

DIRECT POSITION DETERMINATION (DPD) OF MULTIPLE KNOWN AND UNKNOWN RADIO-FREQUENCY SIGNALS

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ABSTRACT

The common methods for position determination of radio signal emitters (communications, radar) are based on measuring a specified parameter of the signal such as Angle-of-Arrival (AOA) or Time-of-Arrival (TOA). The measured parameters are then used to estimate the transmitter's location. Since the measurements are done at each station independently, ignoring the constraint that the AOA/TOA estimates at different stations correspond to the same location, it is a sub-optimal position determination method.

We propose a technique that uses exactly the same data as the common AOA methods but the position determination is direct and can handle more co-channel signals than the common methods. Although there are many stray parameters, only two-dimensional search is required for a planar geometry. The technique provides a natural solution to the measurements-sources association problem that is encountered in AOA based systems and outperforms the AOA methods for unknown as well as known signal waveforms.

1. INTRODUCTION

The problem of emitter location attracts much interest in the Signal-Processing, Communications, and Underwater-Acoustics literature. Defence oriented location systems have been reported since world war I. Civilian systems are in use for the localization of cellular phone callers, spectrum monitoring and law enforcement. Positioning based on AOA or TOA and its derivatives (DTOA, EOTD) is used extensively in cellular phone localization [1], radar systems [2], and underwater acoustics [3, 4].

In this correspondence we discuss a method that solves the localization problem using the data collected at all sensors at all base stations *together*, in contradiction to the traditional AOA/TOA approach that is composed of two separate steps: 1) AOA/TOA independent estimates and 2) triangulation based on the results of the first step. Measuring AOA/TOA at each base station separately and independently is *sub-optimal* approach since it ignores the constraint that the measurements must correspond to the same source position. Moreover, the base stations are geographically separated and therefore often the desired signal appears weak or absent in some of the base stations. Thus the system must ensure that all AOA/TOA measurements correspond to the same source. In the case of co-channel simultaneous sources, the localiza-

tion system confronts an association problem of deciding which of the multiple AOA/TOA estimates correspond to which source.

The Direct Position Determination (DPD) method that we propose takes advantage of the rather simple propagation assumptions that are usually used for Radio Frequency (RF) signals. These assumptions are realistic and have been verified with real data. The proposed method may use both the array response at each station and the time of arrival at each station. We show that for a planar geometry of sources and base stations, a two-dimensional search is sufficient to localize all sources. A side benefit of the DPD is its ability to determine the positions of more sources than the number of sensors at each base station in contrast to AOA. However, the DPD technique requires the transmission of the received signals (possibly sampled) to a central processing location, in contrast to AOA/TOA that require only the transmission of the measured parameters.

2. PROBLEM FORMULATION

Consider L base stations intercepting Q transmitted signals. Each station is equipped with an antenna array consisting of M elements. The bandwidth of the each signal is small compared to the inverse of the propagation time over the array aperture. Denote the q -th transmitter's position by $\mathbf{p}_q$. The complex envelopes of the signals observed by the ℓ -th base station array is given by

$$\mathbf{r}_\ell(t) = \sum_{q=1}^Q b_{\ell q} \mathbf{a}_\ell(\mathbf{p}_q) s_q(t - \tau_\ell(\mathbf{p}_q) - t_q^{(0)}) + \mathbf{n}_\ell(t) \quad (1)$$

where $\mathbf{r}_\ell(t)$ is a time-dependent $M \times 1$ vector, $b_{\ell q}$ is an *unknown* complex scalar representing the channel attenuation between the q -th transmitter and the ℓ -th base station, $\mathbf{a}_\ell(\mathbf{p}_q)$ is the ℓ -th array response to a signal transmitted from position $\mathbf{p}_q$, and $s_q(t - \tau_\ell(\mathbf{p}_q))$ is the q -th signal waveform, transmitted at time $t_q^{(0)}$ and delayed by $\tau_\ell(\mathbf{p}_q)$. The vector $\mathbf{n}_\ell(t)$ represents noise and interference observed by the array.

