DEGRADATION ANALYSIS FOR INTERPOLATED WIENER FILTERING

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ABSTRACT

An analysis is presented of the degradation introduced by using an interpolated FIR filter in the Wiener filtering structure. Applications such as channel equalization and echo cancellation are considered. The proposed degradation measure is a function of the input signal correlation, of the sparseness degree, and of the interpolator. It results from a joint approach combining interpolated transversal filtering and linearly-constrained optimization.

I. INTRODUCTION

The fundamental principles of the interpolated finite impulse response (IFIR) filtering technique were introduced by Neuvo et al. [1]. It consists of exploiting the redundancy in the filter coefficients, removing quite a few impulse response samples, which are recreated using an interpolating scheme.

Adaptive IFIR (AIFIR) filters have been alternatively used in applications that require transversal structures with large order, such as echo and noise cancellation and channel equalization. They show a better convergence rate and a smaller computational complexity for both filtering and coefficient updating operations [2].

However, a study describing the degradation introduced by the IFIR filter in the Wiener filtering structure has not yet been presented in the literature, perhaps, owing to the difficulties in treating the issue at hand as an unconstrained optimization problem.

Through an original and elegant linearly-constrained approach, an appropriate formulation of the interpolated Wiener filtering problem has been introduced by the authors in [3,4], which leads us to more insights in this digital signal processing technique. Our objective here is to extend the theory developed, presenting a degradation analysis of interpolated transversal filters with regard to traditional Wiener filtering. The proposed analysis permits us to evaluate the degradation in terms of the input signal correlation, of the sparseness degree, and of the interpolator.

The paper is organized as follows. In Section II, we briefly review the joint approach combining interpolated transversal filtering and linearly-constrained optimization. Section III presents the degradation analysis of IFIR filters in channel equalization and echo cancellation. Finally, in Section IV, we present our conclusions.

II. INTERPOLATED WIENER FILTERING

Consider the scheme of interpolated Wiener filtering shown in Figure 1, where the classical transversal filter is replaced by the cascade of a sparse FIR filter and an interpolator. Without loss of generality, all the parameters have been assumed to be real-valued.

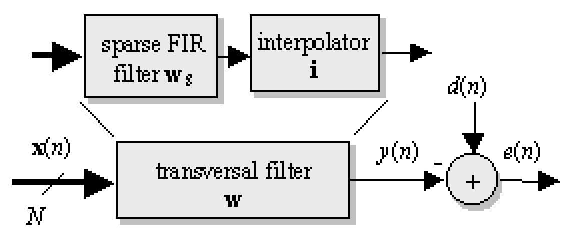

Figure 1: Interpolated Wiener filtering

The error signal is given by

$$\begin{aligned} e(n) &= d(n) - y(n) \\ &= d(n) - \mathbf{w}_S^t \mathrm{X}(n)\mathbf{i} \end{aligned} \tag{1}$$

where:

$$\mathbf{w}_S = [w_{S_0}, w_{S_1}, \ldots, w_{S_{N-1}}]^t \tag{2}$$

denotes the N-by-1 tap-weight vector of the sparse filter;

$$\mathrm{X}(n) = [\mathbf{x}(n), \mathbf{x}(n-1), \ldots, \mathbf{x}(n-M+1)]_{N\mathrm{x}M}\ ; \tag{3}$$

$$\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-N+1)]^t \tag{4}$$

is the N-by-1 tap-input vector; and

$$\mathbf{i} = [i_0, i_1, \ldots, i_{M-1}]^t \tag{5}$$

the M-by-1 coefficient vector of the interpolator.

The sparse filter is characterized by the fact that only one sample of each set of L consecutive samples of its impulse response is nonzero. So, there is a total of $K=\lceil N/L \rceil$ nonzero-valued samples ($\lceil \bullet \rceil$ means to round $\bullet$ to the nearest integer towards infinity) and N-K zero-valued samples. The zero-valued samples of the sparse filter are estimated by the interpolator. The integer L is referred to as sparseness degree or interpolation factor.

Proceeding, the sparseness condition of $\mathbf{w}_S$ can be easily achieved through a linearly-constrained approach [3]. It consists of making (e.g., for L=2 and N odd):

$$\mathbf{C}^{\mathrm{t}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0 \end{bmatrix}_{(N-K)\mathrm{x}N}, \quad (6)$$

$$\mathbf{f} = \begin{bmatrix} 0 & \cdots & 0 \end{bmatrix}^{\mathrm{t}}_{1\mathrm{x}(N\text{-}K)} = \mathbf{0}_{(N-K)\mathrm{x}1}, \quad (7)$$

and of imposing $\mathbf{C}^{\mathrm{t}}\mathbf{w}_S$=$\mathbf{f}$ in a constrained optimization process of the mean-square value of $e(n)$. Furthermore, the generalized sidelobe canceller (GSC) structure can be directly used, and the scheme in Figure 1 turns into the form represented by Figure 2, since $\mathbf{f}$=$\mathbf{0}$ [4].

