APPROXIMATING THE PROTECTION OFFERED BY A CHANNEL CODE IN TERMS OF BIT ERROR RATE

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ABSTRACT

In many joint source-channel coding applications, there is a need to assess the performance of a system on a given channel, with different combinations of channel codes. What is interesting is to know some measure of the overall bit error rate, in order to design the source and/or channel codes accordingly.
We propose here an approximation of the bit error rate of the cascade of a linear block code and a memoryless binary channel. With this approximation, both of these components can be considered in this respect as a single binary symmetric channel, with known transition probability, and can be used as such for further processing.
We also show some possible applications in joint source-channel coding schemes.

1 INTRODUCTION

The field of joint source-channel coding (JSCC) has been studied in many different ways up to now and has lead to numerous different techniques [1, 2, 3, 4, 5, 6, 7, 8, 9, 10].

One of the problems of the joint study of source and channel codes is to obtain a coherent quality measure for these two coders. Many systems rely on the knowledge of a bit or symbol error probability of the channel but fail to take into account the presence of an error correcting code.

We propose here an approximation of the bit error rate of a channel protected by a linear block error correcting code. We will first derive this approximation, together with upper and lower bounds on the bit error rate. Then, we will test the performance of the approximation on different channel codes, and study its properties. Finally, we will outline some possible applications of this simplification on joint source-channel coders.

2 DERIVATION OF THE APPROXIMATION

Given a binary symmetric channel (BSC) of known transition probability p, we will try to find in this section an approximation of the bit error rate on a binary signal transmitted on this channel, after protecting it with an error correcting code.

We will limit ourselves to linear block codes. An (n, k) block code maps a k-bit symbol to an n-bit codeword, adding thus $n-k$ redundancy bits to the original signal. Its rate r is defined as the ratio $\frac{k}{n}$. The quality of the code is measured in general by its error correcting capacity t, which is the maximum number of bit errors that a coded symbol can incur while still being decoded correctly.

Denoting by p' the bit error rate of the combination of the channel and the channel (de)coder, we can first easily derive an inferior bound on p' as follows. If the transmitted word is erroneously decoded, it means that at least one of the k bits of the output of the channel decoder is false, so that the probability that a bit error occurs when a word is decoded incorrectly can be bounded by $\dfrac{1}{k}$. Now the probability p_{word} that a word error occurs is simply the probability that more than t bit errors happen during transmission :

$$p_{word} = \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-1}.$$

The above expression[1] computes the probability p_{word} by simply adding the probabilities of all possible channel error patterns on more than t bits of the same coded symbol. So, we have

$$p' \geq \frac{1}{k} \sum_{i=t+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}. \tag{1}$$

Now an upper bound is given by the following steps : if we decompose the bit error rate (BER) as $p' = \sum_{i=0}^{n} p(e|i).p(i)$, where $p(i)$ is the probability that a received n-bit word contains exactly i bit errors and $p(e|i)$ is the bit error rate of a word containing i errors, we can easily upper bound all terms of this expression. First, $p(i) = \binom{n}{i} p^i (1-p)^{n-i}$, which is an exact expression. Next, let us examine what happens to $p(e|i)$ for different values of i.

[1] As usual, $\binom{a}{b}$ denotes the number of possible combinations of a objects taken b by b.

- If $i \leq t$, the message will be decoded correctly, thus none of the decoded bits will be wrong, so that $p(e|i) = 0$.
- If $i > t$, let us denote by c the sent codeword, by $\hat{r}$ the received message, and by $\hat{c}$ the codeword into which it will be decoded. The process is the following : c is sent over the channel, i bit errors are added to it to give $\hat{r}$, and the channel decoder interprets $\hat{r}$ as an erroneous coded symbol, and corrects it as $\hat{c}$. Then, the hamming distance $d_H(c, \hat{r})$ between $\hat{r}$ and c is i. On the other hand, the decoder can only correct up to t bit errors, so that it will always map a received n-bit word to a codeword which differs at most in t bit places from the received n-bit word, so that
$$d_H(\hat{r}, \hat{c}) \leq t,$$
since $\hat{r}$ will be decoded as $\hat{c}$. This gives us a total of at most $i + t$ bit errors between c and $\hat{c}$. In the worst case, these $i + t$ bit errors will all be located in the k bits of the message, so that we can upper bound $P(e|i)$ by $\dfrac{i+t}{k}$.
- Of course, for $i \geq k - t$, it makes no sense to use the above ratio, which is higher than 1, and is supposed to be a probability. In such a case, we will use 1 instead.

