Adaptive System Identification Using the Normalized Least Mean Fourth Algorithm

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Abstract

In this work we propose a novel scheme for adaptive system identification. This scheme is based on a normalized version of the least-mean fourth (LMF) algorithm. In contrast to the LMF algorithm, this new normalized version of the LMF algorithm is found to be independent of the input sequence autocorrelation matrix. It is also found that it converges faster than the normalized least mean square (NLMS) algorithm for the lowest steady-state error reached by the NLMS algorithm. Simulation results confirm the superior performance of the new algorithm.

1 Introduction

The least mean-square (LMS) [1] algorithm has been used in many applications, this is due to its simplicity. It is robust only if the noise statistics are Gaussian, however, if it deviates from these statistics its performance deteriorates. The least mean-fourth (LMF) [2] algorithm, on the other hand, has better performance than the LMS algorithm only if the statistics of the noise are different from Gaussian.

But for both of these algorithms, their convergence behavior depends on the condition number, i.e., ratio of the maximum to the minimum eigenvalues of the input signal autocorrelation matrix, $\mathbf{R} = E[\mathbf{X}(n)\mathbf{X}^T(n)]$, where $\mathbf{X}(n)$ is the input signal to the adaptive filter.

Recently, new adaptive algorithms based on a mixed-norm criteria [3]-[8] are found to result in better performance than either the LMS or the LMF algorithms in Gaussian and non-Gaussian environments. These have motivated the idea of looking at a normalized version of the LMF algorithm, whereby eliminating its dependency on the statistics of the input signal and therefore increase its speed.

As was the normalized LMS (NLMS) [9]-[12] algorithm advantageous over the LMS algorithm in two respects [13]: 1) potentially faster converging algorithm for both uncorrelated and correlated input data [14]-[15] and 2) stable behavior for a known range of parameter values ($0 < \mu < 2$) independent of the input data correlation statistics [9], [14], in this work a similar approach to that of the NLMS algorithm with equally attractive features is proposed for the LMF algorithm. The resulting algorithm will be called the normalized LMF (NLMF) algorithm. The NLMF algorithm will reduce the effect of the condition number and therefore will increase the convergence speed of the algorithm.

The objective of this paper is to examine the convergence properties of the NLMF algorithm and compare it to those of the LMF algorithm and the NLMS algorithm. In the comparison of the LMF and the NLMF algorithms, only the dependency of the eigenvalue spread of the input data correlation matrix on the convergence behavior is examined. However, when the NLMF and NLMS algorithms are compared, the lowest steady-state error reached by the latter algorithm is desired. As the simulations confirm it, it is found that the NLMF algorithm converges faster than the NLMS algorithm. Of course, this does not contradict the fact the NLMS algorithm converges faster for a gradient step, at the expense of higher misadjustment values.

The paper is organized as follows. Section 2 presents the proposed algorithm. In section 3, simulation results of the new proposed algorithm are compared to those of the LMF and NLMS algorithms.

2 Proposed Algorithm

The LMF algorithm is based on the minimization of the mean-fourth error cost function, that is :

$$J(n) = E[e^4(n)], \tag{1}$$

the error is given by :

$$e(n) = d(n) + w(n) - \mathbf{X}^T(n)\mathbf{C}(n) \tag{2}$$

where $d(n)$ is the desired value, $\mathbf{X}(n)$ is the input signal, $\mathbf{C}(n)$ is the filter coefficient of the adaptive filter and $w(n)$ is the additive noise. This is depicted in Fig. 1.

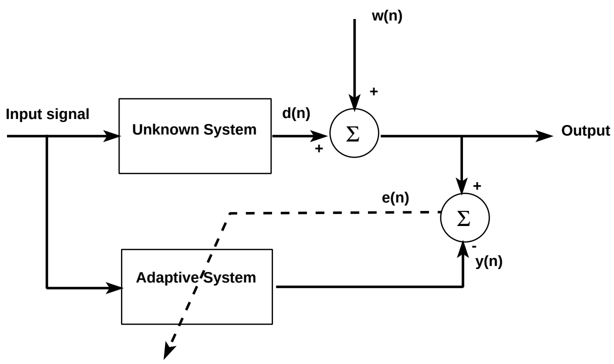

Figure 1: Adaptive system identification.

