HIGHER ORDER BLIND SEPARATION OF NON ZERO-MEAN CYCLOSTATIONARY SOURCES

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ABSTRACT

Most of the current Second Order (SO) and Higher Order (HO) blind source separation (BSS) methods aim at blindly separating statistically independent sources, assumed zero-mean, stationary and ergodic. However in practical situations, such as in radiocommunications contexts, the sources are non stationary and very often cyclostationary. In a previous paper [4] the cumulant-based BSS problem for cyclostationary sources has been analysed assuming zero-mean sources (linear modulations). The non zero-mean case (some non linear modulations) has been considered recently in [5] for the current SO BSS methods. The purpose of this paper is now to analyse the behavior and to propose adaptations of the current HO BSS methods for non zero-mean cyclostationary sources.

1 INTRODUCTION

For more than a decade, SO [1] and HO [2] [3] blind methods have been developed to separate statistically independent sources, assumed *zero-mean*, *stationary* and *ergodic*. However, in many applications such as in radiocommunications contexts, the sources are non stationary and very often *cyclostationary* (digital modulations).

For this reason, in a previous paper [4], the behavior of the current SO and Fourth-order (FO) cumulant-based BSS methods has been analysed for cyclostationary sources, assumed zero-mean, assumption which generally holds for linear modulations in particular. In this context, the current SO methods have been shown to be not affected by the cyclostationarity of the sources contrary to the FO ones which have been shown to be strongly affected in some cases by this property.

Nevertheless, some cyclostationary sources used in practice are not zero-mean but are first order cyclostationary, which is in particular the case for some non linearly modulated digital sources such the CPFSK sources with an integer modulation indice [5]. For this reason, the behavior analysis together with adaptations of the current SO BSS methods for first and second order cyclostationary sources has been presented recently in [5] .

To complete this analysis, the purpose of this paper is now to analyse the behavior and to propose adaptations of the current HO BSS methods [2-3], such as the JADE method [2], for non zero-mean cyclostationary sources.

2 PROBLEM FORMULATION

A noisy mixture of P statistically independent narrow-band (NB) sources is assumed to be received by an array of N sensors. The vector, $\boldsymbol{x}(t)$, of the complex envelopes of the signals at the output of the sensors is thus given by

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_{pc}(t)\, \boldsymbol{a}_p + \boldsymbol{b}(t) \overset{\Delta}{=} A\, \boldsymbol{m_c}(t) + \boldsymbol{b}(t) \qquad (1)$$

where $m_{pc}(t) = m_p(t)e^{j(2\pi\Delta f_p t + \phi_p)}$ is the p-th component of the vector $\boldsymbol{m}_c(t)$, $m_p(t)$, Δf_p, ϕ_p and $\boldsymbol{a}_p$ correspond to the complex envelope, the carrier residu, the phase and the steering vector of the source p respectively, A is the (NxP) matrix whose columns are the vectors $\boldsymbol{a}_p$. The noise vector, $\boldsymbol{b}(t)$, assumed stationary and zero-mean, is normally distributed, spatially white and independent of the sources.

The classical HO blind source separation problem [2-3] consists to find, from both the SO and the HO statistics of the observations, the (NxP) *Linear* and *Time Invariant* source separator W, whose (Px1) output vector $\boldsymbol{y}(t) \overset{\Delta}{=} W^{H}\, \boldsymbol{x}(t)$ corresponds, to within a diagonal matrix Λ and a permutation matrix Π, to the best estimate, $\hat{\boldsymbol{m}}_c(t)$, of the vector $\boldsymbol{m}_c(t)$.

3 HO BLIND SOURCE SEPARATION OF ZERO MEAN STATIONARY SOURCES

3.1 Statistics of the data

For stationary sources, the SO correlation matrix, R_x, of the data is given by

$$R_x \overset{\Delta}{=} \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t)^{H}] = A\, R_{mc}\, A^{H} + \sigma^2\, \mathrm{I} \qquad (2)$$

where I denotes the (NxN) identity matrix, σ^2 is the input noise power per sensor, $R_{mc} \overset{\Delta}{=} \mathrm{E}[\boldsymbol{m}_c(t)\boldsymbol{m}_c(t)^{H}]$ is the correlation matrix of the vector $\boldsymbol{m}_c(t)$, diagonal for zero-mean statistically independent sources.

