BIAS ANALYSIS AND REMOVAL OF A MICROPHONE ARRAY BASED ROAD TRAFFIC SPEED ESTIMATOR

R. López-Valcarce, D. Hurtado, C. Mosquera and F. Pérez-González*

Departamento de Teoría de la Señal y las Comunicaciones
Universidad de Vigo, 36200 Vigo, Spain
valcarce@gts.tsc.uvigo.es

ABSTRACT

Recently, a maximum likelihood estimate has been proposed for road vehicle speed based on two omnidirectional microphones. It is observed that this estimate may be severely biased if the target is moving fast and/or the acoustic signal present high frequency components. We undertake a bias analysis to explain this effect and present a modification which is capable of drastically reducing the bias at the same computational complexity as the original estimate.

1. INTRODUCTION

In order to develop effective management strategies, traffic management systems require accurate estimation of parameters such as traffic density and flow, for which a sensor infrastructure capable of automatic monitoring of traffic conditions must be deployed. Many alternatives exist for collecting data about the transit of road vehicles at a given location. System design must include the choice of a particular sensor as well as the development of adequate signal processing and parameter estimation methods.

Traffic sensors commercially available at present include magnetic induction loop detectors, radar, infrared or ultrasound based detectors, video cameras, and microphones. All of them present different characteristics in terms of robustness to changes in environmental conditions; manufacture, installation and repair costs; safety regulations, etc. A desirable system would be passive, cheap, and easy to install and maintain while able to operate in all-weather day-night conditions. These goals can be achieved with microphone based schemes; however, most commercially available systems tend to be expensive since they use highly directive microphones. The use of cheap (i.e. omnidirectional) sensors must be compensated for with more sophisticated signal processing algorithms. We address the problem of how to directly estimate road vehicle speed from the acoustic signals received at a pair of omnidirectional microphones located next to the traveling path.

Previous work [5, 7] presented an approximate maximum likelihood (ML) estimate of the vehicle speed in such a setting. This estimate requires neither modeling or knowledge of the acoustic source (thus being effectively “blind”), nor intermediate time delay estimation steps, which are potentially troublesome in real applications [4, 5, 6]. Being based on modified crosscorrelations, it is well suited to DSP implementation. However, we have noted that the estimate may be biased for fast speeds and/or high frequency components of the acoustic source. Here we present a bias analysis in order to explain this observation. This will also expose the source of the bias, suggesting a modification of the estimate which will solve the bias problem.

*Supported by a *Ram ́on y Cajal* grant of the Spanish Ministry of Science and Technology.

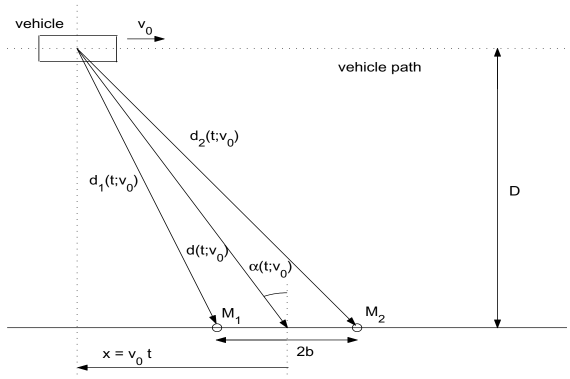

Figure 1: Geometry of the problem

2. PROBLEM DESCRIPTION

As shown in Figure 1, the microphones M_1, M_2 are separated $2b$ m and placed at D m from the lane center. The acoustic source travels at constant speed v_0 on a straight path along the road. The time reference is set at the closest point of approach (CPA), i.e. $t = 0$ when the source is equidistant to M_1 and M_2. The propagation delays from the source to M_1, M_2 are $\tau_i(t;v_0) = d_i(t;v_0)/c$, where $d_i(t;v_0)$ is the distance from the source to microphone M_i,

$$d_{1,2}(t;v_0) = \sqrt{D^2 + (v_0 t \pm b)^2},$$

and c is the sound propagation speed. Define also the angle and distace between the source and the array center

$$\alpha(t;v_0) = \operatorname{atan}\frac{v_0 t}{D}, \qquad d(t;v_0) = \frac{D}{\cos\alpha(t;v_0)}.$$