The observed signal can be partitioned into K sections and each section can be Fourier transformed. The result of this process is

$$\mathbf{r}_\ell(j, k) = \sum_{q=1}^Q b_{\ell q} \mathbf{a}_\ell(\mathbf{p}_q) s_q(j, k) e^{-i\omega_j [\tau_\ell(\mathbf{p}_q) + t_q^{(0)}]} + \mathbf{n}_\ell(j, k) \quad (2)$$

$j = 1, 2, \dots, J; \quad k = 1, 2, \dots, K;$

where $\mathbf{r}_\ell(j, k)$, $s_q(j, k)$, $\mathbf{n}_\ell(j, k)$ are the Fourier coefficient corresponding to frequency ω_j of the k -th section. Define the followings vectors and scalars,

$$\begin{aligned} \bar{s}_q(j, k) &\triangleq s_q(j, k) e^{-i\omega_j t_q^{(0)}} \\ \bar{\mathbf{a}}_\ell(j, \mathbf{p}_q, b_{\ell q}) &\triangleq b_{\ell q} \mathbf{a}_\ell(\mathbf{p}_q) e^{-i\omega_j \tau_\ell(\mathbf{p}_q)} \end{aligned} \quad (3)$$

Using (3) we get the matrix representation of (2),

$$\begin{aligned} \mathbf{r}_\ell(j, k) &= \mathbf{A}_\ell \bar{\mathbf{s}}(j, k) + \mathbf{n}_\ell(j, k) \\ \mathbf{A}_\ell(j) &\triangleq [\bar{\mathbf{a}}_\ell(j, \mathbf{p}_1, b_{\ell 1}), \dots, \bar{\mathbf{a}}_\ell(j, \mathbf{p}_Q, b_{\ell Q})] \\ \bar{\mathbf{s}}(j, k) &\triangleq [\bar{s}_1(j, k), \dots, \bar{s}_Q(j, k)]^T \end{aligned} \quad (4)$$

Since $\bar{\mathbf{s}}(j, k)$ is the same at all base stations, we can concatenate the observed vectors at all stations as follows

$$\begin{aligned} \mathbf{r}(j, k) &= \mathbf{A}(j) \bar{\mathbf{s}}(j, k) + \mathbf{n}(j, k) \\ \mathbf{r}(j, k) &\triangleq [\mathbf{r}_1^T(j, k), \dots, \mathbf{r}_L^T(j, k)]^T \\ \mathbf{n}(j, k) &\triangleq [\mathbf{n}_1^T(j, k), \dots, \mathbf{n}_L^T(j, k)]^T \\ \mathbf{A}(j) &\triangleq [\mathbf{A}_1^T(j), \dots, \mathbf{A}_L^T(j)]^T \end{aligned} \quad (5)$$

3. LOCATION ALGORITHM

In this section we discuss potential location algorithms for the common case of unknown signal waveforms and for the less common case of uncorrelated known signal waveforms.

3.1 Unknown Signals

Assuming that the receivers do not know the signals *a-priori* as is the case in most of the applications. The Maximum Likelihood Estimator of the observations in (5) requires a multidimensional search over the unknown parameter space: $\{\bar{\mathbf{s}}(j, k), b_{\ell q}, \mathbf{p}_q\}$, i.e. an overall of $2QJK + 2(L-1)Q + 2Q$ dimensions. In order to avoid the multidimensional search we use the MUSIC algorithm [5]. Note that

$$\begin{aligned} \mathbf{R}(j) &\triangleq \mathbb{E}\{\mathbf{r}(j, k) \mathbf{r}^H(j, k)\} = \mathbf{A}(j) \mathbf{\Lambda}(j) \mathbf{A}^H(j) + \eta \mathbf{I} \\ \mathbf{\Lambda}(j) &\triangleq \mathbb{E}\{\bar{\mathbf{s}}(j, k) \bar{\mathbf{s}}^H(j, k)\} \\ \mathbb{E}\{\mathbf{n}(j, k) \mathbf{n}^H(j, k)\} &= \eta \mathbf{I} \end{aligned} \quad (6)$$