In a same way, the quadricovariance, Q_x, of the data, such that

$$Q_x[i, j, k, l] \overset{\Delta}{=} \mathrm{Cum}(x_i(t), x_j(t)^*, x_k(t)^*, x_l(t)) =$$
$$\mathrm{E}[x_i(t)x_j(t)^*x_k(t)^*x_l(t)] - \mathrm{E}[x_i(t)x_j(t)^*]\mathrm{E}[x_k(t)^*x_l(t)] -$$

$$\mathrm{E}[x_i(t)x_k(t)^*]\mathrm{E}[x_j(t)^*x_l(t)] - \mathrm{E}[x_i(t)x_l(t)]\mathrm{E}[x_j(t)x_k(t)]^* \quad (3)$$

is given by

$$Q_x = (A \otimes A^*)\, Q_{mc}\, (A \otimes A^*)^{\mathrm{H}} \quad (4)$$

where $\otimes$ is the Kronecker product and Q_{mc} is the quadricovariance of $\boldsymbol{m}_c(t)$.

1.2 Philosophy of the JADE method

The current JADE method [2] aims at separating the sources from the blind identification of the A matrix. This requires the prewhitening of the data which orthonormalizes the sources steering vectors so as to search for the latter through an unitary (PxP) matrix U simpler to handle. If we note $\boldsymbol{z}(t)$ the prewhitened observation vector, the matrix U is chosen so as to jointly diagonalize the P eigenmatrices of Q_z, the quadricovariance of $\boldsymbol{z}(t)$, associated to the P non zero eigenvalues and weighted by the latter, where Q_z is given by

$$Q_z = (A' \otimes A'^*)\, Q_{mc'}\, (A' \otimes A'^*)^{\mathrm{H}} \quad (5)$$

where $\boldsymbol{A'}$ is the (PxP) unitary matrix of the prewhitened source steering vectors and $Q_{mc'}$ is the quadricovariance of $\boldsymbol{m}_{c'}(t)$, the normalized vector $\boldsymbol{m}_c(t)$ such that each component has a unit power. Under some weak conditions [2], it is easy to verify that the unitary matrix $\boldsymbol{A'}$ is, to within a permutation and an unitary diagonal matrix, the only one which jointly diagonalizes the set of previous eigenmatrices.

1.3 Implementation of the JADE method

In practical situations, the SO and FO statistics of the data have to be estimated, by temporal averaging operations, using the ergodicity property of the data. Under this assumption, noting $\boldsymbol{x}(t_k)$ or $\boldsymbol{x}(k)$ the k-th sample of the observation vector $x(t)$, the empirical estimator $\hat{M}_x^{\varepsilon(\nu_n)}[\nu_n]$ of $M_x^{\varepsilon(\nu_n)}[\nu_n] \triangleq \mathrm{E}[x_{i_1}(t)^{\varepsilon_1}\, x_{i_2}(t)^{\varepsilon_2} \ldots x_{i_n}(t)^{\varepsilon_n}]$ from L independent data snapshots, is defined by

$$\hat{M}_x^{\varepsilon(\nu_n)}[\nu_n] \triangleq \frac{1}{L}\sum_{l=1}^{L} x_{i_1}(t_l)^{\varepsilon_1}\, x_{i_2}(t_l)^{\varepsilon_2} \ldots x_{i_n}(t_l)^{\varepsilon_n} \quad (6)$$

where $\nu_n \triangleq (i_1, i_2, \ldots, i_n)$ $(1 \le i_j \le N)$ and $\varepsilon(\nu_n) \triangleq (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)$ with $\varepsilon_i = \pm 1$ and such that $x_i(t)^{-1} = x_i(t)^*$ and $x_i(t)^1 = x_i(t)$. From (6), we deduce the empirical estimators of the SO and FO statistics of the data by replacing in the expressions (2) and (3) respectively the true SO and FO moments by their estimates (6). It is well known that for stationary and ergodic observations the empirical estimators, $\hat{R}_x$ and $\hat{Q}_x$, of R_x and Q_x becomes asymptotically unbiased and consistent.

