MC-CDMA WITH PROLATE SPHEROIDAL FUNCTIONS-BASED ORTHOGONAL CODES

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ABSTRACT

The limiting factor of an uplink Multi-Carrier Code Division Multiple Access (MC-CDMA) performance is Multiple Access Interference (MAI) caused by users' different propagation channels. Even though MAI can be reduced by complex multiuser receiver, our approach is to simplify the receiver choosing more appropriate users' code. We investigated orthogonal codes that concentrate signal energy in desired interval and, to accomplish that, Prolate Spheroidal Functions (PSF), and their time and frequency discrete versions, including periodic sequences, are analyzed. These codes achieve MAI elimination for certain number of users and considerable MAI reduction if their number is increased above this limit, with simple time pre-processing in the receiver. Furthermore, redundancy for Inter Block Interference (IBI) is not necessary in this system. Computer simulation demonstrate superior performance of this MC-CDMA system when compared to the standard one employing Hadamard codes.

1. INTRODUCTION

Orthogonal Frequency Division Multiplex (OFDM) is MC single user modulation whose high efficiency is achieved with overlapped orthogonal signals' spectrums. Several attractive characteristics as robustness to frequency selectivity, common to radio channels, made it suitable for high-rate multimedia systems. However, due to its high requirement for carriers' frequencies accuracy, it was not possible to take advantage of this technique until its digital implementation. [1]

Multiuser systems based on the combination of OFDM and CDMA principles are a challenging issue for researchers at the present. Our focus is MC-CDMA system in which users are assigned by orthogonal codes in frequency domain. However, in the uplink scenario, where each user transmits in a different channel, an orthogonality loss in the receiver is inevitable, causing MAI, and limiting system performance. [2]

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Thus, users' codes design is investigated in order to reduce MAI. One of the possible approaches suggests to evaluate codes by its spectral correlation function, minimizing codes' cross-correlation function leading to MAI minimization [3]. In the same line, in [4], the principles of interferometry are applied to create a code set, choosing different phase shifts (spreading codes) for each user, ensuring a periodic main-lobe in the time domain with side-lobe activity at intermediate times; in this case all users have different main-lobe activity time.

Codes proposed here are based on Prolate Spheroidal Functions because of their energy concentration property. Their time and frequency discrete versions are developed, and infinite set of possible codes is obtained. Further restriction, such as the number of signal samples, or PAR reduction, can be imposed to obtain the appropriate codes set. The limit number of signals' samples, 1, can be regarded as the discrete version of the idea exposed in [4]. This MC-CDMA system takes advantage of the signals' time concentration, leading to MAI reduction and avoidance of IBI with no additional guard time required.

In Section 2, analysis of PSF's discrete version, **P**SF-based **O**rthogonal **C**odes (POC), is presented. POC MC-CDMA system is explained and analyzed in Section 3. Some preliminary computer simulations of POC MC-CDMA system followed by several conclusions are presented in Sections 4 and 5, respectively.

2. PERIODIC DISCRETE PROLATE SPHEROIDAL SEQUENCES

The transmitted i^{th} user baseband signal in MC-CDMA can be expressed as:

$$v_i[n] = \sum_{k=0}^{M-1} V_i[k]\, e^{jk\frac{2\pi}{M}n} \tag{1}$$

where $V_i[k]$ is its assigned code of length M , usually Hadamard code. This model can be interpreted as periodic sequence decomposition in Discrete Fourier Series $V_i[k]$:

$$v_i[n] = \sum_{k=-(M-1)/2}^{(M-1)/2} V_i[k]\, e^{jk\frac{2\pi}{M}n} \tag{2}$$

where $|i| \leq (M-1)/2$ is now the number of different codes and M is odd number of frequency points. Energy concentration of this bandlimited sequence depends on the employed codes (frequency amplitudes). The maximum concentrated bandlimited functions and sequences are found to be those whose frequency charasteristic are Prolate Spheroidal Functions (PSF) [5, 6, 7, 8]. Here, we are interested in periodic sequence concentration where the minimizing criteria is applied to the signal samples in the specified interval; this restriction is much looser than one imposed in [7], since continuous function concentration is not considered, and our only interest is its value at particular samples, multiples of the inverse sampling frequency.

