POWER ALLOCATION TECHNIQUES FOR JOINT BEAMFORMING IN OFDM-MIMO CHANNELS

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ABSTRACT

The use of smart antennas technology permits increasing the data rate and the quality of transmission. Due to this reason, in last years the study of Multi-Input-Multi-Output (MIMO) channels has been given an increasing importance. Simultaneously, the Orthogonal Frequency Division Multiplexing (OFDM) has been proposed for several communications and broadcasting systems due to the easy implementation of the (de)modulator and equalizer. In this paper we present and analyze several techniques that combine OFDM and arrays of antennas at both the transmitter and receiver. It is considered that Channel State Information (CSI) is available at both sides of the system, and a joint beamforming structure is proposed for the design of the front-ends. Within the proposed techniques, MAXMIN is shown to have a good performance with a low computational load.

1 INTRODUCTION

Classically, the use of smart antennas technology has been applied only at the receiver side of the communications system. However, in last years a lot of work has been done on defining techniques able to apply the space diversity simultaneously at the transmitter side, configuring a Multi-Input-Multi-Output (MIMO) channel. The Orthogonal Frequency Division Multiplexing (OFDM) has also been specified for many communications and broadcasting systems because of its efficiency and the easy implementation of the modulation, demodulation and equalization processes.

In this paper we analyze and compare several joint beamforming strategies for the case of combining the OFDM modulation and multiple antennas at the transmitter and receiver. It is assumed that Channel State Information (CSI) and the second-order statistics of the noise plus interferences are available at both sides. For the design of the beamvectors it is applied a constraint over the total transmit power. We summarize classical methods for designing the beamformers and analyze their asymptotic behavior, concluding that the techniques resulting from this analysis have a lower computational load and are directly related to well-known algebraic norms of the Signal-to-Noise and Interference Ratio (SNIR) over the subcarriers of the OFDM modulation.

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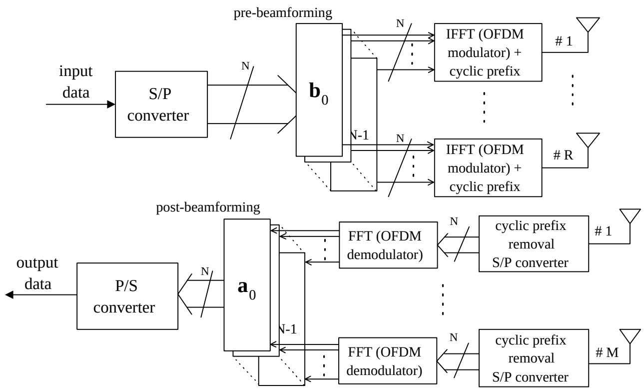

Figure 1: General system structure and configuration for a MIMO channel and OFDM modulation.

This paper is organized as follows. In Section 2 the system and signal models are presented. Section 3 solves the problem for the single-carrier case, whereas Section 4 extends the solution to the multicarrier modulation. Finally, in Section 5 some simulation results and conclusions are presented.

2 SYSTEM AND SIGNAL MODELS

The system design (Fig. 1) is based on beamforming at both the transmitter and receiver, where R and M are the number of transmit and receive antennas. The number of points of the FFT/IFFT is assumed to be N, and $s_k(t)$ is the transmitted symbol at the kth carrier during the tth OFDM period. It is assumed that the mean energy of the symbols is normalized: $E\{|s_k(t)|^2\} = 1$. The Cyclic Prefix (CP) consists on the insertion of L samples at the beginning of each OFDM period after the unitary IFFT operation.

The beamvectors at the transmitter are represented by $\mathbf{b}_k = \left[\begin{array}{ccc} b_1(k) & \cdots & b_R(k) \end{array} \right]^T$, where $(\cdot)^T$ stands for transpose, and whose components are the factors that multiply $s_k(t)$ before the IFFT operation. k is the carrier index.