2 HO BSS FOR NON ZERO-MEAN CYCLOSTATIONARY SOURCES

2.1 Statistics of the data

We now assume that the sources are cyclostationary even at the first order.

4.1.1. First order statistics

The first order statistic of $\boldsymbol{x}(t)$, given by (1), is defined by

$$e_x(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)] = A\,\mathrm{E}[\boldsymbol{m}_c(t)] \triangleq A\,\boldsymbol{e}_{mc}(t) \quad (7)$$

and the first order cyclostationarity property of the sources implies that

$$\boldsymbol{e}_x(t) = \sum_{\gamma \in \Gamma} \boldsymbol{e}_x^{\gamma} e^{j2\pi\gamma t} = \sum_{p=1}^{P} \sum_{\gamma_p \in \Gamma_p} e_{pc}^{\gamma_p} e^{j2\pi\gamma_p t} \boldsymbol{a}_p \quad (8)$$

where $\boldsymbol{e}_x^{\gamma} = \langle \boldsymbol{e}_x(t)\, \mathrm{e}^{-j2\pi\gamma t} \rangle_c$ is called the cyclic mean of $\boldsymbol{x}(t)$ for the cyclic frequency γ, $\langle . \rangle_c$ is the continuous-time temporal mean operation, Γ_p defines the set of cyclic frequencies γ_p of $e_{pc}(t) = \mathrm{E}[m_{pc}(t)]$, $\Gamma = \mathrm{U}_{1\le p\le P}\{\Gamma_p\}$ is the set of the cyclic frequencies γ of $\boldsymbol{e}_x(t)$ and $\boldsymbol{e}_{mc}(t)$.

4.1.2. Second order statistics

The vector $\boldsymbol{x}(t)$ has now two Time Dependent (TD) correlation matrices given by

$$R_x(t,\varepsilon) \triangleq \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t)^{\varepsilon\mathrm{T}}] = A\, R_{mc}(t,\varepsilon)\, A^{\varepsilon\mathrm{T}} + \sigma^2\delta(1+\varepsilon)\, \mathrm{I} \quad (9)$$

where $\varepsilon = \pm 1$, with the notation convention presented in 3.3. Introducing the zero-mean vector $\Delta\boldsymbol{x}(t) = \boldsymbol{x}(t) - \boldsymbol{e}_x(t)$, the correlation matrices $R_x(t,\varepsilon)$ can be written as

$$R_x(t,\varepsilon) = R_{\Delta x}(t,\varepsilon) + E_x(t,\varepsilon) \quad (10)$$

where $E_x(t,\varepsilon) \triangleq \boldsymbol{e}_x(t)\boldsymbol{e}_x(t)^{\varepsilon\mathrm{T}}$ and $R_{\Delta x}(t,\varepsilon) \triangleq \mathrm{E}[\Delta\boldsymbol{x}(t)\Delta\boldsymbol{x}(t)^{\varepsilon\mathrm{T}}]$ is given by

$$R_{\Delta x}(t,\varepsilon) = A\, R_{\Delta mc}(t,\varepsilon)\, A^{\varepsilon\mathrm{T}} + \sigma^2\delta(1+\varepsilon)\, \mathrm{I} \quad (11)$$