Let the sound wave emitted by the vehicle be $s(t)$, assumed deterministic but unknown. Taking into account sound attenuation, we can express the received signal at sensor M_i as

$$r_i(t) = s_i(t) + w_i(t) = \frac{s(t-\tau_i(t;v_0))}{d(t;v_0)} + w_i(t), \qquad (1)$$

with $w_1(\cdot)$, $w_2(\cdot)$ additive noise processes, assumed stationary, independent, zero-mean Gaussian with psd $N_0/2$ W/Hz in the band $|f| < f_s/2$ (f_s = sampling frequency). The problem is to estimate v_0 given the observed signals $r_i(t)$, and without knowledge of the sound wave $s(t)$ or its power spectrum. In [7] an approximate ML estimate was derived, given by $\hat{v}_0 = \arg\max_v \psi(v)$ where, with $(-T/2, T/2)$ the observation window,

$$\psi(v) \triangleq \int_{-T/2}^{T/2} r_1(t - \Delta\tau(t;v)) r_2(t) dt. \quad (2)$$

The differential time delay (DTD) $\Delta\tau(t;v)$ is given by

$$\begin{aligned} \Delta\tau(t;v) &\triangleq \tau_2(t;v) - \tau_1(t;v) & (3) \\ &\approx -\frac{2b}{c}\sin\alpha(t;v) \qquad \text{if } b/D \ll 1. & (4) \end{aligned}$$

This estimate is based on the fact that the noiseless received signals approximately satisfy

$$s_2(t) \approx s_1(t - \Delta\tau(t;v_0)). \quad (5)$$

It exploits knowledge of the DTD parametric dependence with v to accordingly time-compand the signal $r_1(t)$ before performing the crosscorrelation (2), which must be computed over the whole observation window for each candidate speed. Implementation with sampled signals can be done efficiently as described in [7].

3. BIAS ANALYSIS IN THE NARROWBAND CASE

Vehicle-generated acoustic waveforms can be well described by the sum of two components [2]: the first one consists of a series of harmonically related tones, produced by the rotating parts of the engine, while the second is broadband in nature and is due to tire friction noise. Hence, this class of signals does not fit well into a narrowband model. Nevertheless, the analysis of the mean value of the log likelihood $\psi(v)$ in the narrowband scenario, in which $s(t) = A\sin\omega t$, will provide the required insight into the bias problem. Moreover, as shown in [5], the mean log likelihood function in the wideband case can be closely approximated by the superposition of those corresponding to each individual frequency, weighted by the power spectrum of the acoustic waveform.

A similar analysis of $E[\psi(v)]$ was carried out in [7], but the simplifications introduced resulted in an apparent unbiasedness of the estimate. For a more accurate result, we must take into account the fact that the vehicle is moving during the propagation of its acoustic signature to the microphones. To do so, we introduce the following 'delay error' term:

$$\begin{aligned} \xi_\Delta(t;v_0,v) &\triangleq \tau_1(t - \Delta\tau(t;v);v_0) - \tau_1(t;v_0) & (6) \\ &\approx -\frac{2bv_0}{c^2}\left[\sin\alpha(t;v_0) + \frac{b}{D}\cos\alpha(t;v_0)\right]\sin\alpha(t;v), & (7) \end{aligned}$$

the last step being valid for $|v - v_0|$ small. This term becomes necessary for the analysis due to the fact that (5) does not hold with equality, especially for high speed values.

$E[\psi(v)]$ is the value of (2) after replacing r_i by s_i. Note,

$$s_1(t - \Delta\tau(t;v)) = \frac{s(t - \Delta\tau(t;v) - \tau_1(t - \Delta\tau(t;v);v_0))}{d(t - \Delta\tau(t;v);v_0)}. \quad (8)$$

In the denominator of (8), we can make $d(t - \Delta\tau(t;v);v_0) \approx d(t;v_0)$. However, we must be more accurate with the analogous term in the argument of $s(\cdot)$. If $s(t) = A\sin\omega t$, then

$$\begin{aligned} & s_1(t - \Delta\tau(t;v)) \\ & \approx A\frac{\sin[\omega(t - \Delta\tau(t;v) - \tau_1(t;v_0) - \xi_\Delta(t;v_0,v))]}{d(t;v_0)}. \quad (9) \end{aligned}$$

