ANALYSIS AND CONTROL OF THE STABILITY OF A DIFFERENTIAL SEPARATION METHOD FOR UNDERDETERMINED CONVOLUTIVE MIXTURES

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ABSTRACT

Our contributions in this paper are twofold. We first analyze the stability of a differential blind source separation method for underdetermined convolutive mixtures that we proposed elsewhere. This shows that the adaptation gains of this method should be selected depending on the signs of the differential powers of the sources in order to control its stability. As these signs are unknown in a blind context, we then develop a method for deriving them from the observed mixed signals.

1 INTRODUCTION

Blind source separation (BSS) methods aim at restoring a set of N_s source signals $x_j(n)$ from a set of N_o observed signals $y_i(n)$, which are mixtures of these source signals [1]. Only a few authors have investigated the underdetermined case, i.e. when $N_o < N_s$ (see e.g. ref. in [2]). We proposed in [2] a general differential BSS concept, which makes it possible to derive various practical approaches for this case. We defined such an approach intended for convolutive mixtures in [3]. After summarizing its principles, we here analyze its stability. This leads us to introduce a new method for deriving statistical parameters of the source signals from the observed signals, in order to control the stability of the considered BSS approach in practical situations.

2 CONSIDERED BSS METHOD

We consider two observed signals $y_1(n)$ and $y_2(n)$, which are convolutive mixtures of an arbitrary number of unknown source signals $x_1(n)$ to $x_{N_s}(n)$, i.e:

$$Y_i(z) = \sum_{j=1}^{N_s} A_{ij}(z) X_j(z), \qquad \forall\, i \in \{1,2\}. \quad (1)$$

All source signals are supposedly centered for simplicity and uncorrelated. Sources $x_1(n)$ and $x_2(n)$ are assumed to be "long-term non-stationary", whereas $x_3(n)$ to $x_{N_s}(n)$ are "long-term stationary", as defined in [2]-[3]. All sources are "short-term stationary". The mixing filters $A_{ij}(z)$ are requested to be M^{th}-order MA, strictly causal and such that $A_{11}(z) = 1$ and $A_{22}(z) = 1$.

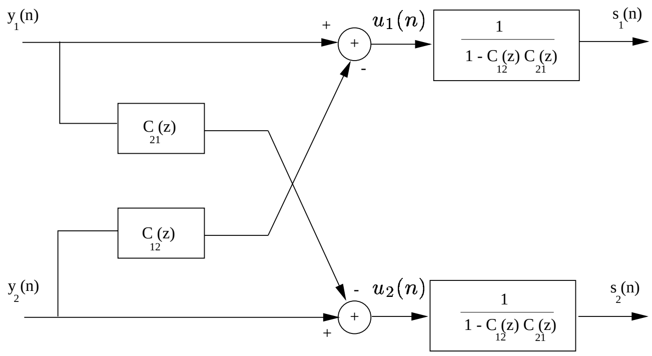

Figure 1: Separating system based on a direct structure.

The method that we proposed in [3] to process such signals uses the structure shown in Fig. 1. Its two M^{th}-order MA strictly causal filters $C_{ij}(z)$ are adapted so as to fulfill the following criterion:

$$\mathcal{DR}_{u_i u_j}(n_1, n_2, 0, k) = 0, i \neq j \in \{1,2\}, k \in [1, M]. \quad (2)$$

This criterion is based on the "differential correlation function", that we defined in [3] as:

$$\begin{aligned} \mathcal{DR}_{vw}(n_1, n_2, l_1, l_2) \;&=\; \mathcal{R}_{vw}(n_2 - l_1, n_2 - l_2) \\ &\quad -\mathcal{R}_{vw}(n_1 - l_1, n_1 - l_2), \quad (3) \end{aligned}$$

where $\mathcal{R}_{vw}(m_1, m_2) = E\{v(m_1)w(m_2)\}$ denotes the standard correlation of any couple of centered signals, n_1 and n_2 are two reference times and l_1 and l_2 are two lags. This criterion leads to a stochastic algorithm which updates the k^{th} coefficient of $C_{ij}(z)$ according to:

$$\begin{aligned} c_{ij}(n+1, k) \;&=\; c_{ij}(n, k) + \mu_i[u_i(n_2)u_j(n_2 - k) \\ &\quad -u_i(n_1)u_j(n_1 - k)], \\ &\quad i \neq j \in \{1,2\}, k \in [1, M]. \quad (4) \end{aligned}$$

It performs a single sweep, indexed by n, over the data. Each step of this sweep involves two points in the data time series, corresponding to the indices n_1 and n_2 (e.g. with $n_1 = n$). The difference between these indices is typically kept constant and "long", as defined in [3].