where $R_{mc}(t,\varepsilon) = R_{\Delta mc}(t,\varepsilon) + E_{mc}(t,\varepsilon)$ with $\Delta\boldsymbol{m}_c(t) \triangleq \boldsymbol{m}_c(t) - \boldsymbol{e}_{mc}(t)$, $R_{\Delta mc}(t,\varepsilon) \triangleq \mathrm{E}[\Delta\boldsymbol{m}_c(t)\Delta\boldsymbol{m}_c(t)^{\varepsilon\mathrm{T}}]$ and $E_{mc}(t,\varepsilon) = \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t)^{\varepsilon\mathrm{T}}$. Moreover, the SO cyclostationary property of the sources implies that the matrices $R_{mc}(t,\varepsilon)$ and $R_{\Delta mc}(t,\varepsilon)$ and thus, the matrices $R_x(t,\varepsilon)$ and $R_{\Delta x}(t,\varepsilon)$ have Fourier serial expansions introducing the SO cyclic frequencies of $\boldsymbol{m}_c(t)$, $\Delta\boldsymbol{m}_c(t)$, $\boldsymbol{x}(t)$ and $\Delta\boldsymbol{x}(t)$ respectively. In particular we obtain

$$R_x(t,\varepsilon) = \sum_{\alpha_\varepsilon} R_x^{\alpha_\varepsilon}(\varepsilon)\, e^{j2\pi\alpha_\varepsilon t} \quad (12)$$

where $R_x^{\alpha_\varepsilon}(\varepsilon) = \langle R_x(t,\varepsilon)\, \mathrm{e}^{-j2\pi\alpha_\varepsilon t} \rangle_c$ is a cyclic correlation matrix for the cyclic frequency α_ε. A similar expression is obtained for $R_{\Delta x}(t,\varepsilon)$ where the cyclic frequenies α_ε are replaced by β_ε.

4.1.3. Third order statistics

The TD third order moments of $\boldsymbol{x}(t)$ which are used in the following are defined by

$$T_x[i, j, k](t) \triangleq \mathrm{E}[x_i(t)x_j(t)^*x_k(t)^*] \quad (13)$$

and have a Fourier serial expansion given by

$$T_x[i, j, k](t) = \sum_{\nu} T_x^{\nu}[i, j, k]\, e^{j2\pi\nu t} \quad (14)$$

where $T_x^{\nu}[i, j, k] = \langle T_x[i, j, k](t)\, \mathrm{e}^{-j2\pi\nu t} \rangle$ is a cyclic third order moment for the cyclic frequency ν.

4.1.4. Fourth order statistics

The TD quadricovariance of $\boldsymbol{x}(t)$, $Q_x(t)$, is now defined by (3) but where the components $x_i(t)$ of $\boldsymbol{x}(t)$ are replaced by the components $\Delta x_i(t)$ of $\Delta\boldsymbol{x}(t)$.

4.2. Behavior analysis of the empirical estimators

For cyclostationary sources, HO BSS methods such as the JADE method has to exploit the information contained in the zero cyclic frequency of the matrices $R_x(t,-1)$ and $Q_x(t)$ [4], i.e. in the temporal means $R_x \triangleq \langle R_x(t,-1)\rangle_c$ and $Q_x \triangleq \langle Q_x(t)\rangle_c$ of the matrices $R_x(t,-1)$ and $Q_x(t)$ respectively, empirically estimated as described in 3.3. Note that the matrices R_x and Q_x are defined by (2) and (4) respectively, where R_{mc} and Q_{mc} are the temporal mean of $R_{mc}(t,-1)$ and $Q_{mc}(t)$ respectively.

For band-limited, cyclo-ergodic and sufficiently oversampled observations, the empirical estimator, $\hat{R}_x$, of R_x becomes asymptotically unbiased and consistent. However, while for zero-mean independent sources, the matrix R_{mc} is diagonal, it is not necessary the case for first order cyclostationary independent sources for which only the matrix $R_{\Delta mc}$ is diagonal while $E_{mc} \triangleq \langle \boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t)^{\mathrm{H}}\rangle_{\mathrm{c}}$ may be not diagonal, which may create an *apparent SO correlation* of the sources in the R_x matrix. It is shown in [5] that the matrix E_{mc} is not diagonal as soon as at least two sources share a first order cyclic frequency. In this case, the whitening process of the data required by the JADE method, and thus the JADE method itself, is affected by the presence of such sources. This problem has been strongly analysed in [5] for SO BSS methods and illustrated for some configurations of CPFSK sources.