Therefore, the product of (9) with

$$s_2(t) = A\frac{\sin[\omega(t - \tau_2(t;v_0))]}{d(t;v_0)}$$

becomes

$$\frac{A^2}{2}\frac{\cos[\omega(\Delta^2\tau(t;v_0,v) - \xi_\Delta(t;v_0,v))]}{d^2(t;v_0)} + (\text{terms in } 2\omega t) \quad (10)$$

where

$$\Delta^2\tau(t;v_0,v) \triangleq \Delta\tau(t;v_0) - \Delta\tau(t;v).$$

When integrating (10), the contribution of the 'double-frequency' term is small compared to that of the other term, so it can be neglected. On the other hand,

$$\begin{aligned} & \cos[\omega(\Delta^2\tau(t;v_0,v) - \xi_\Delta(t;v_0,v))] = \\ & \quad \cos[\omega\Delta^2\tau(t;v_0,v)]\cos[\omega\xi_\Delta(t;v_0,v)] \\ & \quad + \sin[\omega\Delta^2\tau(t;v_0,v)]\sin[\omega\xi_\Delta(t;v_0,v)]. \quad (11) \end{aligned}$$

At this point we need an approximation for the terms involving $\Delta^2\tau(t;v_0,v)$. By visual inspection of this function, the following approximation seems well suited:

$$\Delta^2\tau(t;v_0,v) \approx R\sin[2\,\mathrm{atan}(zt)]. \quad (12)$$

where R and z are found by imposing that the two sides of (12) have the same slope at $t = 0$, and that they peak at the same time instants. These conditions lead to

$$R = \frac{b(v - v_0)}{c\sqrt{2v_0 v}}, \quad z = \frac{\sqrt{2v_0 v}}{D}.$$

Using (12), the sine and cosine terms in (11) can be expanded, in view of the Fourier series

$$f(r\sin x) = \sum_{k=-\infty}^{\infty} J_k(r) f(kx),$$

where $f(\cdot)$ is either $\sin(\cdot)$ or $\cos(\cdot)$ (see e.g. [1]), and where J_k is the kth order Bessel function of the first kind. If we only retain the dominant term in the summations, corresponding to $k = 0$, the following approximation is finally obtained:

$$E[\psi(v)] \approx J_0\left(\frac{\omega b(v - v_0)}{c\sqrt{2v_0 v}}\right)\underbrace{\frac{A^2}{2}\int_{-T/2}^{T/2}\frac{\cos[\omega\xi_\Delta(t;v_0,v)]}{d^2(t;v_0)}dt}_{\triangleq Q_\Delta(v)}. \quad (13)$$

If one makes $\xi_\Delta = 0$ in (13), the expression for $E[\psi(v)]$ given in [7] is recovered. Note that the J_0 factor in (13) has its maximum at $v = v_0$. Thus, if $Q_\Delta(v)$ remained constant with v, the bias of the approximate ML estimate could be expected to be small. From (7), it is seen that if $\omega b v_0/c^2 \ll \pi$, then the product $|\omega\xi_\Delta|$ will remain small so that $\cos\omega\xi_\Delta \approx 1$ and $Q_\Delta(v) \approx$ constant. On the other hand, if $|\omega\xi_\Delta|$ is not small enough, then $Q_\Delta(v)$ cannot be regarded as constant. Although an accurate closed-form expression for $Q_\Delta(v)$ in such case is not available, it can be said that in general it does not peak at $v = v_0$, and therefore neither does the product $J_0(\omega R)Q_\Delta(v)$ in (13). The ensuing bias will increase with source frequency and speed, as well as with sensor spacing.