The filter-coefficient vector update of the LMF algorithm is given by:

$$\mathbf{C}(n+1) = \mathbf{C}(n) + 2\mu e^3(n)\mathbf{X}(n). \tag{3}$$

This algorithm depends on the input signal statistics, that is on the eigenvalues of the input signal. To overcome this dependency, the normalized LMF algorithm is introduced, and its weight update recursion is given by:

$$\mathbf{C}(n+1) = \mathbf{C}(n) + 2\mu e^3(n)\frac{\mathbf{X}(n)}{\gamma + \mathbf{X}^T(n)\mathbf{X}(n)}, \tag{4}$$

where γ is a small positive number.

Examining the mean behavior of Equation (4) under the assumption that the input signal is independent of the noise (both of zero mean) and the odd moments of the noise are zero, and invoking the independence assumption [17], sufficient conditions for convergence of the NLMF algorithm can be shown to be given by [16]:

$$0 < \mu < \frac{1}{3E[w^2(n)]}. \tag{5}$$

It is clear from (5) that the step size, μ, is no longer dependent on input data correlation statistics (largest eigenvalue, λ_{max}, of the autocorrelation matrix of the input data) as it is in the case for the LMF algorithm [2]:

$$0 < \mu < \frac{1}{3E[w^2(n)]\lambda_{max}}. \tag{6}$$

In terms of the computational complexity, the NLMF algorithm when implemented as a shift-input data requires one more multiplication, division, and addition over the LMF algorithm, and only two more multiplications than the NLMS algorithm.

3 Simulation results

In this section, we compare experimentally the performances of the proposed algorithm to those of the LMF and NLMS algorithms. Two experiments are carried out where an unknown system, as depicted in Fig. 1, is to be identified under noisy conditions. The input signal $x(n)$ to the unknown system and to the adaptive filter is obtained by passing a zero mean white Gaussian sequence through a channel used in order to vary the degree of ill-conditioning on the sequence $\{x(n)\}$. The additive white noise, $w(n)$, is a zero-mean and uniformly distributed. The signal to noise ratio is set to be equal to 20dB and the performance measure considered is the normalized weight error norm $10log_{10}||\mathbf{C}(n) - \mathbf{C}_{opt}||^2/||\mathbf{C}_{opt}||^2$, where $\mathbf{C}_{opt}$ is the optimal impulse response of the unknown system. Results are obtained by averaging over 600 independent runs. During the simulations, all the algorithms are obtained for fastest convergence.

Two cases are considered for the sequence $\{x(n)\}$. In the first case $\{x(n)\}$ is uncorrelated while in the second case $\{x(n)\}$ is a correlated sequence.

In the first experiment the eigenvalue spread $(\lambda_{max}/\lambda_{min})$ is equal to 11.8, and this is for the uncorrelated sequence $\{x(n)\}$. Figure 2 shows the convergence characteristics for the NLMF and the LMF algorithms.

It is found that the NLMF algorithm is unaffected by the input data correlation statistics, in contrast to the LMF algorithm. Even though the NLMF algorithm gives a higher weight mismatch over the LMF, as it is the case of the NLMS over LMS [15], the NLMF algorithm outperforms the LMF as far as the convergence behavior is concerned and the increase in the dynamic range of the step size. The low steady state value reached by the LMF algorithm is, however, obtained at the expense of slow convergence. Similar results are obtained for other experiments with different eigenvalue spreads, but due to space limitation only the case of the eigenvalue spread of 11.8 is treated in this paper.

The most attractive feature of the NLMF algorithm is its capability to outperform the NLMS algorithm. This is due to the fact that, when far from the optimum solution, that is $|e(n)| > 1$, the NLMF algorithm exhibits faster convergence than the NLMS algorithm. Figure 3 depicts this behavior where the NLMF clearly outperforms the NLMS for the eigenvalue spread of 68.9. Notice that the two algorithms are compared for the lowest steady-state error reached by the NLMS algorithm.

In the second experiment, both the unknown system and the adaptive filter are of the same order and excited by a correlated signal $x(n)$ that follows the AR model, $x(n) = 0.8x(n-1) + u(n)$, where $\{u(n)\}$ is obtained by passing a zero mean white Gaussian sequence through a channel to give the sequence $\{u(n)\}$ an eigenvalue spread of 68.9. Here in this case only the convergence characteristics of the NLMF and NLMS algorithms are studied to demonstrate the capability of the NLMF algorithm to consistently behave under such a severe situation. As can be seen from Fig 4, the NLMF algorithm still outperforms the NLMS algorithm. As in the case of the uncorrelated input, the comparison index is the lowest steady-state error reached by the NLMS algorithm.