Defining circulant matrix $\mathbf{W}$ as:

$$\mathbf{W}(m,n) = \frac{sin\left(\frac{2\pi(n-m)}{M}\left(N_0 + \frac{1}{2}\right)\right)}{M\,sin\left(\frac{\pi(n-m)}{M}\right)} \qquad (3)$$

the amplitudes of the exponentials, and the energy concentration parameter, λ_i, are obtained from the following set of equations:

$$\sum_{m=-(M-1)/2}^{(M-1)/2} W(m,n)V_i\left[m\right] = \lambda_i V_i\left[n\right] \qquad (4)$$

for $|n| \leq (M-1)/2$, $|i| \leq (M-1)/2$; samples on which the energy is to be concentrated are $-N_0...N_0$.

The set of equation defined by eq. (4) can be expressed in the matrix form as:

$$(\mathbf{W} - \lambda_i\mathbf{I})\mathbf{V}_i = 0 \qquad (5)$$

where λ_i are the eigenvalues and $\mathbf{V}_i$ are the associated eigenvectors. Further analysis can be done observing the fact that

$$\mathbf{W} = \mathbf{F}diag(\mathbf{w})\mathbf{F}^H \qquad (6)$$

with $\mathbf{F}$ denoting Fourier matrix and vector $\mathbf{w}$ rectangular window sequence, specifying the indexes of samples where energy is to be concentrated, with value 1 in those positions and 0 elsewhere. Observing eq. (5) and (6), it can be noticed that eq. (6) presents a possible solution for spectral decomposition of $\mathbf{W}$. The eigenvectors are the columns of the $\mathbf{F}$ and their corresponding eigenvalues are 1 and 0. Let us remark that multiplicity of eigenvalues is the same as the number of samples where $\mathbf{w} = 1$, meaning that infinite basis can be provided. For example, decomposition of $\mathbf{F}$ in real $\mathbf{F}_R$ and imaginary $\mathbf{F}_I$ part can give another solution, as showed in [8].

The analysis of the form of the sequence $\mathbf{v}_i$ that satisfy eq. (5), can also be performed:

$$(\mathbf{F}diag(\mathbf{w})\mathbf{F}^H - \lambda_i\mathbf{I})\mathbf{V}_i = 0 \qquad (7)$$

or,

$$(diag(\mathbf{w}) - \lambda_i\mathbf{I})\mathbf{F}^H\mathbf{V}_i = 0 \qquad (8)$$

$$diag(\mathbf{w} - \lambda_i)\mathbf{v}_i = 0 \qquad (9)$$

Having in mind the structure of vector $\mathbf{w}$, and the fact that λ_i represents the signals energy in the desired interval, (meaning it has value 1 if the signal has the samples inside the interval and 0 otherwise), the set of vectors $\mathbf{v}_i$ that satisfy the Eq. (8) is infinite; $\mathbf{v}_i$ can be any unit energy vector whose zero samples are at the same location points as the $\mathbf{w}$ zeros samples. Their common property, besides the double orthogonality in intervals $[-M, M]$ and $[-N_0, N_0]$, is odd symmetry.

Let us in the sequel label our contribution as Periodic Discrete Prolate Spheroidal Sequences (PDPSS), remarking the periodic character of the sequences involved. Thus, orthogonal codes obtained this way will be denominated as PSF-based Orthogonal Codes - POC. These codes permit generation of users' signals with the same number of samples in the desired interval and with zero samples outside it. If the number of signals' samples is s, a total of s signals, whose samples are in identical positions, will exist. The users sharing the same time location will be denominated as image-users. In the system analyzed here, POCs resulting in 2-samples signals are chosen for the sake of simplicity, although other possibilities are also feasible.

Double orthogonality is very useful when POC are applied to MC-CDMA system. The fact that the signals are orthogonal in the shorter interval, as well as in the whole period, permits zero IBI with no redundancy, leading to more efficient system.

3. MC-CDMA SYSTEM VIA POC

3.1. Transmitter design

The transmitter of user i is similar to the standard MC-CDMA transmitter, except for the necessary time-domain signal rotation. The transmitted signal is

$$\mathbf{x}_i(n) = \mathbf{D}\mathbf{F}_M^T\mathbf{c}_i b_i(n) \qquad (10)$$

where $\mathbf{D}$ is a channel dependent rotation matrix. It rotates symmetric time samples to the form of zero padded vector $\mathbf{x}_i(n) = [\mathbf{y}_i; 0_{L-1,1}]$, concentrating the samples carrying the useful, non-zero information at the beginning of the signal period in order to avoid IBI. It needs to know the channel length, L, in order to adequately rotate the signal. $b_i(n)$ and $\mathbf{c}_i$ are the i^{th} user's symbol transmitted in n^{th} block, and its spreading code, respectively.