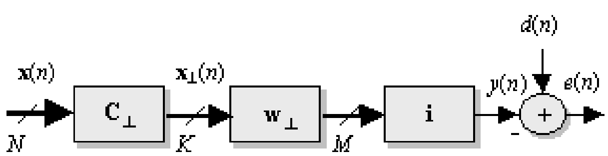

Figure 2: GSC structure for interpolated Wiener filtering.

The signal blocking matrix $\mathbf{C}_\perp$ is given by

$$\mathbf{C}_\perp = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}^{\mathrm{t}}_{K\mathrm{x}N}. \quad (8)$$

It is interesting to observe that the unconstrained filter $\mathbf{w}_\perp$ is composed merely of the nonzero coefficients of the sparse filter $\mathbf{w}_S$. In other words, $\mathbf{w}_\perp$ is not a sparse transversal filter. The premultiplication of $\mathbf{w}_\perp$ by $\mathbf{C}_\perp$ inserts zeros between its elements, making it sparse.

Now, the error signal is given by

$$e(n) = d(n) - \mathbf{w}_\perp^{\mathrm{t}}\mathbf{C}_\perp^{\mathrm{t}}\mathrm{X}(n)\mathbf{i}. \quad (9)$$

In the mean-squared error (MSE) sense, the vector $\mathbf{w}_\perp$ is chosen to minimize the following cost function:

$$\begin{aligned} J(\mathbf{w}_\perp) &= \mathrm{E}\{e^2(n)\} \\ &= \mathrm{E}\{[d(n) - \mathbf{w}_\perp^{\mathrm{t}}\mathbf{C}_\perp^{\mathrm{t}}\mathrm{X}(n)\mathbf{i}]^2\} \\ &= \sigma_d^2 - 2\mathbf{w}_\perp^{\mathrm{t}}\mathbf{p}_{\mathrm{IW}} + \mathbf{w}_\perp^{\mathrm{t}}\mathbf{R}_{\mathrm{IW}}\mathbf{w}_\perp, \quad (10) \end{aligned}$$

where

$$\sigma_d^2 = \mathrm{E}\{d^2(n)\}, \quad (11)$$

$$\begin{aligned} \mathbf{p}_{\mathrm{IW}} &= \sum_{m=0}^{M-1} i_m \mathbf{C}_\perp^{\mathrm{t}} \mathrm{E}\{\mathbf{x}(n-m)d(n)\} \\ &= \sum_{m=0}^{M-1} i_m \mathbf{C}_\perp^{\mathrm{t}} \mathbf{p}_m \quad (12) \end{aligned}$$

and

$$\begin{aligned} \mathbf{R}_{\mathrm{IW}} &= \sum_{j=0}^{M-1}\sum_{m=0}^{M-1} i_j i_m \mathbf{C}_\perp^{\mathrm{t}} \mathrm{E}\{\mathbf{x}(n-j)\mathbf{x}^{\mathrm{t}}(n-m)\}\mathbf{C}_\perp \\ &= \sum_{j=0}^{M-1}\sum_{m=0}^{M-1} i_j i_m \mathbf{C}_\perp^{\mathrm{t}} \mathbf{R}_{m-j} \mathbf{C}_\perp. \quad (13) \end{aligned}$$

In expanded forms, we have:

$$\mathbf{C}_\perp^{\mathrm{t}}\mathbf{p}_m = [p(m), p(L+m), \ldots, p((K-1)L-m)]^{\mathrm{t}}_{1\mathrm{x}K}, \quad (14)$$

and

$$\mathbf{C}_\perp^{\mathrm{t}}\mathbf{R}_{m-j}\mathbf{C}_\perp = \begin{bmatrix} r(m-j) & \cdots & r((K-1)L+m-j) \\ \vdots & \ddots & \vdots \\ r((K-1)L+m-j) & \cdots & r(m-j) \end{bmatrix}_{K\mathrm{x}K}, \quad (15)$$

where $p(m)$=E$\{d(n)x(n\text{-}m)\}$ and $r(m\text{-}j)$=E$\{x(n\text{-}j)x(n\text{-}m)\}$. Finally, the MSE optimum interpolated Wiener filter is given by

$$\mathbf{w}_{\perp\mathrm{opt}} = \mathbf{R}_{\mathrm{IW}}^{-1}\mathbf{p}_{\mathrm{IW}}. \quad (16)$$