The MIMO channel is represented by the collection of time-impulse responses $h_{m,r}(n)$ with length $N_t + 1$ taps, which models the response of the channel between the rth transmit and mth receive antenna, with a frequency response equal to: $H_{m,r}(k) = \sum_{n=0}^{N_t} h_{m,r}(n) e^{-j\frac{2\pi}{N}kn}$. We assume that $L \geq N_t$, therefore neither Inter Carrier Interference (ICI) nor

Inter Block Interference (IBI) are present in the received signal. Under these assumptions, we can represent in a single snapshot vector $\mathbf{y}_k(t)$ all the samples corresponding to the kth subcarrier at all the receive antennas:

$$\begin{aligned} \mathbf{y}_k(t) &= \mathbf{H}(k)\mathbf{b}_k s_k(t) + \mathbf{n}_k(t) && (1) \\ [\mathbf{H}(k)]_{m,r} &= H_{m,r}(k), \quad 1 \le m \le M,\ 1 \le r \le R && (2) \end{aligned}$$

where $\mathbf{H}(k)$ is the MIMO channel frequency response matrix at the kth carrier, $\mathbf{n}_k(t)$ is the noise plus interferences contribution, with associated covariance matrix $\mathbf{R}_n(k) = E\left\{\mathbf{n}_k(t)\mathbf{n}_k^H(t)\right\}$, $(\cdot)^H$ stands for the complex conjugate transpose and $E\{\cdot\}$ for the mathematical expectation. The receiver applies a different beamvector $\mathbf{a}_k = \begin{bmatrix} a_1(k) & \cdots & a_M(k) \end{bmatrix}^T$ for each subcarrier after extracting the CP and calculating the unitary FFT. The output sample $r_k(t)$ of the kth receive beamformer is as follows:

$$r_k(t) = \mathbf{a}_k^H \mathbf{y}_k(t) = \mathbf{a}_k^H \mathbf{H}(k)\mathbf{b}_k s_k(t) + \mathbf{a}_k^H \mathbf{n}_k(t) \tag{3}$$

Finally, the estimated symbol for the kth carrier is based on a decision taking into account the soft-estimate $r_k(t)$: $\widehat{s}_k(t) = dec\{r_k(t)\}$.

3 SUBCARRIER OPTIMIZATION OF SNIR

In this section we present the design of the receive and transmit beamvectors subject to a carrier power constraint. The SNIR at the kth carrier is as follows:

$$\mathrm{SNIR}_k = \frac{E\left\{\left|\mathbf{a}_k^H \mathbf{H}(k)\mathbf{b}_k s_k(t)\right|^2\right\}}{E\left\{\left|\mathbf{a}_k^H \mathbf{n}_k(t)\right|^2\right\}} = \frac{\left|\mathbf{a}_k^H \mathbf{H}(k)\mathbf{b}_k\right|^2}{\mathbf{a}_k^H \mathbf{R}_n(k)\mathbf{a}_k} \tag{4}$$

The beamvector $\mathbf{a}_k$ that maximizes SNIR_k is the matched-filter: $\mathbf{a}_k = \alpha_k \mathbf{R}_n^{-1}(k)\mathbf{H}(k)\mathbf{b}_k$, where the constant α_k is arbitrary and does not affect SNIR_k. When using this design for the receiver, the SNIR is maximized and equal to: $\mathrm{SNIR}_k \mid_{\max} = \mathbf{b}_k^H \mathbf{H}^H(k)\mathbf{R}_n^{-1}(k)\mathbf{H}(k)\mathbf{b}_k$.

In a real system, the power constraints at the transmitter side must be taken into account. If the transmitted power over all the antennas at the kth carrier is a prefixed value $\|\mathbf{b}_k\|^2 = p_k$, then it can be shown, that the best transmit beamvector $\mathbf{b}_k$, the one that maximizes SNIR_k, is a scaled version of the normalized eigenvector $\mathbf{u}_k$ associated to the maximum eigenvalue $\lambda_{\max}(k)$ of the following expression:

$$\lambda_{\max}(k)\mathbf{u}_k = \mathbf{H}^H(k)\mathbf{R}_n^{-1}(k)\mathbf{H}(k)\mathbf{u}_k,\ \|\mathbf{u}_k\| = 1,\ \mathbf{b}_k = \sqrt{p_k}\mathbf{u}_k$$

Under these assumptions, the SNIR can be expressed as: $\mathrm{SNIR}_k = \lambda_{\max}(k)p_k$. In this section, it has been assumed that it is known which is the transmitted power at each subcarrier p_k. However, in a real system, the power constraint refers to all the available power at the transmitter side. In the following section we treat this design problem.