Selection of user's code ($\mathbf{c}_i$) depends on the number of users that are simultaneously transmitting. Improved system performance is achieved when chosen code interferes with the minimum number of already active users. With adequate selection, if the number of active users is smaller than N/L, where *N=M-L+1*, the codes can be chosen to have MAI-free TDMA system in the limit case (*1*-sample signals). If the signals consists of more samples, this number represents the maximum number of users for which the interference can be completely eliminated in the receiver.

The number of possible codes without IBI is also channel dependent; the number of desirable IBI free codes is not equal to the number of used carriers (*M*), but to *N*. This fact could be interpreted as a spectral efficiency loss of this scheme. However, having in mind that multiuser systems are interference-limited, maximum number of users is not achievable with satisfactory performances in a standard scenario, and this restriction is already naturally imposed by MAI. Even though, further modifications can be done, like introducing guard interval between symbols, that way ensuring IBI free transmission for *M* users.

3.2. Receiver design

The received signal, in a full load uplink scenario that assures IBI-free transmission, with N users, is:

$$\mathbf{y}(n) = \sum_{j=1}^{N} \mathbf{H}_j \mathbf{x}_j(n) + \mathbf{n}(n) \tag{11}$$

where $\mathbf{H}_j$ is the Toeplitz channel matrix and vector $\mathbf{n}(n)$ AWGN samples.[2]

Some time pre-processing, before equalization at the receiver, is possible thanks to the signal concentration, leading to improved system performance.

As a logical consequence of signal concentration on a few samples, time windowing is possible in POC MC-CDMA system. This window let pass only the samples carrying the desired user information, while other users' signals, whose samples do not overlap with those of the desired user, are blocked. This reduces both, MAI generated by those users, and noise.

Other possibility for MAI reduction is blocking of the image users, possible because of the image-users orthogonality in the time domain. Furthermore, this does not require complex receivers since those users' signals are also concentrated on few samples. This blocking matrix, $\mathbf{B}_{im}$ depends on the image-user's code and channel length:

$$\mathbf{B}_{im} = \mathbf{I} - pinv(\mathbf{X}_j) \tag{12}$$

where $\mathbf{X}_j$ is a matrix whose columns are formed from the signal of the user that is to be blocked, $\mathbf{x}_j$, and its *L-1* shifts.

After this pre-processing, the channel equalization both in time and frequency domain can be performed.

The zero forcing (ZF) equalization in the time domain is complex because it requires channel matrix inversion, and can be inaccurate if the path that the receiver is synchronized on, is weaker than the others. However, it guarantees the channel invertibility. [2]

The frequency equalization shown in Fig. 1 is simpler and is usually performed in MC-CDMA systems. ZF equalization also suffers from the problems common to MC systems; if the channel frequency response has deep fades on the frequency that should be equalized, the noise is enhanced and the carried information may be lost.

In POC MC-CDMA system, frequency equalization is performed in a way similar to the one in [2], for zero padded transmission, converting the linear convolutive time model into multiplicative in frequency domain. Once in the time domain, signal is zero padded and rotated again to the symmetric form. Finally, in the frequency domain user's code despreading is performed.

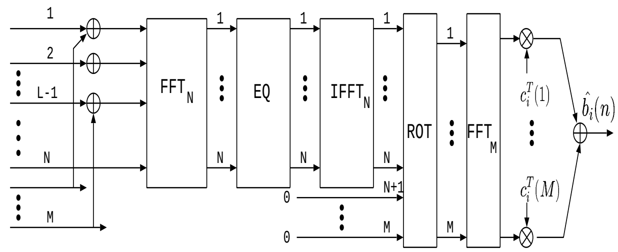

Fig. 1. Frequency equalizer

3.2.1. *Improved receivers by blocking*

More sophisticated receivers for slowly fading channels that are based on time processing of the received signal are also developed, as suggested in [9]. The Generalizes Side Lobe Canceller (GSLC) like structure is implemented for MAI reduction.

The signal model that results adequate for this type of receiver is the signal $\mathbf{x}_i$, representated in matrix form, with its *L-1* delayed versions, and channel representation in a vector form. Rewriting eq. (11) this way, it becomes:

$$\mathbf{y}(n) = \sum_{j=1}^{N} \mathbf{X}_j(n) \mathbf{h}_j + \mathbf{n}(n) \tag{13}$$

where $\mathbf{h}_j$ is the channel finite impulse response.