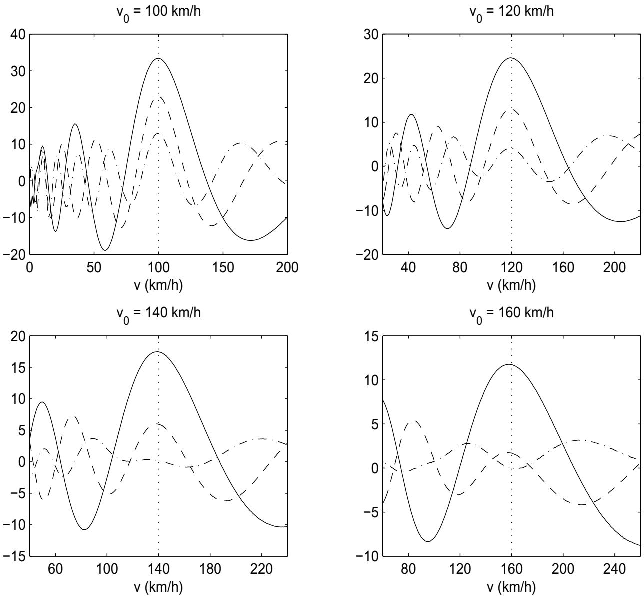

Figure 2: Plots of $E[\psi(v)]$ for a narrowband source of 1 (solid), 1.5 (dashed) and 2 kHz (dashdot). $D = 10$ m, $2b = 1$ m, $T = 2$ s, $f_s = 10$ kHz.

Fig. 2 plots $E[\psi(v)]$ for a fast narrowband source with frequency 1, 1.5 and 2 kHz moving at 100, 120, 140 and 160 km/h. It is clear how, as the frequency of the source increases, the lobe of $E[\psi(v)]$ associated to $v = v_0$ becomes less and less prominent. Eventually, its peak is overtaken by that of a secondary lobe, in which case the resulting bias is rather severe (possibly of several tens of km/h). Since power spectra of vehicle acoustic signatures contain significant components in the range 20 Hz - 2 kHz [3], this behavior is clearly unacceptable, especially in scenarios such as freeways where vehicles are expected to travel at high speeds.

4. BIAS REMOVAL

As seen in section 3, the bias in the estimate is due to the delay error term $\xi_\Delta(t; v_0, v)$ defined in (6), whose origin is traced back to the approximation (5) becoming less accurate for high frequencies and speeds. In theory, the modified crosscorrelation that should be maximized is that of $r_2(t)$ and $r_1(u(t; v))$, where $u(t; v)$ is a warping of the time axis, parameterized by v, such that the noiseless received signals satisfy $s_2(t) = s_1(u(t; v_0))$. Here we have neglected the effect of time-axis warping in the attenuation term $1/d(t; v_0)$ affecting the received signals, since this envelope varies much more slowly with time than the acoustic signature $s(t)$.

Therefore, from (1), it is seen that the function $u(t; v)$ that we would like to determine must satisfy

$$t - \tau_2(t; v_0) = u(t; v_0) - \tau_1(u(t; v_0); v_0) \tag{14}$$

which is an implicit equation in $u(t; v_0)$. We can use a first-order approximation of $\tau_1(u; v_0)$ around $u = u(t; v_0) = t$,

$$\tau_1(u; v_0) \approx \tau_1(t; v_0) + \frac{\partial \tau_1(t; v_0)}{\partial t}(u - t),$$

which, once substituted in (14), leads to the expression

$$u - t \approx -\frac{\Delta\tau(t; v_0)}{1 - \frac{\partial \tau_1(t; v_0)}{\partial t}},$$

where $\Delta\tau(t; v_0)$ is the DTD as before, defined in (3). Therefore we find that $u = u(t; v_0) \approx t - \delta\tau(t; v_0)$, where

$$\delta\tau(t; v) \triangleq \frac{\Delta\tau(t; v)}{1 - \frac{\partial \tau_1(t; v)}{\partial t}}. \tag{15}$$

This suggests replacing the DTD $\Delta\tau(t; v)$ by the modified DTD $\delta\tau(t; v)$ in the formulation of the log likelihood function (2). Observe that, since the specific shape of the time-axis warping function does not affect the implementation of the estimate (see [5, 7]), this modification does not introduce any additional complexity in the computation of $\psi(v)$.

As in section 3, we can introduce the delay error term $\xi_\delta(t; v_0, v)$ in such a way so as to have

$$s_1(t - \delta\tau(t; v)) = \frac{s(t - \Delta\tau(t; v) - \tau_1(t; v_0) - \xi_\delta(t; v_0, v_0))}{d(t; v_0)},$$

ignoring again the effect of time delays in the attenuation factor. Therefore, this delay error term is defined as follows:

$$\begin{aligned} \xi_\delta(t; v_0, v) \quad &\triangleq \quad \delta\tau(t; v) - \Delta\tau(t; v) - \tau_1(t; v_0) \\ &\quad + \tau_1(t - \delta\tau(t; v); v_0). \end{aligned} \tag{16}$$