4 JOINT BEAMFORMING TECHNIQUES

We consider the power allocation problem resulting from the distribution of all the available power at the transmitter side P_0. This global power constraint is expressed as follows:

$$\sum_{k=0}^{N-1} \|\mathbf{b}_k\|^2 = \sum_{k=0}^{N-1} p_k = P_0 \tag{5}$$

We now summarize three classical and well-known strategies (CAP, MMSE and Chernoff) for allocating the power depending on different design criteria. For all these techniques, some carriers, the ones most degraded by frequency selective channels and/or high level noise or interferences, may be unused. As a direct consequence, it is necessary to calculate a parameter α by means of iterative mechanisms, which increases the computational load. An alternative to avoid this problem is found by defining new strategies (GEOM, HARM and MAXMIN), that do not cancel any carrier, and which correspond to the asymptotic behavior of the classical techniques. By making use of this mechanism, three new algorithms are found with a lower computational cost and a performance that may improve the original ones. Besides, these new techniques are directly related to different norms of the SNIR at the subcarriers.

4.1 Maximization of the Capacity (CAP)

For the case of the N-carrier modulation, the system capacity, assuming that the interferences and noise are Gaussian distributed and for the set $\{\mathrm{SNIR}_k\}_{k=0}^{N-1}$ is [1]:

$$C = \sum_{k=0}^{N-1} \log_2\left(1 + \mathrm{SNIR}_k\right) \tag{6}$$

In this definition we have implicitly assumed a constraint referring to the structure of the transmitter and receiver: for each frequency k only one spatial subchannel is used, the one with the highest eigenvualue; thus we do not permit multiple beamforming. The maximization of the capacity subject to the global power constraint results in the "water-filling" power allocation, whose expression is shown as follows:

$$p_{k,\mathrm{CAP}} = \max\left\{0, \alpha - \frac{1}{\lambda_{\max}(k)}\right\} \tag{7}$$

where α is a constant calculated iteratively to fulfill the constraint (5). From (7) it is deduced that more power is injected in those carriers with a better quality, that is, with a higher $\lambda_{\max}(k)$. As explained before, for certain transmit power conditions, some carriers may be unused ($p_k = 0$).

4.1.1 *Asymptotic Behavior (GEOM)*

In this subsection we deduce the technique corresponding to the asymptotic behavior of $p_{k,\mathrm{CAP}}$ when the transmitted power tends to infinity. When P_0 is high enough, no subcarrier is unused and, therefore, $\alpha = \frac{P_0}{N} + \frac{1}{N}\sum_{i=0}^{N-1}\frac{1}{\lambda_{\max}(i)}$. Asymptotically, the power allocation parameters are approximated as follows:

$$p_{k,\mathrm{GEOM}} = \lim_{P_0 \to \infty} p_{k,\mathrm{CAP}} = \frac{P_0}{N} \tag{8}$$

It can be easily demonstrated that this solution is equivalent to the maximization of the geometric mean of the SNIR evaluated at all the carriers ($\prod_{k=0}^{N-1}\mathrm{SNIR}_k^{1/N}$), subject to the global power constraint (5). We call this technique GEOM. Indeed, the same power is transmitted in all the carriers, and therefore, no power allocation is carried out. This solution is the same as that presented in [2].

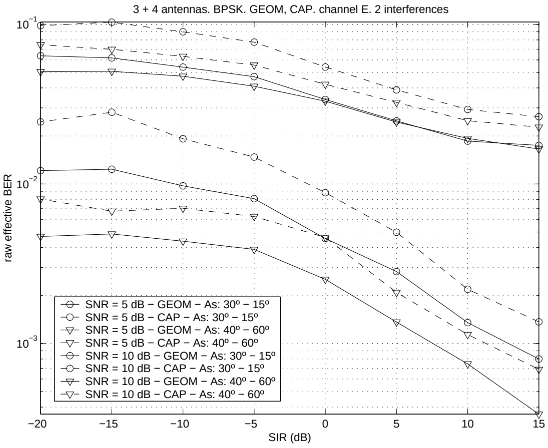

Figure 2: 3 + 4 ant. (chan. E). BER vs SIR. 2 interferences (SNR = 5, 10 dB). GEOM and CAP.