Substituting (16) in (10), we find the minimum MSE produced by the interpolated Wiener filter in Figure 2:

$$\begin{aligned} J_{\mathrm{IWmin}} &= \sigma_d^2 - \mathbf{p}_{\mathrm{IW}}^{\mathrm{t}}\mathbf{w}_{\perp\mathrm{opt}} \\ &= \sigma_d^2 - \mathbf{p}_{\mathrm{IW}}^{\mathrm{t}}\mathbf{R}_{\mathrm{IW}}^{-1}\mathbf{p}_{\mathrm{IW}}. \quad (17) \end{aligned}$$

Moreover, the unique optimality of $\mathbf{w}_{\perp\mathrm{opt}}$ can be explicitly shown by

$$\begin{aligned} J(\mathbf{w}_\perp) &= \sigma_d^2 - \mathbf{p}_{\mathrm{IW}}^{\mathrm{t}}\mathbf{R}_{\mathrm{IW}}^{-1}\mathbf{p}_{\mathrm{IW}} + \\ &\quad + (\mathbf{w}_\perp - \mathbf{R}_{\mathrm{IW}}^{-1}\mathbf{p}_{\mathrm{IW}})^{\mathrm{t}}\mathbf{R}_{\mathrm{IW}}(\mathbf{w}_\perp - \mathbf{R}_{\mathrm{IW}}^{-1}\mathbf{p}_{\mathrm{IW}}) \\ &= J_{\mathrm{IWmin}} + (\mathbf{w}_\perp - \mathbf{w}_{\perp\mathrm{opt}})^{\mathrm{t}}\mathbf{R}_{\mathrm{IW}}(\mathbf{w}_\perp - \mathbf{w}_{\perp\mathrm{opt}}). \quad (18) \end{aligned}$$

Utilizing the eigenanalysis of $\mathbf{R}_{\mathrm{IW}}$, we have:

$$\begin{aligned} J(\mathbf{w}_\perp) &= J_{\mathrm{IWmin}} + \\ &\quad + (\mathbf{w}_\perp - \mathbf{w}_{\perp\mathrm{opt}})^{\mathrm{t}}\mathbf{Q}_{\mathrm{IW}}\Lambda_{\mathrm{IW}}\mathbf{Q}_{\mathrm{IW}}^{\mathrm{t}}(\mathbf{w}_\perp - \mathbf{w}_{\perp\mathrm{opt}}), \quad (19) \end{aligned}$$

where Λ_{IW} is the K-by-K diagonal matrix consisting of the eigenvalues, whose associated eigenvectors compose the columns of the K-by-K matrix $\mathbf{Q}_{\mathrm{IW}}$. Let

$$\nu_{\mathrm{IW}} = \mathbf{Q}_{\mathrm{IW}}^{\mathrm{t}}(\mathbf{w}_\perp - \mathbf{w}_{\perp\mathrm{opt}}) \quad (20)$$

be a transformed version of the difference between $\mathbf{w}_\perp$ and $\mathbf{w}_{\perp\mathrm{opt}}$. Thus, the canonical form of (19) is defined by

$$\begin{aligned} J(\mathbf{w}_\perp) &= J_{\mathrm{IWmin}} + \nu_{\mathrm{IW}}^{\mathrm{t}}\Lambda_{\mathrm{IW}}\nu_{\mathrm{IW}} \\ &= J_{\mathrm{IWmin}} + \sum_{k=1}^{K}\lambda_k\nu_k^2, \quad (21) \end{aligned}$$

where ν_k is the kth component of ν_{IW} and λ_k the kth eigenvalue.

III. DEGRADATION ANALYSIS

A quantitative measure of the degradation introduced by the IFIR filter in the Wiener filtering structure can be defined as

$$\gamma \equiv \frac{J_{\min}}{J_{\text{IWmin}}}, \tag{22}$$

where:

$$J_{\min} = \sigma_d^2 - \mathbf{p}^t \mathbf{w}_{\text{opt}}$$
$$= \sigma_d^2 - \mathbf{p}^t \mathbf{R}^{-1} \mathbf{p} \tag{23}$$

denotes the minimum MSE of the optimum Wiener filter

$$\mathbf{w}_{\text{opt}} = \mathbf{R}^{-1}\mathbf{p}, \tag{24}$$

$$\mathbf{R} = \mathrm{E}\{\mathbf{x}(n)\mathbf{x}^t(n)\} \tag{25}$$

is the N-by-N correlation matrix of $\mathbf{x}(n)$, and

$$\mathbf{p} = \mathrm{E}\{\mathbf{x}(n)d(n)\} \tag{26}$$

the N-by-1 cross-correlation vector between $\mathbf{x}(n)$ and $d(n)$ [5,6]. Thus, the proposed degradation measure depends on the input signal correlation, the sparseness degree L and the interpolator.