If we now substitute in (16) the first-order approximation

$$\tau_1(t - \delta\tau(t; v); v_0) \approx \tau_1(t; v_0) - \delta\tau(t; v)\frac{\partial \tau_1(t; v_0)}{\partial t},$$

then, using (15), a little algebra shows that

$$\xi_\delta(t; v_0, v) \approx \delta\tau(t; v)\left[\frac{\Delta\tau(t; v_0)}{\delta\tau(t; v_0)} - \frac{\Delta\tau(t; v)}{\delta\tau(t; v)}\right], \tag{17}$$

suggesting that at $v = v_0$ this delay error term must be small. In fact, from (15), one has the following:

Property 1. The delay error term $\xi_\delta(t; v_0, v)$ defined in (16) satisfies $\xi_\delta(t; v_0, v_0) = O[(\delta\tau(t; v_0))^2]$ for all t. ■

Now, using (16), one can proceed to find the expected value of the log likelihood function when $\delta\tau(t; v)$ is used instead of $\Delta\tau(t; v)$. Following the steps of section 3, it is found that, similarly to (13),

$$E[\psi(v)] \approx J_0\left(\frac{\omega b(v - v_0)}{c\sqrt{2v_0 v}}\right) \underbrace{\frac{A^2}{2}\int_{-T/2}^{T/2} \frac{\cos[\omega\xi_\delta(t; v_0, v)]}{d^2(t; v_0)} dt}_{\triangleq Q_\delta(v)}.$$

Hence, as a consequence of Property 1, $Q_\delta(v)$ is approximately maximized for $v = v_0$, since in that case $\cos[\omega\xi_\delta(t; v_0, v_0)] \approx 1$ for all t. Thus, the bias of the estimate obtained with the modified DTD $\delta\tau(t; v)$ will be much smaller than that of the original approach. Note that a property analogous to Property 1 does not hold for the delay error term $\xi_\Delta(t; v_0, v)$ of (6), for which a first-order approximation

$$\tau_1(t - \Delta\tau(t; v); v_0) \approx \tau_1(t; v_0) - \Delta\tau(t; v)\frac{\partial \tau_1(t; v_0)}{\partial t}$$

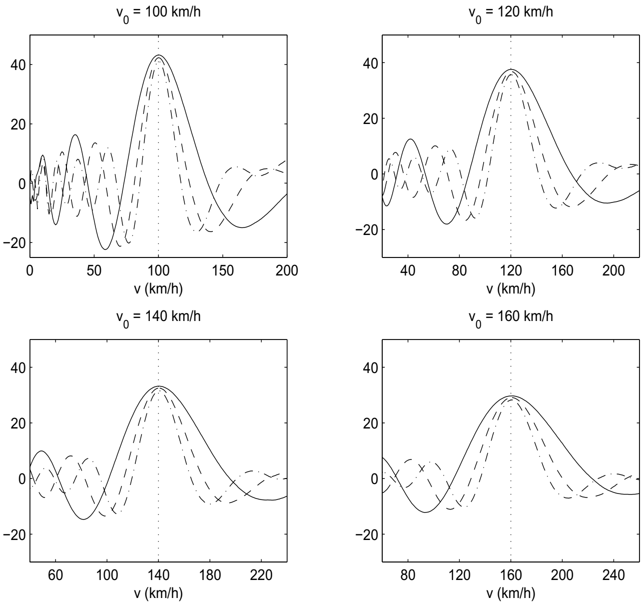

Figure 3: $E[\psi(v)]$ for a narrowband source of 1 (solid), 1.5 (dashed) and 2 kHz (dashdot), using the modified DTD function. $D = 10$ m, $2b = 1$ m, $T = 2$ s, $f_s = 10$ kHz.

