \in C$ if and only if

$$\sum_{j=\max\{0,Mn-I\}}^{\min\{Mn,I\}} h_i^d(j)\, h_k^t(Mn-j) = c\,\delta_{i,k}\delta_{n,\tau} \tag{4}$$

for $n = 0,1,\ldots,[2I/M]$, where [.] means an integer part of $2I/M$.

4. FILTER DESIGN

Conditions (4) can be written in matrix notation

$$\left(h_i^t\right)^T A_n h_k^d = c\,\delta_{i,k}\delta_{n,\tau}, \tag{5}$$

where $h_i = \left[h_i(0) \quad h_i(1) \quad \cdots \quad h_i(I)\right]^T \in \Re^{I+1}$,

$A_n = \left\{a_{n,j,l}\right\} \in \Re^{(I+1)\times(I+1)}$

and $a_{n,j,l} = \begin{cases} 1 & \text{if} \quad j+l = Mn+2 \\ 0 & \text{if} \quad j+l \neq Mn+2 \end{cases}, \quad n = 0,\ldots,\left[2I/M\right]$.

For each pair $i,k \in \{1,2,\cdots,M\}$ of filter numbers, condition (5) gives $\left[2I/M\right]+1$ equations. Therefore it results in the system of $M^2\left(\left[2I/M\right]+1\right)$ equations.

To accomplish the perfect reconstruction with the shifting τ, conditions (5) have to be followed while filters designing. There is no simple method to find a solution of the set of bilinear equations (5). This makes the optimization methods useful. To find the FIR filters, a computer minimizes the quantity criterion

$$Q = \sum_{i=1}^{M}\sum_{k=1}^{M}\sum_{n=0}^{[2I/M]}\left(h_i^t A_n h_k^d - c_i\delta_{i,k}\delta_{n,\tau}\right)^2 \tag{6}$$

with respect to h_i^t and h_k^d. Each solution obtained in such way depends on the starting point of minimization procedure and usually reaches a local minimum of (6) only.

5. EXAMPLE

To verify the presented in Fig.1 method of 1-D filters application to image transmultiplexing, some examples were computed and analyzed. One of them is presented in this paper. Images consist of 200×200 pixels coded on 8 bit grayscale.

To design 1-D filters the following was assumed:

- 1-D transmultiplexer consists of two channels, i.e. $M = 2$,
- all filters are of 10-th order,
- shifting $\tau = 2$,
- amplification $c = 1$.

For the considered case ($M = 2$, $I = 10$), the set of conditions (5) consists of twenty-four equations. All values of filters parameters were equal to one to start the minimization procedure. A minimum value $Q_{\min} = 0.26333\cdot10^{-6}$ of the quantity criterion (6) was obtained. It means that for each equation, the obtained average value $\sqrt{Q_{\min}/24}$ of residuum is approximately equal to 10^{-4}. The results of minimization are presented in Table 1 (filter coefficients for transmultiplexing) and in Table 2 (filter coefficients for detransmultiplexing). The amplitude and the phase characteristics of all filters are presented in Fig.2 and Fig.3. Amplification *c* was assumed to be one, so the high amplification of transmultiplexing filters is balanced by the low amplification of detransmultiplexing filters.

According to the scheme presented in Fig.1, the filters defined in Table 1 were used to combine four images, each of size 200×200 pixels, into one image of size 400×400 pixels. The obtained combine image is presented in Fig.4 and its spectrum is presented in Fig.5. The filters presented in Fig.3 and Table 2 were used to recover the four original images.

In Fig.6 and Fig.7 the input and the output images obtained for four-channel transmultiplexer are presented. When compare the input and output signals, only boundary effects are visible. The Root Mean Square Errors (RMSE) for the tested signals are presented in Table 3. Taking into consideration the 256 levels of picture's gray, we notice that transmission errors constitute usually less than 4% but for some images can rise even to 9%. Such errors are not noticeable for a human eye (compare Fig.6 and Fig.7).

Table 3. Transmission errors.

Channel No.	RMSE
1	2.03
2	22.55
3	1.35
4	3.98

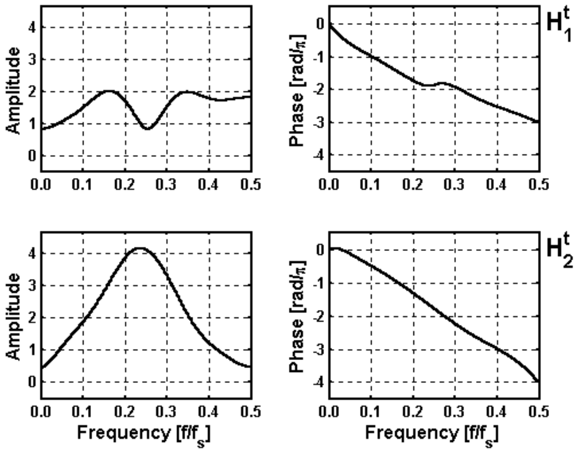

Figure 2: Characteristics of filters for the 2-channel transmultiplexing.

