Robust Performance of the Adaptive Periodic Noise Canceller in a Closed-Loop system

J. Timoney and J.B. Foley
Department of Electronic and Electrical Engineering,
Trinity College, Dublin 2, IRELAND
e-mail: jtimoney@ee.tcd.ie

Abstract
This paper addresses the issue of robustness of the LMS-driven Adaptive Periodic Noise Canceller (APNC) in a closed-loop system. The concept of robustness provides a framework within which it is possible to make a useful assessment of algorithm performance. By adopting an analysis based on H^{∞}-theory, conditions are shown under which the APNC, driven by the LMS algorithm, will exhibit robust performance properties. Results are presented for the case of a broadband signal input to a one-dimensional closed-loop system. They display the relationship between the algorithm stepsize, the magnitude of the feedback coefficient and their bounds for robust performance. This result can be directly related to the use of the APNC in an echo control application.

Keywords
Adaptive Periodic Noise Canceller, echo cancellation, robust performance

1. Introduction

Adaptive filtering is the most commonly proposed solution in applications where some form of feedback echo attenuation is required. The dynamic nature of the echo requires a time-variant filter structure for continuous tracking and cancellation. The unknown nature of the echo suggests that the evaluation of a criterion of robustness would be a most relevant measure under which algorithm performance could be assessed. As the echo may exhibit rapid fluctuations in magnitude, an adaptive filter with robust performance properties would be one whose adaption process would not be disturbed by such fluctuations.

The LMS-driven Adaptive Periodic Noise Canceller (APNC) can be used for acoustic echo suppression [1]. A theoretical proof of the robustness of the LMS algorithm in the sense of it being H^{∞}-optimal was given in [2]. The value of algorithm stepsize was shown to be the crucial parameter for ensuring algorithm robustness. Following from this, a determination of the necessary conditions for robust performance of the APNC in an open-loop feedback system was given in [3]. This contribution aims to extend the work presented in [3], and to ascertain the criteria for robust performance of the APNC when incorporated into a closed-loop feedback system. The complete closed-loop system is a realistic representation of the practical echo control situation. The block diagram shown in Fig.1 below provides an outline A fraction of the loudspeaker output is fed back towards the microphone through the path defined by the transfer function of *H*. The purpose of the APNC is to remove or to attenuate this echo component in the input speech signal.

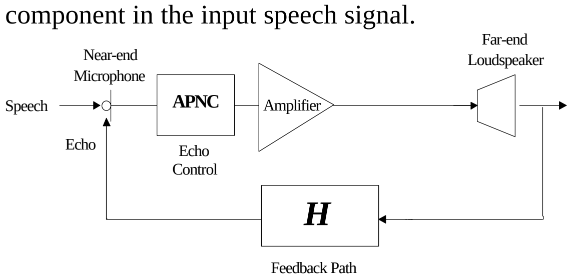

Fig.1 Block Diagram Of Closed-loop Environment with Feedback Echo Control

Given the non-deterministic nature of the input signal, robust performance of the APNC is an important practical objective. Thus, the aim of this work is to discover the maximum allowable stepsize value for a range of model parameters that guarantees robust performance of the APNC in this application.

2. Method Outline

A model of the APNC-based echo control system outlined above is shown in Fig.2. $S(n)$ denotes the input and the return echo is represented by $S_e(n)$. The feedback environment is modelled by the transfer function $H(z) = h_1 z^{-1}$. A one-weight APNC, driven by the LMS algorithm, is also assumed. As with [2], the purpose of the reduced dimensionality is to keep the analysis tractable while still capturing the essential core of the problem.

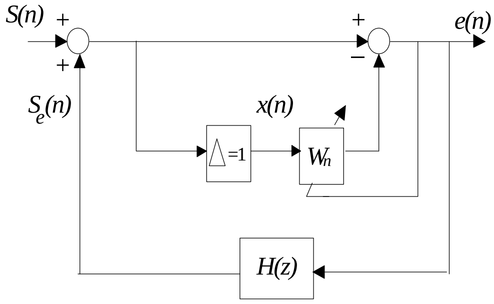

Fig.2 Block Diagram of APNC in the Closed-Loop System

The output of the system $e(n)$ is given by

$$e(n) = S(n) + h_1 e(n-1) - W_{\text{opt}} x(n) - V_n x(n) \qquad (1)$$

where the filter input is given by,

$$x(n) = S(n-1) + h_1 e(n-2) \qquad (2)$$

and

$$V_n = W_n - W_{\text{opt}} \qquad (3)$$

(i.e. the difference between the filter weight and its optimum value).

Thus, an output error or error due to filter misadjustment, $\varepsilon(n)$, can be defined as

$$\varepsilon(n) = e(n)\big|_{W_{\text{opt}}} - e(n)\big|_{W_n} \qquad (4)$$

giving

$$\varepsilon(n) = V_n\, x(n) \qquad (5)$$

Examining (5), the term V_n can be interpreted as an error gain. It is a function of all previous inputs and outputs. Robust performance would imply that the magnitude of this gain is minimised over time and that fluctuations of $x(n)$, due to abrupt disturbances in $S(n)$, will not disturb this minimisation procedure. The LMS solution is dependent on the choice of stepsize μ and therefore, to ensure robust performance it is necessary to determine a maximum upper limit.