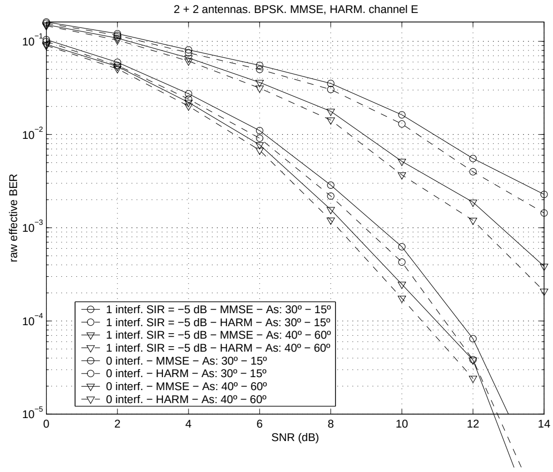

Figure 3: 2 + 2 ant. (chan. E). BER vs SNR. 1 interference (SIR = -5 dB) and no interferences. MMSE and HARM.

4.2 Mimimization of the MSE (MMSE)

The minimization of the Mean Square Error (MSE) ξ is a well-known technique [3]. In this case, we can express the MSE in the frequency domain as follows: $\xi = \frac{1}{N}\sum_{k=0}^{N-1}\left|\mathbf{a}_k^H\mathbf{H}(k)\mathbf{b}_k - 1\right|^2 + \mathbf{a}_k^H\mathbf{R}_n(k)\mathbf{a}_k$. The minimization of this expression subject to the global power constraint, is obtained with the following power allocation parameters:

$$p_{k,\mathrm{MMSE}} = \max\left\{0, \frac{\alpha}{\sqrt{\lambda_{\max}(k)}} - \frac{1}{\lambda_{\max}(k)}\right\} \quad (9)$$

where α is calculated iteratively to fulfill the power constraint, due to the fact that some carriers may be inactive.

4.2.1 Asymptotic Behavior (HARM)

The asymptotic behavior of the MMSE technique is analyzed in this subsection to define a new strategy. When the transmitted power is high enough, the constant α is calculated as follows: $\alpha = \frac{P_0 + \sum_{i=0}^{N-1}\lambda_{\max}^{-1}(i)}{\sum_{i=0}^{N-1}\lambda_{\max}^{-1/2}(i)}$. Asymptotically, the power allocation parameters can be approximated as shown next:

$$p_{k,\mathrm{HARM}} = \lim_{P_0\to\infty} p_{k,\mathrm{MMSE}} = \frac{P_0}{\sum_{i=0}^{N-1}\lambda_{\max}^{-1/2}(i)}\frac{1}{\sqrt{\lambda_{\max}(k)}} \quad (10)$$

This approximation is equivalent to the maximization of the harmonic mean of the SNIR: $H\{\mathrm{SNIR}\} = N\left(\sum_{k=0}^{N-1}\mathrm{SNIR}_k^{-1}\right)^{-1}$, and is the same as the Zero Forcing design criterion, as presented in [4]. We call this simplified technique HARM. In this case, more power is injected in the most degraded carriers.

4.3 Minimization of the Chernoff Bound of the Effective Probability of Error

There are power allocation strategies suitable for the minimization of the effective probability of error. It is defined as $P_{e,eff} = \frac{1}{N}\sum_{k=0}^{N-1}Q\left(\sqrt{k_m\mathrm{SNIR}_k}\right)$ if we consider that both the interferences and the noise are Gaussian distributed, where k_m is a constant that depends on the modulation applied to the subcarriers. The direct minimization of $P_{e,eff}$ is difficult, and therefore, we propose here the minimization of the Chernoff upper bound: $Q(x) \leq e^{-x^2/2}$. The minimization of this upper bound subject to the global power constraint (5) results in the following power allocation [5]:

$$p_{k,\mathrm{Chernoff}} = \frac{2}{k_m}\frac{\max\{0, \log(\lambda_{\max}(k)) - \alpha\}}{\lambda_{\max}(k)} \quad (11)$$

where α must be calculated iteratively to fulfill the global power constraint, as some carriers may be unused.