Under the same assumptions, it is possible to show that the empirical estimator, $\hat{Q}_x$, of Q_x becomes asymptotically biased and generates an apparent quadricovariance temporal mean, noted Q_{xa}, and such that

$$
\begin{aligned}
Q_{xa}(i,j,k,l) &= Q_x(i,j,k,l) + \sum_{\alpha_1 \neq 0} C_x^{\alpha_1}(i,l)\; C_x^{\alpha_1}(j,k)^* \\
&+ \sum_{\alpha_{-1}\neq 0} \{ R_x^{\alpha_{-1}}(i,k)\, R_x^{-\alpha_{-1}}(l,j) + R_x^{\alpha_{-1}}(i,j)\, R_x^{-\alpha_{-1}}(l,k) \} \\
&- 2\sum_{\gamma}\sum_{\delta} \{ R_x^{-(\gamma+\delta)}(i,j) e_x^{\gamma}(k)^* e_x^{\delta}(l) + R_x^{-(\gamma+\delta)}(i,k)\, e_x^{\gamma}(j)^* e_x^{\delta}(l) \\
&\quad + R_x^{-(\gamma+\delta)}(l,j)\, e_x^{\gamma}(k)^* e_x^{\delta}(i) + R_x^{-(\gamma+\delta)}(l,k)\, e_x^{\gamma}(j)^* e_x^{\delta}(i) \\
&\quad + C_x^{(\gamma+\delta)}(i,l)\, e_x^{\gamma}(j)^* e_x^{\delta}(k)^* + C_x^{(\gamma+\delta)}(j,k)^*\, e_x^{\gamma}(i)\, e_x^{\delta}(l) \} \\
&+ \sum_{\gamma} \{ T_x^{-\gamma}(l,j,k)\, e_x^{\gamma}(i) + T_x^{-\gamma}(k,i,l)^*\, e_x^{\gamma}(j)^* \\
&\quad + T_x^{-\gamma}(j,i,l)^*\, e_x^{\gamma}(k)^* + T_x^{-\gamma}(i,j,k)\, e_x^{\gamma}(l) \} \\
&+ 6 \sum_{\gamma}\sum_{\delta}\sum_{\omega} e_x^{\gamma}(i)\, e_x^{\delta}(j)^* e_x^{\omega}(k)^* e_x^{\delta+\omega-\gamma}(l)
\end{aligned}
\tag{15}
$$

where $R_x^{\alpha_\varepsilon}(-1)$ and $R_x^{\alpha_\varepsilon}(1)$ are noted $R_x^{\alpha_\varepsilon}$ and $C_x^{\alpha_\varepsilon}$ respectively. The matrix Q_{xa}, whose coefficient are given by (15), can also be written as

$$
Q_{xa} = (A \otimes A^*)\, Q_{mca}\, (A \otimes A^*)^{\mathrm{H}} \tag{16}
$$

where Q_{mca} is the apparent quadricovariance of $\boldsymbol{m}_c(t)$, defined by (15) with the indice *mc* replacing *x*. In a similar way, the apparent quadricovariance of the whitened observation vector $\boldsymbol{z}(t)$ has also the form depicted by (16) but where the *mca* indice is replaced by the *mca'* one. The previous result shows that $Q_{xa}(i,j,k,l) = Q_x(i,j,k,l)$ if $Q_{mca}(i,j,k,l) = Q_{mc}(i,j,k,l)$, which obviously occurs in particular for zero-mean stationary sources. In a same way, $Q_{xa}(i,j,k,l) \neq Q_x(i,j,k,l)$ if $Q_{mca}(i,j,k,l) \neq Q_{mc}(i,j,k,l)$, which obviously occurs when the sources have some non zero cyclic frequencies, i.e when the sources are cyclostationary. In this latter case, some FO cross-cumulants of the Q_{mca} matrix, i.e some $Q_{mca}(i,j,k,l)$ terms with $(i,j,k,l) \neq (i,i,i,i)$, may be non zero and may create *an apparent FO correlation* of some sources in the Q_{xa} matrix, which may also affect the behavior of the JADE method. We verify from (15) with *mc* instead of *x* that this situation occurs when at least two sources are such that the intersection between their sets of first, third and non-zero SO cyclic frequencies is not empty. For zero-mean sources, this condition means that the two considered sources have to share some non zero SO cyclic frequencies, results already found in [4]. However for non zero mean sources such as the CPFSK sources with an integer modulation indice [5], the previous condition is obtain in particular when the two sources share at least one first order cyclic frequency. These results are illustrated in section 5.