(5) can also be represented as a time-varying matrix

$$\mathbf{e}(n) = \boldsymbol{V_n X}(n) \qquad (6)$$

where $\mathbf{e}(n)$ denotes a vector of error outputs to time *n*.

The H^∞-norm of $\boldsymbol{V}$ is the peak value of the gain over the time interval $n = 1, \ldots, N$, and is defined as

$$\|\boldsymbol{V}\|_\infty = \sup_{x \neq 0} \frac{\|\mathbf{e}\|_2}{\mu^{-1}|V_1| + \|\boldsymbol{X}\|_2} \qquad (7)$$

where the input disturbance is $\left\{\mu^{-1/2}(V_1), \{x(n)\}_{n=1}^{N}\right\}$ with $\mu^{-1/2}(V_1)$ being the (weighted) energy of the weight error due to the initial guess and the H^2- norm of $x(n)$ is given as $\|\boldsymbol{X}\|_2^2 = \sum_{k=1}^{\infty} x(n)x(n)$.

For robust performance the energy of the residual error should be upper bounded by the energy of the disturbances and the initial uncertainty. This translates into ensuring that the H^∞-norm of the error gain matrix (***V***) must be less then one,

The H^∞-norm is actually calculated by finding the maximum singular values $\sigma_{\boldsymbol{n}\max}$ of ***V*** at each time instant, which are given by

$$\sigma_{n\max} = \sqrt{\max\left(\lambda_n\left(\boldsymbol{VV}^T\right)\right)} \qquad (8)$$

where λ_n denotes the eigenvalues of the matrix $\boldsymbol{VV}^T$.

3. Results

Simulations of the closed-loop APNC system were made with a broadband white noise input to measure the relationship between the robustness of the LMS algorithm, the value of feedback coefficient and the choice of stepsize. Broadband white noise was chosen as an input because similarly to speech, it has a short correlation time relative to the dynamics of the adaptive filter. Simulations were carried out for a range of coefficient values of h_1 between -0.1 and -1. A semi-empirical approach was adopted for the selection of a maximum allowable stepsize parameter $\mu_{\max}$ and resulted in an expression of the form (see Appendix)

$$\mu_{\max} = C\left(abs\left(W_{\text{opt}}\right) \Big/ \left(2 \times S_{\max}^2\right) \right) \qquad (9)$$

where $S_{\max}^2$ is the square of the maximum input noise value, W_{opt} is the optimum adaptive filter weight vector and *C* is a multiplication factor. W_{opt} is calculated as follows:

The optimum Wiener weight value is given by

$$W_{\text{opt}} = R^{-1}P \qquad (10)$$

where

$$R = E\left[x(n)x(n)\right] \qquad (11)$$

and

$$P = E\left[x(n+1)x(n)\right] \qquad (12)$$

Assuming that for a broadband input,

$$E\left[S(n)e(n-k)\right] = \sigma^2, \quad k = 0 \qquad (13)$$
$$= 0, \quad \text{otherwise}$$

where σ^2 is the power of the white noise input, and

$$E\left[e(n)e(n-k)\right] = \sigma^2, \quad k = 0 \qquad (14)$$
$$= 0, \quad \text{otherwise}$$

Then, solving (11) and (12) gives,

$$R = \left(1 + h_1^2\right)\sigma^2 \qquad (15)$$

$$P = h_1\sigma^2 \qquad (16)$$

then, by substituting (15) and (16) into (10), the optimum weight value is given by

$$W_{\text{opt}} = \frac{h_1\sigma_s^2}{\left(1 + h_1^2\right)\sigma_s^2} \qquad (17)$$

The results presented in Fig.2 demonstrate the relationship between the choice of stepsize multiplication factor and the magnitude of the maximum singular values for a fixed magnitude of the feedback coefficient of h_1 = -0.7. The stepsize multiplication factor of *C* was varied over the range 0.2 to 1 in steps of 0.2. The maximum singular values of ***V*** for the first 200 iterations of the LMS algorithm were calculated in each case.

From Fig.2, it can be seen that all values of *C* provide robust system performance. In addition, it is clear that better performance in term of robustness is achieved for lower values of *C*.

Fig.3 shows simulation results for a constant value *C*=1 and when h_1 is incremented in steps of -0.1, from -0.4 to -0.8. Again, the maximum singular values for 200 iterations of the LMS algorithm are shown. From Fig.3 it can be seen that the maximum singular values are very close for each case. In all cases. robust performance of the system performance is exhibited. This demonstrates that there is a stronger relationship between robust performance and the value of stepsize rather than the magnitude of the feedback coefficient.