Following the MUSIC algorithm we propose the following cost function,

$$\begin{aligned} F(\mathbf{p}, \mathbf{b}) &\triangleq \sum_j \bar{\mathbf{a}}^H(j, \mathbf{p}, \mathbf{b}) \mathbf{U}_s(j) \mathbf{U}_s^H(j) \bar{\mathbf{a}}(j, \mathbf{p}, \mathbf{b}) \\ \bar{\mathbf{a}}(j, \mathbf{p}, \mathbf{b}) &\triangleq [\bar{\mathbf{a}}_1^T(j, \mathbf{p}, b_1), \dots, \bar{\mathbf{a}}_L^T(j, \mathbf{p}, b_L)]^T \\ \mathbf{b} &\triangleq [b_1, \dots, b_L]^T \end{aligned} \quad (7)$$

where $\mathbf{U}_s(j)$ is a $ML \times Q$ matrix consisting of the eigenvectors of $\mathbf{R}(j)$ corresponding to the Q largest eigenvalues and $\mathbf{p}$ and $\mathbf{b}$ are variable vectors representing the unknown position and unknown attenuations. The minimum points of $F(\mathbf{p}, \mathbf{b})$ depend on all unknowns and therefore require only a $2(L-1)+D$ dimensional search.

In order to reduce this search we propose to represent $\bar{\mathbf{a}}(j, \mathbf{p}, \mathbf{b})$ as follows,

$$\begin{aligned} \bar{\mathbf{a}}(j, \mathbf{p}, \mathbf{b}) &= \Gamma(j) \mathbf{H} \mathbf{b} \\ \Gamma(j) &\triangleq \text{diag}\{\mathbf{a}_1^T(\mathbf{p}) e^{-i\omega_j \tau_1(\mathbf{p})}, \dots, \mathbf{a}_L^T(\mathbf{p}) e^{-i\omega_j \tau_L(\mathbf{p})}\} \\ \mathbf{H} &\triangleq \mathbf{I}_L \otimes \mathbf{1}_M \end{aligned} \quad (8)$$

where $\mathbf{I}_L$ stands for the $L \times L$ identity matrix, $\mathbf{1}_M$ stands for a $M \times 1$ column vector of ones, and $\otimes$ stands for the kronecker product. Substituting equation (8) in (7) yields

$$F(\mathbf{p}, \mathbf{b}) = \mathbf{b}^H \mathbf{H}^H \left[\sum_j \Gamma^H(j) \mathbf{U}_s(j) \mathbf{U}_s^H(j) \Gamma(j) \right] \mathbf{H} \mathbf{b} \quad (9)$$

In order to facilitate a unique solution we assume that the norm of $\mathbf{b}$ is one. Hence, for any assumed position $\mathbf{p}$ the maximum of $F(\mathbf{p}, \mathbf{b})$ corresponds to the maximal eigenvalue of the matrix $\mathbf{D}(\mathbf{p})$ defined by,

$$\mathbf{D}(\mathbf{p}) \triangleq \mathbf{H}^H \left[\sum_j \Gamma^H(j) \mathbf{U}_s(j) \mathbf{U}_s^H(j) \Gamma(j) \right] \mathbf{H} \quad (10)$$

thus, equation (9) reduces to

$$F(\mathbf{p}) = \lambda_{\max}[\mathbf{D}(\mathbf{p})] \quad (11)$$

where the right side of (11) denotes the largest eigenvalue of $\mathbf{D}(\mathbf{p})$. The maximization of (11) requires only a D -dimensional search. Moreover, the dimensions of $\mathbf{D}(\mathbf{p})$ are $L \times L$ which are usually rather small.