By using the differential statistics of the signals, this adaptation scheme is only sensitive to the "long-term non-stationary" sources, i.e. $x_1(n)$ and $x_2(n)$. It thus aims at adapting the filters of the BSS system so as to reach the partial separating state [3] associated to $x_1(n)$ and $x_2(n)$, i.e. the state when each of these "useful sources" only appears in one of the outputs of the BSS system (together with some residual contributions of the "noise sources" $x_3(n)$ to $x_{N_s}(n)$). We proved in [3] that (2) is actually met at this partial separating state. This state is therefore an equilibrium point of the proposed algorithm. This equilibrium point must be stable in addition. The current paper first aims at investigating in which conditions this requirement is met.

3 STABILITY ANALYSIS

The Ordinary Differential Equation (ODE) method [4] makes it possible to analyze the local asymptotic behavior of adaptive systems whose updating algorithm reads in vector form:

$$\theta_{n+1} = \theta_n + H(\theta_n, \xi_{n+1}), \tag{5}$$

where θ_n, $H(\theta_n, \xi_{n+1})$ and ξ_{n+1} are the column vectors resp. composed of: i) the adaptive parameters of the system, which define its state, ii) the updating terms for the parameters contained in θ_n, and iii) the signal values required to define the above updating terms.

Stability is then analyzed for stationary sources by approximating the discrete-time recurrence (5) by the continuous-time differential system

$$\frac{d\theta}{dt} = \lim_{n\to+\infty} E_\theta[H(\theta, \xi_{n+1})], \tag{6}$$

where $E_\theta[.]$ denotes the mathematical expectation with respect to the probability law of the vector ξ_{n+1} for a given vector θ. This differential system is locally stable in the vicinity of an equilibrium point θ^* if and only if (iff) the associated tangent linear system:

$$\frac{d\theta}{dt} = J(\theta^*)(\theta - \theta^*) \tag{7}$$

is stable, i.e. iff all the eigenvalues of $J(\theta^*)$ have negative real parts. For any state θ, $J(\theta)$ denotes the corresponding Jacobian matrix of the system, i.e. the matrix composed of the partial derivatives

$$J_{ij}(\theta) = \lim_{n\to+\infty} \frac{\partial(E_\theta[H(\theta, \xi_{n+1})]^{(i)})}{\partial\theta^{(j)}}, \tag{8}$$

where $E_\theta[H(\theta, \xi_{n+1})]^{(i)}$ is the i^{th} component of $E_\theta[H(\theta, \xi_{n+1})]$ and $\theta^{(j)}$ is the j^{th} component of θ.

We first have to apply this ODE approach to a slightly extended version of the classical decorrelation approach intended for 2 mixtures of only 2 sources (see [3]), where we introduce two independent adaptation gains μ_1 and μ_2 resp. for adapting the filters $C_{12}(z)$ and $C_{21}(z)$, i.e:

$$c_{ij}(n+1,k) = c_{ij}(n,k) + \mu_i u_i(n) u_j(n-k) \quad i \neq j \in \{1,2\}, k \in [1, M]. \tag{9}$$

This extended rule falls in the class of algorithms defined by (5) and corresponds to:

$$\theta_n = [c_{12}(n,1), \ldots, c_{12}(n,M), c_{21}(n,1), \ldots, c_{21}(n,M)]^T \tag{10}$$

and $H(\theta_n, \xi_{n+1})$ and ξ_{n+1} derived accordingly from (5) and (9). The corresponding Jacobian matrix $J(\theta^s)$ at the separating state θ^s may then be derived from (10) and the associated $H(\theta_n, \xi_{n+1})$ and ξ_{n+1} by means of (8). It has a complex expression, which does not allow one to easily derive its eigenvalues. It gets simpler when: i) the coefficients of the mixing filters $A_{12}(z)$ and $A_{21}(z)$ are very small and ii) the sources are temporally white (at order 2), i.e:

$$\mathcal{R}_{x_i}(m) = 0 \quad \text{if } m \neq 0 \tag{11}$$

where correlation functions $\mathcal{R}_{x_i}(.)$ have a single argument in this part of the discussion, as the sources are supposedly stationary. The powers or variances $\mathcal{R}_{x_i}(0)$ of these centered sources are denoted $\mathcal{P}_{x_i}$ hereafter. $J(\theta^s)$ then consists of four simple sub-matrices, i.e:

$$J(\theta^s) \simeq \begin{pmatrix} -\mu_1 \mathcal{P}_{x_2} I_M & 0 \\ 0 & -\mu_2 \mathcal{P}_{x_1} I_M \end{pmatrix} \tag{12}$$

where I_M is the M^{th}-order identity matrix. Although these calculations only concern the (extended) classical algorithm, we presented them because they are a required first step of our analysis, which has not been reported in the literature to our knowledge. This also yields the following stability condition for the classical algorithm. The matrix $J(\theta^s)$ obtained in (12) is diagonal and its eigenvalues are $-\mu_1 \mathcal{P}_{x_2}$ and $-\mu_2 \mathcal{P}_{x_1}$. The separating state is a stable equilibrium point for this BSS algorithm iff these eigenvalues are negative. As the source signal powers $\mathcal{P}_{x_2}$ and $\mathcal{P}_{x_1}$ are always positive, this stability condition reads:

$$\mu_1 > 0 \quad \text{and} \quad \mu_2 > 0. \tag{13}$$

This is the reason why the classical algorithm uses $\mu_1 = \mu_2 = \mu > 0$.

The original algorithm (4) studied in this paper also falls in the class of adaptation rules defined by (5). Moreover, it is related in a simple linear way to the algorithm (9) that we just analyzed: its function value $H(\theta_n, \xi_{n+1})$ is the difference between the two values, resp. at times n_2 and n_1, of the function $H(\theta_n, \xi_{n+1})$ corresponding to the algorithm (9). Moreover, the ODE approach itself is also linear with respect to $H(\theta_n, \xi_{n+1})$. Therefore, when applying this approach to the proposed algorithm (4), the expressions obtained in the successive steps of this analysis are straighforwardly derived from those obtained above for the classical algorithm (9): the previous expressions are replaced by the difference of their values between times n_2 and n_1. Especially, the eigenvalues of the Jacobian matrix here become $-\mu_1 \mathcal{D}\mathcal{P}_{x_2}(n_1, n_2)$ and $-\mu_2 \mathcal{D}\mathcal{P}_{x_1}(n_1, n_2)$, where we define the differential power of any signal $v(n)$ for times n_1 and n_2 as:

$$\mathcal{DP}_v(n_1,n_2) = \mathcal{R}_v(n_2,n_2) - \mathcal{R}_v(n_1,n_1). \tag{14}$$

The considered differential algorithm is then locally stable at the partial separating state iff:

$$\mu_1 \mathcal{DP}_{x_2}(n_1,n_2) > 0 \quad \text{and} \quad \mu_2 \mathcal{DP}_{x_1}(n_1,n_2) > 0. \tag{15}$$

The differential powers cannot be removed from this condition, as they may be positive or negative, depending on the considered source signals. This should be contrasted with their classical, i.e. non-differential, counterparts which appeared in the classical approach and which are always positive. The signs of the adaptation gains μ_1 and μ_2 should therefore be selected according to the signs of the differential powers of the useful source signals (and, for the sake of simplicity, both adaptation gains may have the same absolute value, i.e: $|\mu_1| = |\mu_2| = \mu > 0$). However, the latter signs are unknown in a blind context ! A method for estimating them should therefore be developed for the proposed BSS approach to be really applicable. We solve this problem in the next section.

4 DETERMINING THE SIGNS OF THE DIFFERENTIAL POWERS OF THE SOURCES

We consider the same source signals and mixing conditions as in Section 2, except that no restrictions on the mixing filters $A_{ij}(z)$ are needed here. Using only the mixed signals $y_1(n)$ and $y_2(n)$, we aim at determining the signs of the differential powers of the useful sources $x_1(n)$ and $x_2(n)$, while being insensitive to the presence of the noise sources $x_3(n)$ to $x_{N_s}(n)$ in $y_1(n)$ and $y_2(n)$. To this end, we here extend the approach that we proposed in [5] for a partly related problem. We first filter the observed signals $y_i(n)$ by means of narrow-band filters, whose transfer function is denoted $B(z)$. The two resulting signals $y_1'(n)$ and $y_2'(n)$ may be expressed as mixtures of the narrow-band versions of the sources, i.e. of $X_i'(z) = B(z)X_i(z)$. Moreover these mixtures are thus approximately restricted to a simplified form only involving attenuations α_{ij} and time delays m_{ij}, i.e:

$$y_i'(n) = \sum_{j=1}^{N_s} \alpha_{ij} x_j'(n - m_{ij}), \qquad \forall\, i \in \{1,2\}. \tag{16}$$