Fig.4 shows a collection of results for different values of stepsize multiplication factor and magnitude of feedback coefficient. Over the each set of simulations, the value of *C* was varied from 0.1 to 1 in steps of 0.1. The maximum singular values of the gain matrix were calculated for the first 400 iterations of the LMS algorithm and then the mean of the last 50 values taken.

From Fig.4, it can be seen that for all the values of feedback coefficient, h_1 = -0.1 to -1, and all the values of stepsize multiplication factor *C* = 0.1 to 1, the APNC in the closed-loop system exhibits robust performance. For h_1 =-1, the APNC exhibits robust performance for C = 0.1 to 0.8, and otherwise the APNC system is unstable. It can be observed that in all cases for a low value of stepsize multiplication factor, the best performance in terms of robustness is achieved.

4. Conclusion

For a broadband input signal, the APNC in a closed-loop configuration, was shown to exhibit robust performance properties in all cases of a range of values of feedback coefficient from -0.1 to -1, when the maximum allowable stepsize value given by (9) had a multiplication factor of 0.8 or smaller. Thus, this provides a more useful quantitative measure of APNC performance which is applicable to practical adaptive filtering echo cancelling situations. A possible suggestion for further work would be to extend this analysis for a multidimensional closed-loop feedback system. This would provide insight into the best choice for algorithm stepsize value to ensure robust performance when considering practical applications of greater complexity.

5. References

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Appendix - Stepsize Selection

The choice of stepsize is the crucial factor in determining the level of system performance. The weight vector update equation in term of V is given by

$$V_n = V_{n-1}$$

$$-2\mu\left(x^2(n-1)V_{n-1} + x(n)\,x(n-1) - W_{opt}x^2(n-1)\right) \qquad (1A)$$

Examining (2), to ensure that the energy of the disturbance $x(n)$ is not amplified, the stepsize must be chosen so that

$$\left|2\mu\left(W_{opt}\; x^2(n-1)\right)\right| \leq 1 \qquad (2A)$$

and

$$\left|-2\mu\left(x(n)\; x(n-1)\right)\right| \leq 1 \qquad (3A)$$

and

$$\left|-2\mu\left(\; x^2(n-1)V_{n-1}\right)\right| \leq 1 \qquad (4A)$$

In practice, the only signal that can be measured is the input speech $S(n)$. Therefore, a possible choice of stepsize which may ensure robust performance is

$$\mu_{max} \leq abs\left(W_{opt}\right) \Big/ \left(2 \times S^2_{max}\right) \qquad (5A)$$

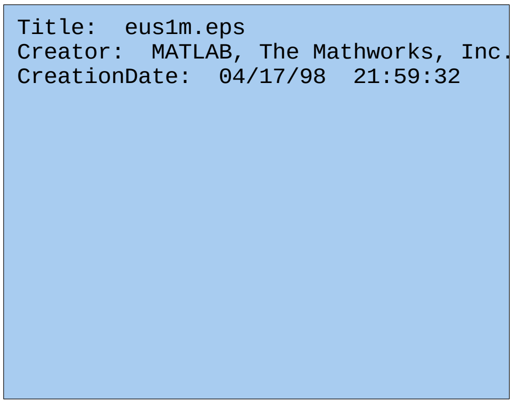

Fig.2 Max. Singular Values for Different Multiplication Coefficient ***C***

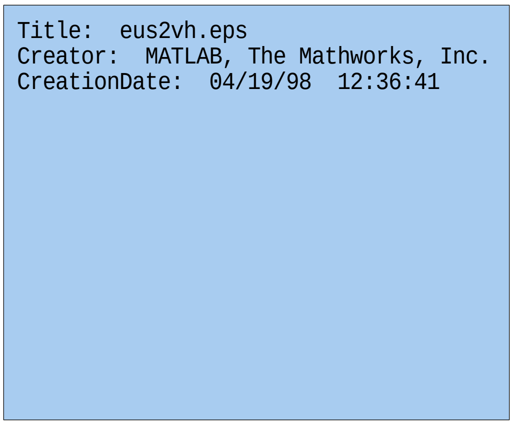

Fig.3 Max. Singular Values for Different Coefficient h_1

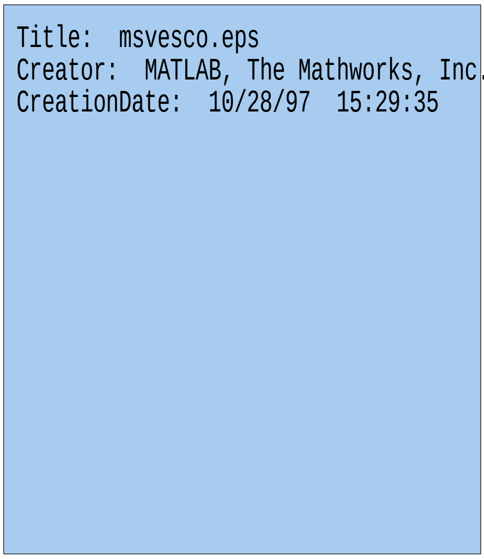

Fig.4 Relationship between Robust Performance, Stepsize Multiplication Factor and Feedback Coefficient Value