Firstly, in order to become aware the IFIR filter degradation, consider the equalization system shown in Figure 3 [5, chap. 9]. The random sequence $b(n)$ applied to the channel input has zero mean and unit variance. The impulse response of the channel is described by the raised cosine:

$$c_j = \begin{cases} \frac{1}{2}(1+\cos(\frac{2\pi}{S}(j-2))), & j = 1,2,3 \\ 0, & \text{otherwise} \end{cases}, \tag{27}$$

where S controls the eigenvalue spread $\chi(\mathbf{R})$ of the correlation matrix of the tap inputs in the equalizer (Table I). The sequence $v(n)$ is an additive white noise that corrupts the channel output with zero mean and variance $\sigma_v^2=0.001$. The equalizer has eleven coefficients and $\delta=7$.

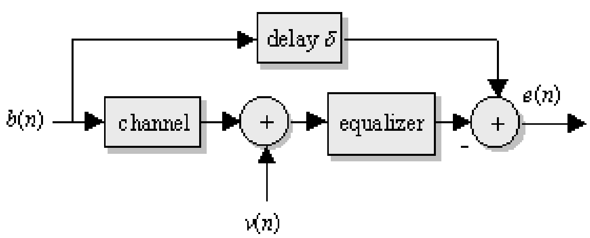

Figure 3: Block diagram of equalization scheme.

Table I: $\chi(\mathbf{R})$ versus S

S	3.5	3.3	3.1	2.9
$\chi(\mathbf{R})$	46.8216	21.7132	11.1238	6.0782

Figure 4 shows the degradation of the interpolated Wiener filter varying with the sparseness degree L (linear interpolation), for each eigenvalue spread $\chi(\mathbf{R})$. We can observe that more the equalizer input is ill conditioned, the smaller the degradation. Another observation is the very slight improvement from L=4 to L=5. Since the channel has an impulse response that is symmetric, the arrangement of the nonzero-valued coefficients for L=5 has a better agreement.

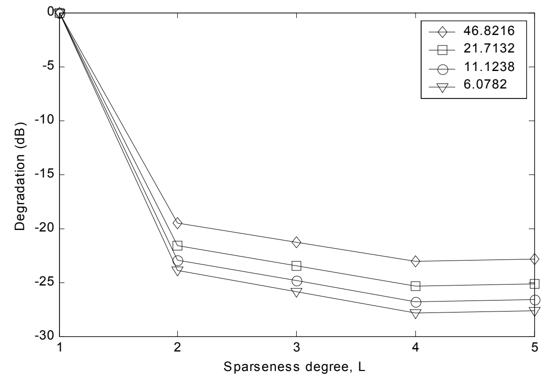

Figure 4: IFIR filter degradation.

In Table II, we give the degradation by fixing L=2 and varying eigenvalue spread.

Table II: Degradation versus $\chi(\mathbf{R})$, for L=2

$\chi(\mathbf{R})$	46.8216	21.7132	11.1238	6.0782
γ in dB	-19.4818	-21.6327	-23.0124	-23.9273

A second example that has been considered is the application of AIFIR filters in echo cancellation. Figure 5 shows the principle of the echo canceller for one direction of transmission [5].

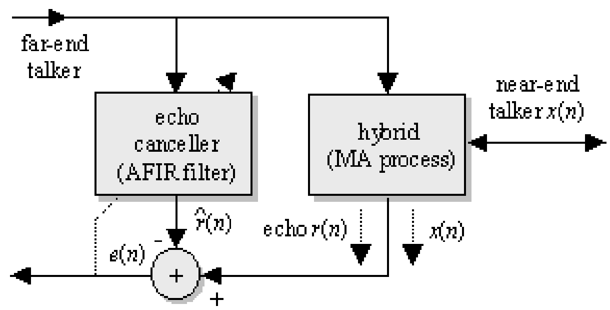

Figure 5: Echo cancellation scheme.