3.2 Known Uncorrelated Signals

In certain applications the transmitted waveforms are known to the location system. For example, in cellular systems synchronization and training sequences are transmitted periodically and are known *a-priori*. Moreover, it is possible to detect the data sequence of a digitally modulated signal and then restore the complex signal envelope based on the known modulation scheme.

We follow the algebraic steps in [6]. Assume that the noise, $\mathbf{n}(j, k)$, is circularly symmetric complex Gaussian random vector with zero-mean and second orders statistics given by,

$$E\{\mathbf{n}(i, k)\mathbf{n}^H(j, \ell)\} = \eta \mathbf{I} \delta_{ij} \delta_{k\ell}; \quad E\{\mathbf{n}(i, k)\mathbf{n}^T(j, \ell)\} = 0 \quad (12)$$

We further assume that the signals are uncorrelated, *i.e.* $\mathbf{\Lambda}(j)$ is diagonal. The log-likelihood function of the array output vectors, $\mathbf{r}(j, k)$, is proportional to

$$F_1 = \sum_{j=1}^J \frac{1}{K} \sum_{k=1}^K \|\mathbf{r}(j, k) - \mathbf{A}(j)\bar{\mathbf{s}}(j, k)\|^2 \quad (13)$$

Define the following matrices

$$\begin{aligned} \hat{\mathbf{R}}_{sr}(j) &\triangleq \frac{1}{K} \sum_{k=1}^K \bar{\mathbf{s}}(j, k) \mathbf{r}^H(j, k) \\ \hat{\mathbf{R}}_{ss}(j) &\triangleq \frac{1}{K} \sum_{k=1}^K \bar{\mathbf{s}}(j, k) \bar{\mathbf{s}}^H(j, k) \\ \hat{\mathbf{A}}(j) &= \hat{\mathbf{R}}_{sr}(j) \hat{\mathbf{R}}_{ss}^{-1}(j) \end{aligned} \quad (14)$$

Substituting (14) in (13) it can be verified that minimizing F_1 is equivalent to minimizing

$$F_2 = \text{tr} \left\{ \sum_{j=1}^J [\mathbf{A}(j) - \hat{\mathbf{A}}(j)]^H [\mathbf{A}(j) - \hat{\mathbf{A}}(j)] \hat{\mathbf{R}}_{ss}(j) \right\} \quad (15)$$

Since we assumed that the signals are uncorrelated, $\hat{\mathbf{R}}_{ss}(j)$ is asymptotically diagonal and F_2 can be decoupled as

$$\begin{aligned} F_3 &= \sum_{q=1}^Q F_3(q) \\ F_3(q) &\triangleq \sum_{j=1}^J \|\bar{\mathbf{a}}(j, \mathbf{p}_q, \mathbf{b}_q) - \hat{\mathbf{a}}_q(j)\|^2 \end{aligned} \quad (16)$$

where $\bar{\mathbf{a}}(j, \mathbf{p}_q, \mathbf{b}_q)$, $\hat{\mathbf{a}}_q(j)$ represent the q -th column of $\mathbf{A}(j)$ and $\hat{\mathbf{A}}(j)$, respectively. Using (8) in (16) yields

$$F_3(q) = \sum_{j=1}^J \|\Gamma_q(j) \mathbf{H} \mathbf{b}_q - \hat{\mathbf{a}}_q(j)\|^2 \quad (17)$$

The vector $\mathbf{b}_q$ that minimizes the cost function F_3 is

$$\hat{\mathbf{b}}_q = \left(\sum_j \mathbf{H}^H \Gamma_q^H(j) \Gamma_q(j) \mathbf{H} \right)^{-1} \mathbf{H}^H \sum_j \Gamma_q^H(j) \hat{\mathbf{a}}_q(j) \quad (18)$$