Proper blocking matrix for user i is now defined as:

$$\mathbf{B}_i(n) = \mathbf{X}_i(\mathbf{X}_i^H \mathbf{X}_i)^{-1} \mathbf{X}_i^H \tag{14}$$

This matrix blocks the desired user's signal in the lower branch, and the other users' interference projected to the subspaces defined with the columns of $\mathbf{X}_i$, can be eliminated.

4. COMPUTER SIMULATIONS

Simulations are performed to compare different proposed MC-CDMA systems. Systems with spreading factor (SF) are 16 (for Hadamard Codes) and M=17 (for POCs). QPSK symbols are transmitted via every 2-ray Rayleigh channels with power profile (0,-9)dB, and the average of 1000 channels' Bit Error Rate (BER) is depicted. Perfect synchronization is considered. The abbreviations used are: f and t for frequency and time equalization, respectively; $\mathbf{J}$ for the time windowing, $\mathbf{Bim}$ for the blocking of image user and $\mathbf{B}$ for the Interference Cancelation (IC) based on GSLC structure.

In Fig. 2, systems' behavior, simulating 12 active users, with different AWGN levels is presented. Simulations confirm the improvement when more sophisticated receivers, based on GSLC, are used. Furthermore, the better performance of POC´s MC-CDMA is demonstrated; although, it does suffer from the same problem as MC-CDMA with Hadamard codes: high interference (not the noise-level) is the limiting factor of its performance. Thus, Interference Canceller (IC) structures are necessity, since they enhance the systems performance significantly as the noise level gets lower.

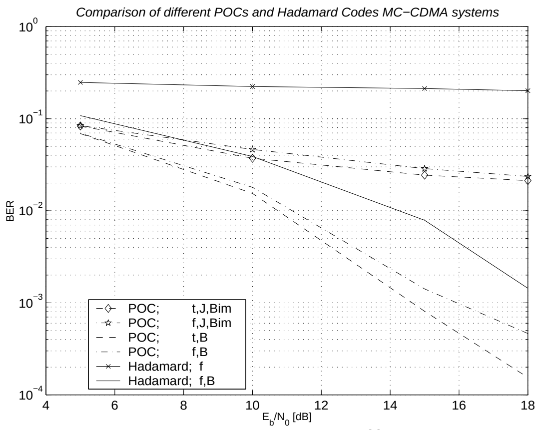

Fig. 2. MC-CDMA schemes with different noise levels

In Fig. 3, different receiver's schemes are analyzed depending on the number of active users. The superior performance of POC vs Hadamard codes without IC is presented. It can be observed that the performances of the system with POC codes is constant and that there is no MAI if the number of users is smaller than 8, as was stated in Subsection 3.1. On the other hand, MC-CDMA with Hadamard codes is saturated if there is more than 7 active users. IC has sense for standard MC-CDMA systems no matter how many active users there is. However, if POC codes are used, there is no interference till a certain number of active users, and IC should be applied only if there are many active users. That is the hybrid solution, presented in the Fig. 3. It can be observed that if there is only one user transmitting, the proposed system performs superior to the standard one, because of the noise suppression with applied time pre-processing. This can be considered as confirmation of its superior performance in the downlink channel.

5. CONCLUSIONS

MC-CDMA system with codes based on Prolate Spheroidal Functions is presented in this paper. This system performance was shown to be superior to the MC-CDMA system employing Hadamard codes both in downlink and uplink channel; its time domain energy concentration property avoids the need for guard interval, leading to more efficient system, and also permits simple interference reduction processing in the receiver. Further codes investigation should consider its selection for PAR reduction.

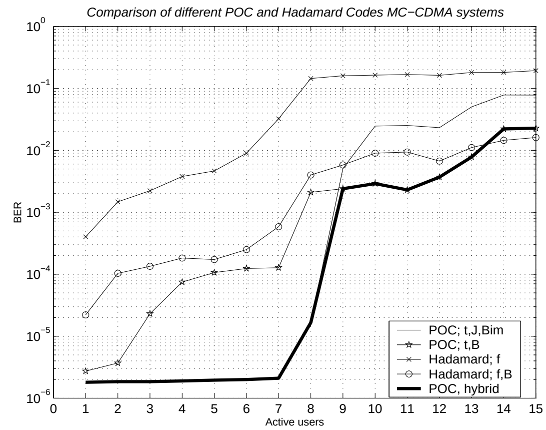

Fig. 3. MC-CDMA with different number of active users

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