Combining the above arguments, we get the upper bound :

$$\begin{aligned} p' &\leq \sum_{i=t+1}^{k-t-1} \frac{i+t}{k} \binom{n}{i} p^i (1-p)^{n-i} \\ &\quad + \sum_{i=k-t}^{n} \binom{n}{i} p^i (1-p)^{n-i} \qquad (2) \end{aligned}$$

Finally, to obtain the approximation we are looking for, we follow the same path as above, but instead of making a worst-case assumption on $p(e|i)$, we make an assumption that is a bit more fair : when $i > t$, we consider that the $i + t$ errors will be spread equally among the n bits of $\hat{c}$, so that $p(e|i) \simeq \dfrac{i+t}{n}$ and

$$\begin{aligned} p' &\simeq \sum_{i=t+1}^{n-t-1} \frac{i+t}{n} \binom{n}{i} p^i (1-p)^{n-i} \\ &\quad + \sum_{i=n-t}^{n} \binom{n}{i} p^i (1-p)^{n-i} \qquad (3) \end{aligned}$$

3 QUALITY AND PROPERTIES OF THE APPROXIMATION

3.1 Comparisons with simulations

We compare here the bit error rate computed by (3) to the one obtained through simulations. For each given code, we have simply encoded a sequence of random bits,

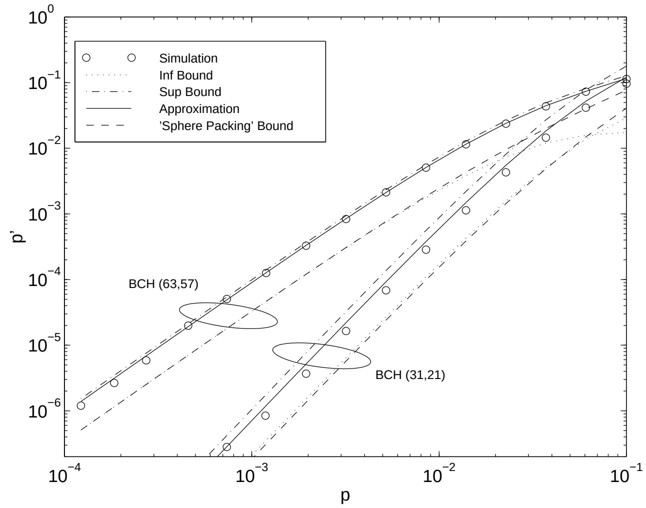

Figure 1: Performance of the approximation and bounds for a (63,57) and a (31,21) BCH code.

made a computer simulation of the channel, and counted the bit errors at the output of the channel decoder. Figure 1 presents these results for a binary $(63, 57)$ BCH code and a $(31, 21)$ BCH code. The horizontal axis represents the channel transition probability (p) and the vertical axis is the BER after channel coding p', represented by the approximation (3) and the bounds (1) and (2). The figure also shows a bound on p' obtained by the sphere packing bound, as proposed in [1]. Clearly, our approximation is good, and our bounds are relatively tight. In general, our lower bound on p' shows a similar behavior as the one derived from the sphere packing bound.

3.2 Case of repetition codes

One property that we would like to point out is that the approximation error is exactly zero for repetition codes. Repetition codes are $(2m + 1, 1)$ codes, whose generator matrix is simply given by

$$\mathcal{G} = [111 \cdots 1] .$$

This means that one simply sends $2m + 1$ copies of each bit instead of only one copy. On the decoder side, the decision is taken on the majority, i.e. the decoding will be correct as long as $m + 1$ bits out of the $2m + 1$ of the codeword remain unchanged. Thus the repetition code can correct exactly $t = m$ errors. Since each input bit corresponds to one word, the word error probability is the same as the bit error probability, and is given exactly by :

$$p' = \sum_{i=t+1}^{2t+1} \binom{2t+1}{i} p^i (1-p)^{2t+1-i}.$$

Now expressing the approximation from above with this particular code, we see that our computations give

$$p' \simeq \sum_{i=n-t=t+1}^{2t+1} \binom{2t+1}{i} p^i (1-p)^{2t+1-i},$$

which clearly is the exact expression.