Replacing $\mathbf{b}_q$ with $\hat{\mathbf{b}}_q$ in (17) we get a cost function that depends on $\mathbf{p}$ only. This cost function can be simplified by

assuming that $\|\mathbf{a}_\ell(j)\| = 1; \forall j, \ell, \mathbf{p}$. Thus, it can be shown that minimizing (17) is equivalent to maximizing

$$F_4(q) = \sum_{\ell=1}^L \left| \sum_{j=1}^J e^{j\omega_j \tau_\ell(\mathbf{p})} \mathbf{a}_\ell^H(\mathbf{p}) \hat{\mathbf{a}}_q^{(\ell)}(j) \right|^2 \quad (19)$$

where $\hat{\mathbf{a}}_q^{(\ell)}(j)$ is the ℓ -th sub-vector of $\hat{\mathbf{a}}_q(j)$. Note that (19) indicates that the cost function is in fact a sum of L distinct cost functions, each associated with a distinct base station. This is the reason that DPD outperforms methods that maximize the cost function at each station independently.

4. NUMERICAL RESULTS

We performed Monte-Carlo simulations in order to compare the traditional AOA approach with DPD. We applied two different techniques to locate the transmitters: 1) AOA estimation at each base station independently, 2) DPD according to the algorithms described in the previous section. The performance evaluation is based on the RMS error given by

$$\text{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^N (\hat{x}_i - x_i)^2 + \frac{1}{N} \sum_{i=1}^N (\hat{y}_i - y_i)^2} \quad (20)$$

where (x_i, y_i) is the emitter location and $(\hat{x}_i, \hat{y}_i)$ is the i -th location estimate.

Test Case 1: Consider 3 base-stations placed at co-ordinates, (2, -2), (2, 0), (2, 2) [Km] and two emitters placed at (0, +1.5), (0, -1.5) [Km]. The signals are unknown to the receivers. Each base-station is equipped with a uniform linear array of only 3 antenna elements. The attenuation vector is $\mathbf{b} = [1, 0.8, 0.4]^T$. Each location determination is based on 200 snapshots at a single frequency. The SNR is varied between 3 [dB] and 23 [dB] and we performed 100 experiments at each SNR. The results for one of the sources are shown in Figure 1. As can be seen, DPD outperforms AOA at low SNR while both methods are equivalent at high SNR.

Test Case 2: we kept the base stations at the same locations and used two emitters placed at (0, +Y), (0, -Y) [Km], 100 snapshots and SNR=20 [dB]. The channel attenuation to all stations is equal. We changed Y from 0.2 [Km] to 1.2 [Km] and the results are plotted in Figure 2. As expected, the accuracy of the traditional AOA is very sensitive to sources that are not well separated as opposed to the DPD method.

Test Case 3: we kept the base stations at the same location and placed 3 transmitters at (0, 1.5), (0, -1.5), (-1, 0) [Km]. Each base station collects 1000 snapshots and the attenuation is equal at all base stations. Since each base station is equipped with an array of 3 elements, traditional AOA based on MUSIC fails while DPD works fine as shown in Figure 3.

Test Case 4: we used 4 base stations located at (-2, -2), (-2, +2), (2, -2), (2, +2) [Km] and a single source at (1, 1) [Km]. Each base station is equipped with a circular array of 5 elements. The waveforms are known. The number of snapshots is 1000. The attenuation to two base stations is 0 dB and for

the other two is -10 dB. The accuracy results are plotted in Figure 4.

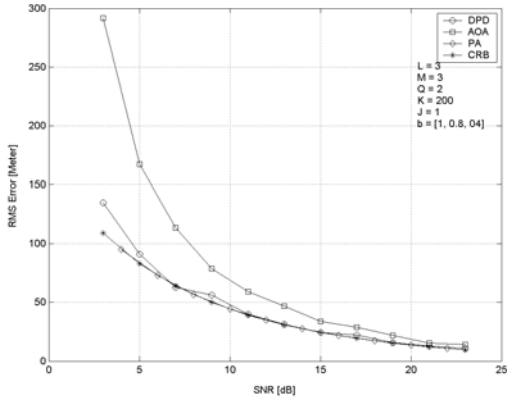


Figure 1: RMS error of DPD and AOA, Cramér-Rao Bound and Performance Analysis results for 3 base stations, two sources.