4.3.1 Asymptotic Behavior (MAXMIN)

In case that the transmitted power P_0 is high enough, no subcarrier is nulled and the constant can be calculated directly as follows: $\alpha = \frac{\sum_{i=0}^{N-1}\log(\lambda_{\max}(i))\lambda_{\max}^{-1}(i) - P_0 k_m/2}{\sum_{i=0}^{N-1}\lambda_{\max}^{-1}(i)}$. As the transmitted power tends to infinity, the power allocation parameters can be approximated as shown next:

$$p_{k,\mathrm{MAXMIN}} = \lim_{P_0\to\infty} p_{k,\mathrm{Chernoff}} = \frac{P_0}{\sum_{i=0}^{N-1}\lambda_{\max}^{-1}(i)}\frac{1}{\lambda_{\max}(k)} \quad (12)$$

This criterion is equivalent to the maximization of the minimum SNIR_k over all the subcarriers. This simplified technique, called MAXMIN, has a lower computational cost as no constant must be calculated iteratively. Also in this case, more power is injected in the most degraded subcarriers.

5 SIMULATION RESULTS AND CONCLUSIONS

The simulation parameters are those corresponding to HIPERLAN/2 [6]. The transmitter and receiver have perfect estimates of the CSI and the second-order statistics of the noise plus interferences, although this is not foreseen in the standard. 52 carriers are active based on 64-points IFFT, and the length of the cyclic prefix is $L = 16$ (sampling frequency = 20 MHz). We simulate normalized MIMO channels $\left(E\left\{\sum_{n=0}^{N_t}|h_{m,r}(n)|^2\right\} = 1\right)$ with standardized delay profiles [7], and BPSK modulated carriers. The only

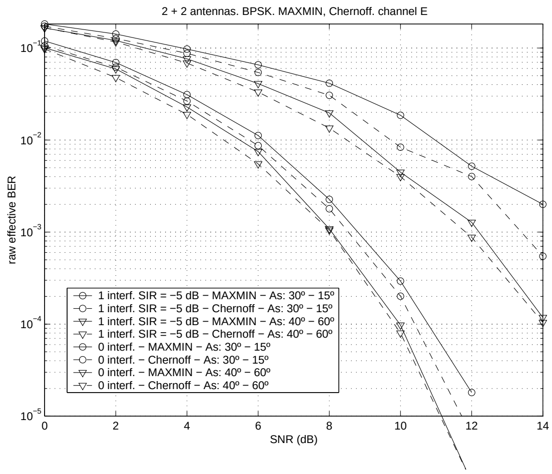

Figure 4: 2 + 2 ant. (chan. E). BER vs SNR. 1 interference (SIR = -5 dB) and no interferences. MAXMIN and Chernoff.

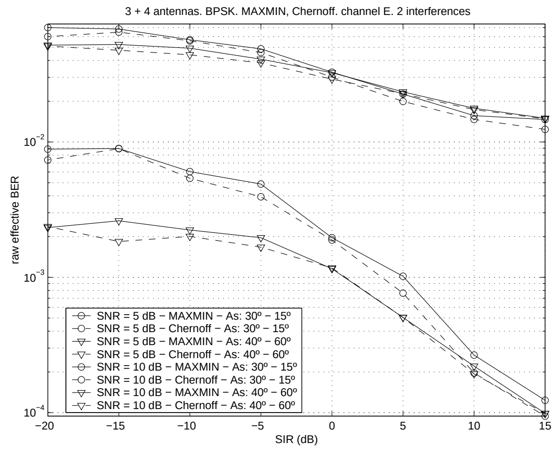

Figure 5: 3 + 4 ant. (chan. E). BER vs SIR. 2 interferences (SNR = 5, 10 dB). MAXMIN and Chernoff.