4.3. Adaptation : new estimators introduction

4.3.1. SO statistics

Since the averaged correlation matrice R_{mc} may be non diagonal in the presence of first order cyclostationary sources, we have to exploit the information contained in the averaged covariance matrice $R_{\Delta mc}$ which is always diagonal for statistically independent sources, zero-mean or not. In other words, we have to implement the whitening step of indirect HO BSS methods from the averaged covariance matrix $R_{\Delta x} \triangleq \langle R_{\Delta x}(t,-1)\rangle_{\mathrm{c}}$ defined from (10) for $\varepsilon = -1$, where $E_x \triangleq \langle E_x(t,-1)\rangle_{\mathrm{c}}$ is given by

$$
E_x \triangleq \langle \boldsymbol{e}_x(t)\, \boldsymbol{e}_x(t)^{\mathrm{H}}\rangle_{\mathrm{c}} = \sum_{\gamma\in\Gamma} \boldsymbol{e}_x^{\gamma} \boldsymbol{e}_x^{\gamma H} \tag{17}
$$

where Γ is the set of the first order cyclic frequencies of $\boldsymbol{x}(t)$. So, for first order and SO cyclostationary and band-limited vectors $\boldsymbol{x}(t)$ having a SO cyclo-ergodicity property and for sufficiently oversampled data, after a preliminary step of first order cyclic frequencies estimation, we define an asymptotic unbiased and consistent estimator $\hat{R}_{\Delta x}(L)$ of the averaged covariance matrix $R_{\Delta x}$ by [5]

$$
\hat{\boldsymbol{R}}_{\Delta x}(L) \triangleq \hat{\boldsymbol{R}}_x(L) - \sum_{l=1}^{L} \hat{\boldsymbol{e}}_x^{\gamma(l)} \hat{\boldsymbol{e}}_x^{\gamma(l)H} \tag{18}
$$

where $\hat{R}_{\Delta x}(L)$ is defined in 3.3 and

$$
\hat{\boldsymbol{e}}_x^{\gamma} \triangleq \frac{1}{L}\sum_{l=1}^{L} \boldsymbol{x}(l)\, e^{-j2\pi l T_e \gamma} \tag{19}
$$

4.3.2. FO statistics

The exploitation of the information contained in the true data quadricovariance matrix in the presence of first order cyclostationary sources requires to take into account the cyclic statistics of the data up to the third order as it is shown by (15). In this context, after a preliminary step of data's first and SO cyclic frequencies estimation, we have to compute an estimate of $Q_x(i, j, k, l)$ from (15), replacing $Q_{xa}(i, j, k, l)$ by its empirical estimation presented in 3.3 and the true data cyclic statistics up to the third order, $e_x^\gamma(i)$, $R_x^\alpha(i, j)$, $C_x^\beta(i, j)$ and $T_x^\nu(i, j, k)$ by their estimates given respectively by

$$\hat{e}_x^\gamma(i) = \frac{1}{L} \sum_{l=1}^{L} x_i(l)\, e^{-j2\pi\gamma l T_e} \tag{20}$$

$$\hat{R}_x^\alpha(i, j) = \frac{1}{L} \sum_{l=1}^{L} x_i(l)\, x_j(l)^*\, e^{-j2\pi\alpha l T_e} \tag{21}$$

$$\hat{C}_x^\beta(i, j) = \frac{1}{L} \sum_{l=1}^{L} x_i(l)\, x_j(l)\, e^{-j2\pi\beta l T_e} \tag{22}$$

$$\hat{T}_x^\nu(i, j, k) = \frac{1}{L} \sum_{l=1}^{L} x_i(l)\, x_j(l)^*\, x_k(l)^*\, e^{-j2\pi\nu l T_e} \tag{23}$$