We then introduce the "conceptual" [5] signal

$$s(n) = y_1'(n) - c y_2'(n-k), \tag{17}$$

where c and k are resp. real-valued and integer-valued tunable coefficients. Now consider the differential power $\mathcal{DP}_s(n_1,n_2)$ of this signal, defined by (14). It may be shown easily that the long-term stationary sources $x_3(n)$ to $x_{N_s}(n)$ yield no contributions in $\mathcal{DP}_s(n_1,n_2)$. Moreover, $x_1'(n)$ and $x_2'(n)$ are here assumed to be MA processes, i.e:

$$x_i'(n) = \sum_{m=-L_1}^{L_2} d_i(m) p_i(n-m) \qquad \forall\, i \in \{1,2\}, \tag{18}$$

where $p_i(n)$ is a short-term stationary white signal. The above equations then yield, for $n_2 - n_1$ long enough:

$$\begin{aligned} \mathcal{DP}_s(n_1,n_2) \quad = \quad & \mathcal{DP}_{p_1}(n_1,n_2) \sum_{m=-\infty}^{+\infty} h_1^2(m) \\ & + \mathcal{DP}_{p_2}(n_1,n_2) \sum_{m=-\infty}^{+\infty} h_2^2(m) \end{aligned} \tag{19}$$

with:

$$h_1(m) = \alpha_{11} d_1(m - m_{11}) - c\alpha_{21} d_1(m - m_{21} - k) \tag{20}$$
$$h_2(m) = \alpha_{12} d_2(m - m_{12}) - c\alpha_{22} d_2(m - m_{22} - k) \tag{21}$$

This leads to the following properties concerning the variations of the sign of $\mathcal{DP}_s(n_1,n_2)$ vs c and k, when $\mathcal{DP}_{p_1}(n_1,n_2)$ and $\mathcal{DP}_{p_2}(n_1,n_2)$ have given signs:

1. First consider the case when:

   $$\mathcal{DP}_{p_1}(n_1,n_2) > 0 \quad \text{and} \quad \mathcal{DP}_{p_2}(n_1,n_2) > 0. \tag{22}$$

   If there existed a value (c_z, k_z) of (c,k) such that:

   $$\forall\, m, \; h_1(m) = 0 \;\text{ and }\; h_2(m) = 0, \tag{23}$$

   this value would meet the conditions:

   $$c_z = \alpha_{11}/\alpha_{21} \quad , \quad k_z = m_{11} - m_{21} \tag{24}$$
   $$c_z = \alpha_{12}/\alpha_{22} \quad , \quad k_z = m_{12} - m_{22} \tag{25}$$

   due to (20)-(21). But this requires:

   $$\frac{\alpha_{11}}{\alpha_{21}} = \frac{\alpha_{12}}{\alpha_{22}} \quad \text{and} \quad m_{11} - m_{21} = m_{12} - m_{22}. \tag{26}$$

   $y_1'(n)$ and $y_2'(n)$ are then only a scaled and time-shifted version of one another as for the contributions of $x_1'(n)$ and $x_2'(n)$ that they contain. We exclude this degenerate case here, so that:

   $$\forall\, (c,k), \; \sum_{m=-\infty}^{+\infty} h_1^2(m) > 0 \text{ or } \sum_{m=-\infty}^{+\infty} h_2^2(m) > 0. \tag{27}$$

   Combining this with (19) and (22) yields

   $$\forall\, (c,k), \quad \mathcal{DP}_s(n_1,n_2) > 0. \tag{28}$$

2. It may be shown in the same way that

   $$\mathcal{DP}_{p_1}(n_1,n_2) < 0 \quad \text{and} \quad \mathcal{DP}_{p_2}(n_1,n_2) < 0 \tag{29}$$

   leads to

   $$\forall\, (c,k), \quad \mathcal{DP}_s(n_1,n_2) < 0. \tag{30}$$

3. Now assume that $\mathcal{DP}_{p_1}(n_1,n_2)$ and $\mathcal{DP}_{p_2}(n_1,n_2)$ have opposite signs. The values of (c,k), defined by (24) and (25) resp. result in $\mathcal{DP}_s(n_1,n_2) = \mathcal{DP}_{p_2}(n_1,n_2)\sum_{m=-\infty}^{+\infty} h_2^2(m)$ and $\mathcal{DP}_s(n_1,n_2) = \mathcal{DP}_{p_1}(n_1,n_2)\sum_{m=-\infty}^{+\infty} h_1^2(m)$, which here have opposite signs. $\mathcal{DP}_s(n_1,n_2)$ therefore takes positive and negative values when (c,k) is varied.