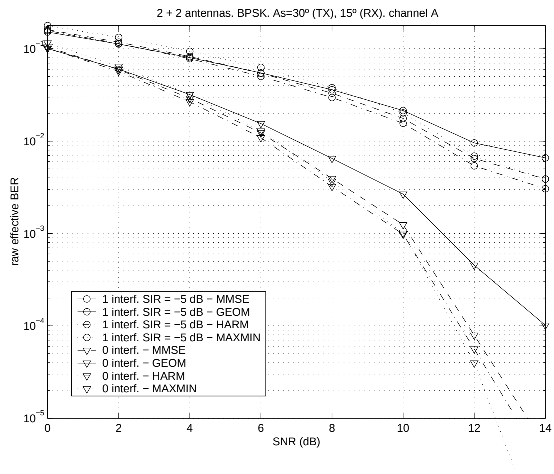

Figure 6: 2 + 2 ant. (chan. A). BER vs SNR. 1 interference (SIR = -5 dB) and no interferences.

channel model that has a maximum delay lower than the length of the CP is model A, although for the other channel delay profiles, the multiplicative model in the frequency domain (1) is almost true. We use the parameters SNR and SIR per branch: SNR = $\frac{P_0 RM}{P_N}$, SIR = $\frac{P_0 R}{P_I}$, where P_N and P_I are the mean power of the noise and interferences at each receive antenna. Several number of antennas $(R + M)$ are considered, where the arrays are uniform and linear $d = \lambda/2$.

Fig. 3, 4 and 6 show the results for 2+2 antennas in different scenarios: one interference (SIR = -5 dB) and no interference in the channel A (delay spread = 50 ns) and E (delay spread = 250 ns), and for different angular spreads. The conclusion is that the increase of the angular and delay spread results in an improvement of the system performance. The technique based on the Chernoff upper bound of the probability of error is the one with the lowest BER, followed by MAXMIN; however the MAXMIN technique has a lower computational load, concluding that this technique has a good performance-complexity trade-off.

Fig. 2 and 5 show the evaluation for a 3+4 antennas configuration and 2 equal level interferences in two different SNR conditions (5 and 10 dB). The techniques GEOM and MAXMIM are compared with CAP and Chernoff.

In general it is concluded that the asymptotic techniques GEOM and HARM perform better than the original ones; and that MAXMIN has a low computational cost and a performance very near from the optimum Chernoff technique.

References

[1] G. G. Raleigh and J. M. Cioffi, "Spatio-Temporal Coding for Wireless Communication," *IEEE Trans. on Communic.*, vol. 46, pp. 357–366, March 1998.

[2] K. K. Wong, R. S. K. Cheng, K. B. Letaief, and R. D. Murch, "Adaptive Antennas at the Mobile and Base Stations in an OFDM/TDMA System," *IEEE Trans. on Communic.*, vol. 49, pp. 195–206, Jan. 2001.

[3] J. Yang and S. Roy, "On Joint Transmitter and Receiver Optimization for Multiple-Input-Multiple-Output (MIMO) Transmission Systems," *IEEE Trans. on Communic.*, vol. 42, no. 12, pp. 3221–3231, Dec. 1994.

[4] A. Pascual Iserte, A. I. Pérez-Neira, and M. A. Lagunas Hernández, "Pre- and Post-Beamforming in MIMO Channels applied to HIPERLAN/2 and OFDM," in *IST Mobile Communic. Summit*, Barcelona, Sept. 2001.

[5] E. N. Onggosanusi, B. D. Van Veen, and A. M. Sayeed, "Efficient Signaling Schemes for Wideband Space-Time Wireless Channels Using Channel State Information," *Submitted to IEEE Trans. on Communic.*, 2001.

[6] ETSI, *ETSI TS 101 475 v1.1.1: Broadband Radio Access Networks (BRAN); HIPERLAN Type 2; Physical (PHY) layer*, April 2000.

[7] ETSI, *ETSI EP BRAN 3ERI085B: Channel Models for HIPERLAN/2 in Different Indoor Scenarios*, March 1998.