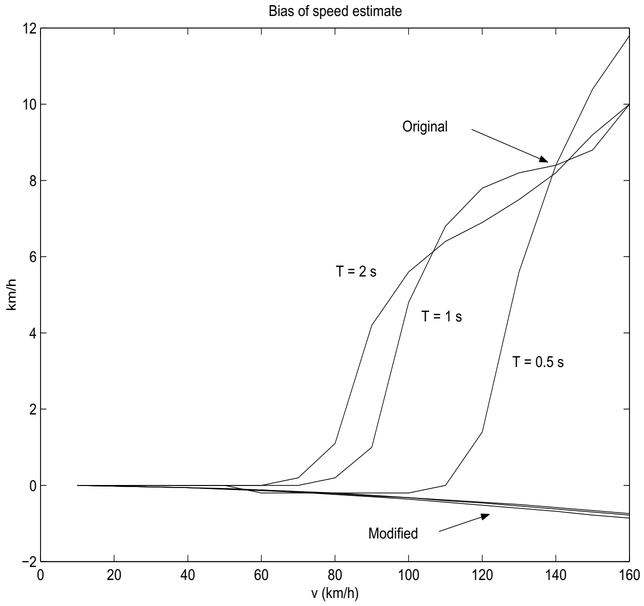

Figure 4: Bias of the speed estimate for the original and modified DTD functions for a wideband Gaussian source and several observation windows. $D = 10$ m, $2b = 1$ m, $f_s = 10$ kHz.

shows that

$$\xi_\Delta(t; v_0, v) \approx -\Delta\tau(t; v)\frac{\partial \tau_1(t; v_0)}{\partial t}.$$

Since

$$\frac{\partial \tau_1(t; v)}{\partial t} = \frac{v}{c}\frac{vt + b}{\sqrt{D^2 + (vt + b)^2}},$$

we see that unless $|v_0| \ll c$, $\xi_\Delta(t; v_0, v_0)$ cannot be neglected.

Finally, if $b/D \ll 1$, then one can make

$$\frac{\partial \tau_1(t; v)}{\partial t} \approx \frac{v}{c}\frac{vt/D}{\sqrt{1 + (vt/D)^2}} = \frac{v}{c}\sin\alpha(t; v).$$

Thus, using this and (4), we arrive at the following expression of the modified DTD:

$$\delta\tau(t; v) \approx -\frac{2b}{c}\frac{\sin\alpha(t; v)}{1 - \frac{v}{c}\sin\alpha(t; v)}. \qquad (18)$$

Fig. 3 plots the function $E[\psi(v)]$ obtained under the same conditions as in Fig. 2, but using the modified DTD function (18) in the crosscorrelation. Clearly, the undesirable behavior of the original estimate observed in Fig. 2 has been avoided.

To confirm the efectiveness of the modified estimate in a wideband scenario, we tested it using a synthetic signal $s(t)$ generated as a realization of a Gaussian process with flat power spectrum in the range $|f| < f_s/2$. Fig. 4 shows the bias of the original and modified speed estimates as a function of source speed, averaged over 10^3 runs. Although a modest amount of bias is present in the modified scheme (less than 1 km/h at $v_0 = 160$ km/h), it is much smaller than that of the original approach, as desired.

5. CONCLUSIONS

The modification presented effectively reduces the bias of the speed estimate to tolerable levels, at the same computational cost as the original approach. The algorithm is amenable to implementation in off-the-shelf digital signal processors that can be deployed at the sensor locations, so that only the value of the estimated parameters need be transmitted to the control center. This reduces dramatically the bandwidth requirements of the sensor network.

REFERENCES

[1] M. Abramowitz and I. Stegun (eds.), *Handbook of mathematical functions,* New York: Dover, 1964.

[2] Y. Ban *et al.*, ‘Synthesis of car noise based on a composition of engine noise and friction noise’, *Proc. ICASSP*, vol. 2, pp. 2105-2108, 2002.

[3] J. C. Chen, K. Yao and R. E. Hudson, ‘Source localization and beamforming’, *IEEE Signal Proc. Mag.*, vol. 19, pp. 30-39, Mar. 2002.

[4] S. Chen, Z. P. Sun and B. Bridge, ‘Traffic monitoring using digital sound field mapping’, *IEEE Trans. Vehicular Technology*, vol. 50, pp. 1582-1589, Nov. 2001.

[5] R. López-Valcarce, C. Mosquera, F. Pérez-González, ‘Estimation of road vehicle speed using two omnidirectional microphones: a maximum likelihood approach’, *EURASIP J. Applied Signal Processing*, to appear.

[6] K. W. Lo and B. Ferguson, ‘Broadband passive acoustic technique for target motion parameter estimation’, *IEEE Trans. Aerosp. Elect. Syst.*, vol. 36, pp. 163-175, Jan. 2000.

[7] F. Pérez-González, R. López-Valcarce, C. Mosquera, ‘Road vehicle speed estimation from a two-microphone array’, *Proc. ICASSP*, vol. 2, pp. 1321-1324, 2002.