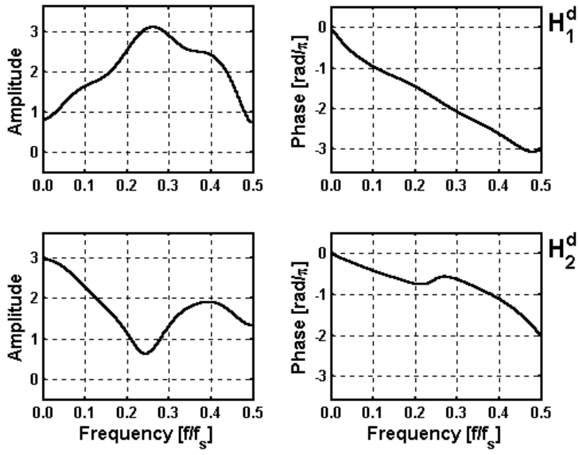

Figure 3. Characteristics of filters of the 2-channel detransmultiplexing.

Table 1. Values of H_k^t coefficients, $k \in \{1,2\}$.

No.	H_1^t	H_2^t
0	0.0237	0.0058
1	-0.0863	-0.0211
2	-1.0307	-0.2492
3	0.9830	1.2160
4	0.3036	1.4178
5	0.4566	-1.3306
6	0.3232	-0.8861
7	-0.1633	0.2540
8	-0.1888	0.2585
9	0.1231	-0.1318
10	0.0723	-0.0758

Table 2. Values of H_i^d coefficients, $i \in \{1,2\}$.

No.	H_1^d	H_2^d
0	-0.0106	-0.0694
1	-0.2320	1.1175
2	-1.2930	1.0563
3	1.4245	0.1259
4	0.9054	0.9395
5	-0.3472	-0.3784
6	0.2018	0.1412
7	0.0389	-0.0134
8	0,2189	0.0750
9	-0.1100	-0.0415
10	0.0075	-0.0011

Figure 4. Image in the transmission line.

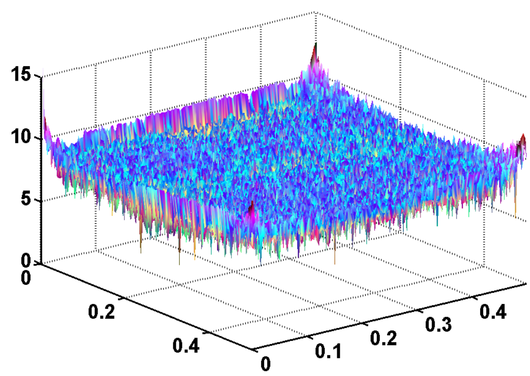

Figure 5. Spectrum of the combine image.

6. CONCLUSIONS

Transmutilplexers allow transmitting a large number of signals by one transmission line. This reduces the operating costs essentially. Moreover transmultiplexing increases the protection against some kind of disturbances. For example, the loss of some parts of transmitted image, even though causing disturbances in all output images, usually leads to small and invisible effects. If the loss of information were limited to the one image only, it could lead to noticeable effects.

It is an important observation that the perfect reconstruction can be obtained without assumption that filters have the separate frequency bands. It protects the transmitted images against some kind of frequency interferences.

Upon the comparison of input and output images presented in this paper, no distortions were noticeable. It means that minimization procedures enable us to find the filters which realize the perfect image reconstruction. It is possible to find more than one such solution. This results in an opportunity to fulfil the additional conditions.

It is easy to prove that causal FIR filters lead to at least $\tau = 1$ shifting. The greatest admissible shifting $\tau_{\max} = \left[2I / M\right] - 1$ is an increasing function of filters range and a decreasing function of number of channels.

Figure 6. An example of the input images.

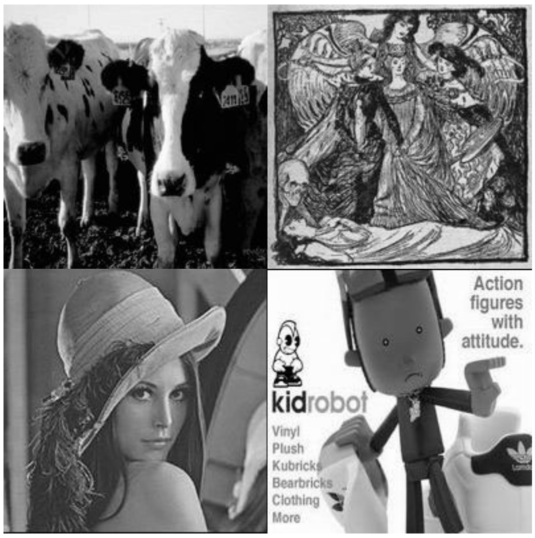

Figure 7. An example of the output images transmitted by 4-channel transmultiplexer.

7. ACKNOWLEDGMENTS

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