Thus, under the assumption of non zero-mean cyclostationary, cyclo-ergodic and band-limited vectors $\boldsymbol{x}(t)$, provided the data are sufficiently oversampled, we obtain an unbiased and consistent estimate of $Q_x(i, j, k, l)$.

3 SIMULATIONS

To illustrate the previous results, we assume that two statistically independent NB and orthogonal ($A^H A = N$ I) 2-CPFSK sources are received by an array of N =5 sensors. These two sources have the same input SNR (Signal Noise Ratio) of 10 dB and are synchronized. Their symbol durations and their modulation indices are such that $h_1/T_1=h_2/T_2=(4T_e)^{-1}$ for h_1=2 and h_2=4. We apply the JADE method and the SINRMk (Maximal Signal to Interference plus Noise Ratio of the source k), used in [4], at the output of the JADE separator for k=1,2 are computed and averaged over 200 realizations.

Under the previous assumptions, the figure 1 shows the variations of the SINRM1 of the first source at the output of the JADE separator as a function of the number of snapshots, when the JADE method is implemented from four couples of statistics estimators : (EMP_SO, EMP_FO), (EMP_SO, NEW_FO), (NEW_SO, EMP_FO) and (NEW_SO, NEW_FO), where EMP and NEW corresponds to the Empirical and the New statistics estimators. Taking zero carrier frequencies, $\Delta f_1=\Delta f_2=0$, the first and SO cyclic frequencies of the observations belong to $\{0, -h_1/2T_1, h_1/2T_1\}$ and are such that the two sources become apparently SO and FO correlated. As planned, the figure 1 shows the poor separation of the sources when the JADE method uses at least one of the two empirical estimators. On the contrary, the use of the NEW SO and FO statistics estimators allows the optimal separation of the two 2-CPFSK sources.

4 CONCLUSION

In this paper, we showed that the current HO BSS methods, such as the JADE method, may be affected by the presence of statistically independent NB sources which are both non zero-mean and cyclostationary, such as some CPFSK sources. This problem is directly related to the fact that, in the previous context, the current HO BSS methods aim at exploiting the information contained in biased estimation of the SO and FO cumulant matrix of the data, both generated by the SO and FO empirical statistics estimators respectively.

To solve this problem, the HO BSS methods have to exploit the information contained in the temporal mean of both the true SO and FO cumulant matrices of the data. For this purpose, we introduced in the paper unbiased and consistent estimators of the two previous matrices for non zero-mean cyclostationary observations, assuming that the first and second order cyclic frequencies of the latter have been estimated previously. These estimators require the estimation of the cyclic statistics of the observations up to the third order.

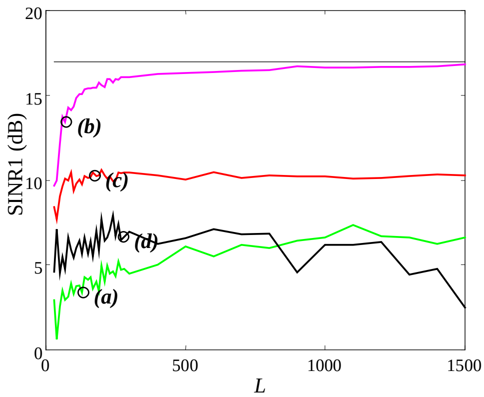

Fig.1 - *SINRM1 as a function of L, (a) (EMP_SO, EMP_FO), (b) (NEW_SO, NEW_FO), (c) (NEW_SO, EMP_FO), (d) (EMP_SO, NEW_FO)*

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