4 SOME POSSIBLE APPLICATIONS

Optimal quantizers for noisy channels. The design of optimal quantizers for noisy channels [5] relies on the knowledge of the input signal pdf as well as the transition probabilities between states of the quantizer due to the channel. Our approximation can be used as is to provide channel coding capabilities to this system, without major changes in the quantizer design algorithm : one only has to replace the parameter p by p'.

Enhanced decoding by residual correlation. The use of residual redundancy in the transmitted signal to improve the quality of the reconstruction at the decoder side has been used in some joint source-channel coding systems [2, 3]. Once again, we can introduce easily channel codes in these schemes by the above approximation and come to a really joint design : it allows to know what quality improvement we can expect from the addition of a channel code.

Integration into a complete JSCC system. As a last application, we would like to point out the work we have carried out [11, 12] on the optimization of a complete image coding chain *including Huffman coding* based on the wavelet transform. Thanks to the approximation developed here, we were able to design the whole transmission chain by taking into account bit error probabilities as well as desynchronization probabilities. The reader is referred to [11] for more information.

5 CONCLUSION

We have presented here an approximation of the bit error rate of a channel protected by a linear block code, which can be useful for numerous joint source-channel coding applications. We have also shown that its performance is superior to the one of the bound proposed in [1]. Another advantage is that the approximation is coherent, in the sense that it gives the true BER for repetition codes. Possible applications have been outlined and can be simply summarized as a possibility to design a system in terms of BER even when a linear block error-correcting code is used.

References

[1] Nam Phamdo, Nariman Farvardin, and Takehiro Moriya. A unified approach to tree-structured and multistage vector quantization for noisy channels. *IEEE Transactions on Information Theory*, 39(3):835–850, May 1993.

[2] Khalid Sayood and Jay C. Borkenhagen. Use of residual redundancy in the design of joint source/channel coders. *IEEE Transactions on Communications*, 39(6):838–846, June 1991.

[3] Nam Phamdo and Nariman Farvardin. Optimal detection of discrete markov sources over discrete memoryless channels—applications to combined source-channel coding. *IEEE Transactions on Information Theory*, 40(1):186–193, January 1994.

[4] Shahriar Emami and Scott L. Miller. DPCM picture transmission over noisy channels with the aid of a markov model. *IEEE Transactions on Image Processing*, 4(11):1473–1481, November 1995.

[5] Nariman Farvardin and Vinay Vaishampayan. Optimal quantizer design for noisy channels : an approach to combined source-channel coding. *IEEE Transactions on Information Theory*, 33(6):827–838, November 1987.

[6] Nariman Farvardin. A study of vector quantization for noisy channels. *IEEE Transactions on Information Theory*, 36(4):799–809, July 1990.

[7] David G. Daut and James W. Modestino. Two-dimensionnal DPCM picture transmission over fading channels. *IEEE Transactions on Communications*, 31(3):315–328, March 1983.

[8] Michael J. Ruf and James W. Modestino. Rate-distortion performance for joint source and channel coding of images. In *International Conference on Image Processing—ICIP'95*. IEEE, 1995.

[9] James W. Modestino, David G. Daut, and Acie L. Vickers. Combined source-channel coding of images using the block cosine transform. *IEEE Transactions on Communications*, 29(9):1261–1274, September 1981.

[10] James W. Modestino and David G. Daut. Combined source-channel coding of images. *IEEE Transactions on Communications*, 27(11):1644–1659, November 1979.

[11] Claude Desset, Fabrice Labeau, Benoît Macq, and Luc Vandendorpe. Joint source-channel coding for optimal transmission of multiresolution encoded images. In *Proceedings of the IEEE Fifth symposium on communications and vehicular technology in the Benelux*, pages 210–217, October 1997.

[12] Benoît Macq, Fabrice Labeau, Claude Desset, Luc Vandendorpe, and Jean-Jacques Quisquater. Image processing for multimedia terminals and related architectures. In *Proceedings of the COST 254 Workshop on Emerging Techniques for Communications Terminals*, July 1997.