The measure of echo cancellation performance is defined as

$$\zeta \equiv \frac{\sigma_r^2}{\sigma_e^2}, \tag{28}$$

where

$$\sigma_r^2 = \mathrm{E}\{r^2(n)\} \tag{29}$$

and

$$\sigma_e^2 = \mathrm{E}\{e^2(n)\}$$
$$= \mathrm{E}\{[r(n) - \hat{r}(n)]^2\} \tag{30}$$

denote the total and residual ($x(n)$=0) echo variances, respectively.

The transfer function of the echo path (hybrid) was modeled as a moving average (MA) process of order 256, whose impulse response is shown in Figure 6. As far as the speech signal from far-end talker is concerned, we used a white noise of zero mean and unitary variance, and an asymptotically stationary autoregressive (AR) process of order 2 (colored noise). The AR(2) process is governed by the difference equation:

$$u(n) - 0.1u(n-1) - 0.8u(n-2) = v(n)\ , \qquad (31)$$

where $v(n)$ is a white noise of zero mean and variance σ_v^2=0.27, which is chosen to make the variance of $u(n)$ equal unity [5, chap. 2]. The eigenvalue spread of the correlation matrix of $u(n)$ is $\chi(\mathbf{R})$=313,3256.

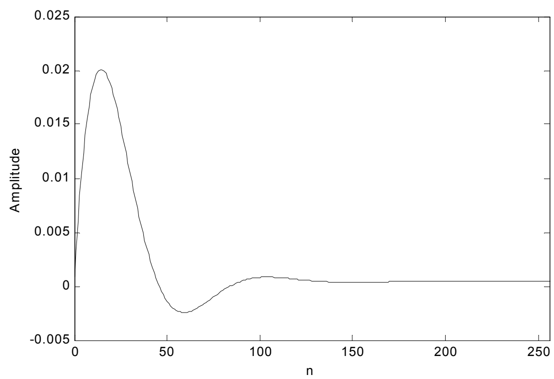

Figure 6: Echo impulse response.

The performance degradation of the AIFIR echo canceller in relation to the sparseness degree L can be verified in Figure 7. The echo impulse response identified by the interpolated Wiener filter is plotted in Figure 8, for L=16. Again, a linear interpolation scheme was used.

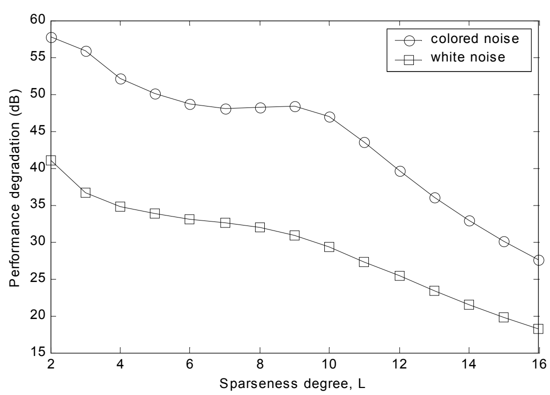

Figure 7: Minimum MSE evolution.

Finally, it is worth pointing out that the above degradation analysis is done for an interpolated transversal filter, in which all its coefficients are sparse. However, the proposed approach can also be applied to other interpolated transversal filtering schemes. For instance, the two-stage echo canceller in [2].

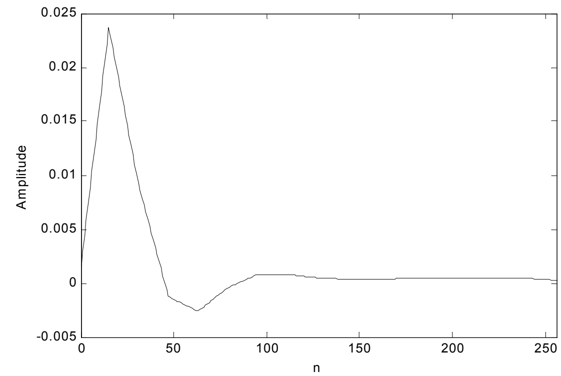

Figure 8: Model identified using the IFIR filter with L=16.

IV. CONCLUSION

It has been shown that we can estimate the degradation introduced by an IFIR filter in the Wiener filtering structure, and that such a degradation depends on the input signal correlation, the sparseness degree, and the interpolation scheme. It was possible, thanks to a linearly-constrained approach to the interpolated Wiener filtering problem. The approach is generic, since it can be extended to filters with different forms of sparseness. Actually, such an approach is a powerful and interesting tool of analysis for interpolated Wiener filtering.

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