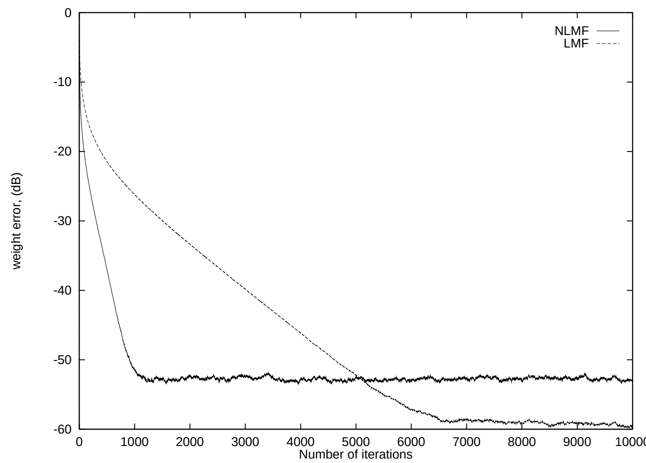

Figure 2: Convergence characteristics for LMF and NLMF algorithms.

4 Conclusion

This paper presents a detailed investigation of the implications of using the normalized LMF algorithm for the development of system identification based algorithms. The NLMF algorithm is found to be independent of the input data correlation statistics. The NLMF algorithm is also found to outperform the LMF algorithm. When the NLMF algorithm is compared to the NLMS algorithm for the the lowest steady-state error, it is found that the former outperforms the latter.

On the whole, the paper examines the advantages of the utilization of the NLMF algorithm in terms of enhancing both convergence time and steady-state error in developing linear transversal filter based system identification algorithms.

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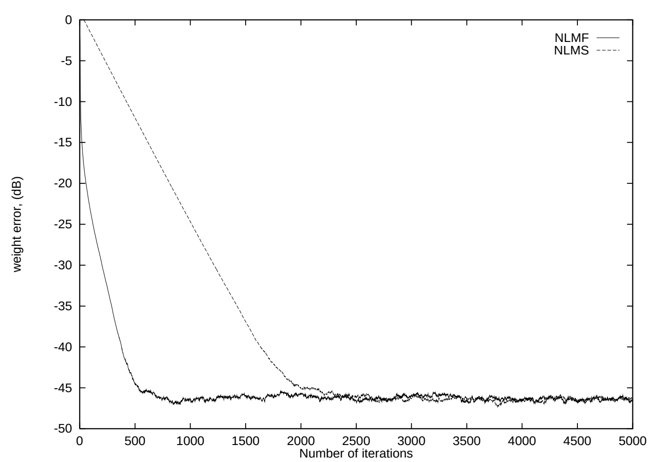

Figure 3: Convergence characteristics for NLMF and NLMS algorithms for $\lambda_{max}/\lambda_{min} = 68.9$, uncorrelated input.

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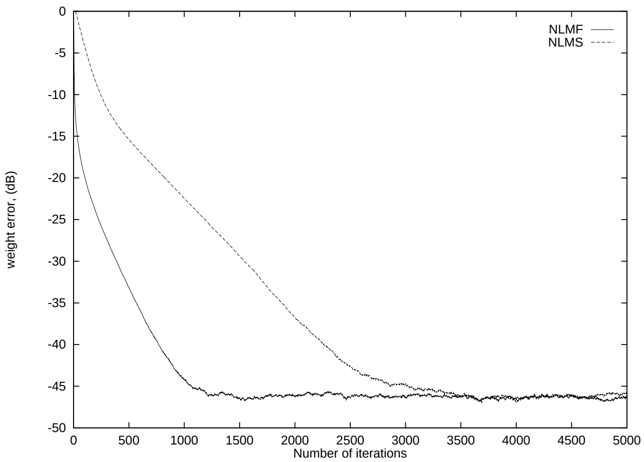

Figure 4: Convergence characteristics for NLMF and NLMS algorithms for $\lambda_{max}/\lambda_{min} = 68.9$, correlated input.

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