The above results define the properties of the differential power $\mathcal{DP}_s(n_1,n_2)$ of the signal $s(n)$ with respect to those of the differential powers $\mathcal{DP}_{p_i}(n_1,n_2)$ of the innovations processes $p_i(n)$ of the source signals. Conversely, one then easily derives from this analysis the following properties of $\mathcal{DP}_{p_i}(n_1,n_2)$ vs those of $\mathcal{DP}_s(n_1,n_2)$ [1]:

1. If the sign of $\mathcal{DP}_s(n_1,n_2)$ changes when (c,k) is varied, then $\mathcal{DP}_{p_1}(n_1,n_2)$ and $\mathcal{DP}_{p_2}(n_1,n_2)$ have opposite signs[2].
2. Otherwise, $\mathcal{DP}_{p_1}(n_1,n_2)$ and $\mathcal{DP}_{p_2}(n_1,n_2)$ have the same sign, which may be determined as follows. $\mathcal{DP}_s(n_1,n_2)$ then has the same sign whatever c, and this sign is the same as that of both $\mathcal{DP}_{p_i}(n_1,n_2)$. This sign is e.g. the sign of $\mathcal{DP}_s(n_1,n_2)$ obtained when setting $c=0$. But in this case
$$s(n) = y_1'(n), \tag{31}$$
so that
$$\mathcal{DP}_s(n_1,n_2) = \mathcal{DP}_{y_1'}(n_1,n_2). \tag{32}$$
Therefore, in this case the common sign of both $\mathcal{DP}_{p_i}(n_1,n_2)$ is obtained as the sign of $\mathcal{DP}_{y_1'}(n_1,n_2)$ which is an observable quantity.

The above criterion provides the signs of the differential powers of the innovation processes of the source signals. The same criterion applies to the sources signals themselves and to $x_i'(n)$: for short-term stationary signals and for n_2-n_1 long enough, (18) yields:

$$\mathcal{DP}_{x_i'}(n_1,n_2) = \mathcal{DP}_{p_i}(n_1,n_2)\sum_{m=-L_1}^{L_2} d_i^2(m), \tag{33}$$

so $\mathcal{DP}_{x_i'}(n_1,n_2)$ and $\mathcal{DP}_{p_i}(n_1,n_2)$ have the same sign, and the same principle applies to $\mathcal{DP}_{x_i}(n_1,n_2)$.

The above criterion provides no practical means for determining if the sign of $\mathcal{DP}_s(n_1,n_2)$ varies with (c,k): (19) cannot be used to this end, as the $h_i(m)$ are unknown in practice. This problem is solved by considering $s(n)$ with respect to the filtered mixed signals. We then derive $\mathcal{DP}_s(n_1,n_2)$ from (17), which yields

$$\begin{aligned}\mathcal{DP}_s(n_1,n_2) &= \mathcal{DP}_{y_2'}(n_1,n_2)c^2 - 2\mathcal{DR}_{y_2'y_1'}(n_1,n_2,k,0)c \\ &\quad + \mathcal{DP}_{y_2'}(n_1,n_2).\end{aligned} \tag{34}$$

$\mathcal{DP}_s(n_1,n_2)$ thus appears as a 2^{nd}-order polynomial of c. Its coefficients, which depend on k, may be estimated from the filtered observed signals. The method that we propose for determining the signs of the differential powers of the source signals therefore consists in successively considering all integer values of k situated in a domain which contains the two values defined by (24)-(25). For each such value of k, we :

- Estimate the coefficients of the polynomial of c defined by (34).
- Determine if the sign of this polynomial changes when c is varied, i.e. if this polynomial has at least one real-valued root. This only requires one to check the sign of its determinant.
- Derive the signs of $\mathcal{DP}_{x_i}(n_1,n_2)$ from the above criterion, i.e. briefly: they have opposite signs iff the sign of $\mathcal{DP}_s(n_1,n_2)$ changes vs c for a given k or from one value of k to the next one.

5 CONCLUSIONS AND FUTURE WORK

In this paper, we first analyzed the stability of the BSS approach for underdetermined convolutive mixtures that we proposed in a previous paper. This showed that the adaptation gains of this approach should be selected depending on the signs of the differential powers of the useful sources to control its stability. This led us to develop a method for deriving these signs from the observed mixed signals in order for the approach to be applicable to a blind context. We now plan to apply this overall approach to real signals.

[1] For the sake of brevity, we omit the case when one or both $\mathcal{DP}_{p_i}(n_1,n_2)$ are zero.

[2] There is no sense wondering which of the innovation processes has a positive differential power, as the order of the sources in the considered mixed signals is arbitrary.

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