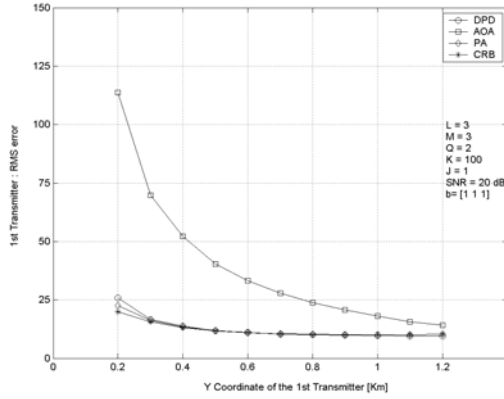


Figure 2: RMS error of DPD and AOA, Cramér-Rao Bound and Performance Analysis results for 3 base stations, two sources with increasing separation.

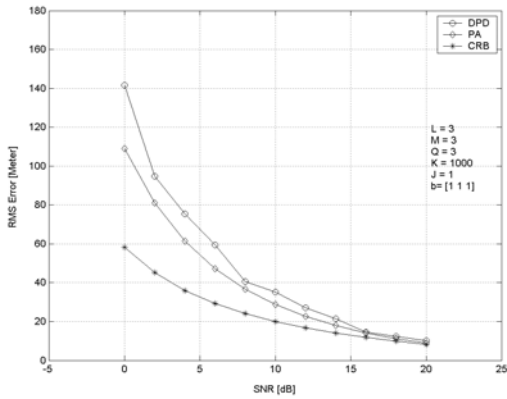


Figure 3: RMS error of DPD and AOA, Cramér-Rao Bound and Performance Analysis results for 3 base stations, 3 sources.

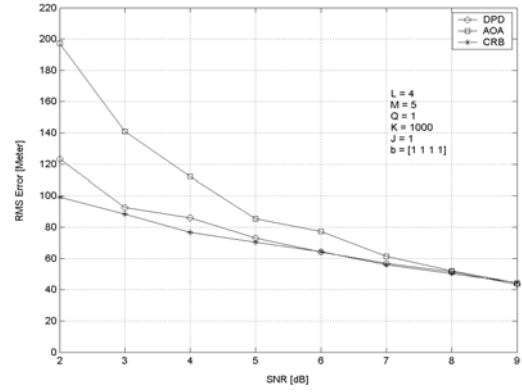


Figure 4: RMS error of DPD and traditional AOA for *known* waveforms. Four base stations each equipped with 5 element circular array.

5. CONCLUSIONS

We have proposed a direct position determination (DPD) technique for localizing multiple narrowband radio frequency sources. The technique can locate more sources than the traditional AOA approach. Moreover, DPD provides better accuracy than traditional AOA and it does not encounter the association problem of independent AOA measurements at each base station. The proposed technique uses the MUSIC approach in order to reduce the complexity of the algorithm. However, DPD requires raw signal data to be transferred to a common processor.

REFERENCES

- [1] J. C. Liberti, and T. S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications*, Prentice Hall 1999.
- [2] M. I. Skolnik, *Introduction to Radar Systems*, 3rd edition, McGraw-Hill, 2000.
- [3] G. C. Carter (editor), *Coherence and Time Delay Estimation*, IEEE press, 1993.
- [4] J. A. Shorey, and L.W. Nolte, "Wideband optimal a posteriori probability source localization in an uncertain shallow ocean environment", *J. Acoust. Soc. Am.* 103(1), Jan 1998.
- [5] R. O. Schmidt, "Multiple emitter location and signal parameter estimation", *IEEE Transactions on Antennas and Propagation*, Vol. 34, No. 3, pp. 276–280, Mar 1986.
- [6] J. Li, B. Halder, P. Stoica, and M. Viberg, "Computationally Efficient Angle Estimation for Signals with Known Waveforms", *IEEE Trans. On Signal Processing*, Vol. 43, No. 9, pp. 2154-2163, September 1995.