A DESIGN METHOD OF 2-D AXIAL-SYMMETRIC PARAUNITARY FILTER BANKS WITH A LATTICE STRUCTURE

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ABSTRACT

A lattice structure of 2-D axial-symmetric paraunitary filter banks (ASPUFBs) is proposed, which makes it possible to design such filter banks in a systematic manner. ASPUFBs consist of non-separable axial-symmetric (AS) filters, and can be regarded as a subclass of non-separable linear-phase paraunitary (PU) ones. The AS property is desirable for image processing, because it enables us to use the symmetric extension method. Since our proposed system structurally restricts both the PU and AS properties, it can be designed by using an unconstrained optimization process. A design example will be given to show the significance of the lattice structure.

1 INTRODUCTION

The application of filter banks to data compression, known as subband coding (SBC), has been studied as an effective coding scheme in audio and visual communications [1]. Recently, multidimensional (MD) multirate signal processing has increasingly been used in video processing [2], and interest in MD filter banks has risen. MD filter banks used in practical applications are often constructed with 1-D ones and implemented as separable systems because of their simplicity. For SBC, the PU property of the employed filter banks is significant, since the average energy of quantization error energy in subbands is preserved in the error energy of the reconstructed signal. On the other hand, the LP property is also of interest, since 1-D filter banks with this property can handle finite-duration signals by means of the symmetric extension method to avoid the size-increasing problem [3]. Thus, 1-D linear-phase paraunitary filter banks (LPPUFBs) have been well studied so far [4–9]. The lattice structure in particular has received a lot of attention, because it enables us to design LPPUFBs in a systematic manner.

MD signals, however, are generally non-separable, and separable systems have limitation in exploiting their characteristics. In order to overcome this disadvantage, non-separable MD filter banks are required. The extension of 1-D LPPUFBs to MD non-separable systems has already been discussed [10–13]. The lattice structure has firstly presented by Kovačević *et al.* for some simple cases [14]. Then, the structure was generalized and shown to achieve higher coding gain than that of separable one [13]. In compensation for this advantage, there is a drawback that the symmetric extension method can not be applied to them due to the point-wise symmetry of their filters. To use the method, filters have to be axial-symmetric (AS) for each dimension. Recently, Stanhil *et al.* stated this fact, where the word "*four-fold symmetry*" is used instead of "*axial-symmetry*" in the article [15]. It proposes a design method of ASPUFBs. However, it requires us to solve a matrix equation under some conditions. Independently, we studied a special type of ASPUFBs, where the filter coefficients are restricted to be binary-valued [16].

In light of this fact, we propose a design method of ASPUFBs with a lattice structure. The structure guarantees for filter banks to consist of non-separable AS filters and to be PU during the design phase. It just requires us to characterize some orthogonal matrices, and provides us filter banks with continuous-valued coefficients.

All through this paper, $\mathbf{I}_M$ and $\mathbf{J}_M$ denote the $M\times M$ identity and counter-identity matrices, respectively, and $\mathbf{\Gamma}_M$ is the $M\times M$ diagonal matrix which has $+1$ and -1 elements alternatively on the diagonal [1]. In addition, $\hat{\mathbf{I}}^{\{0\}}=\mathrm{diag}(1,-1)$ and $\hat{\mathbf{I}}^{\{1\}}=\mathrm{diag}(-1,1)$.

2 AXIAL-SYMMETRIC FILTER BANKS

Figure 1 shows the parallel structure of filter banks, where $H_k(\mathbf{z})$ and $F_k(\mathbf{z})$ are the k-th analysis and synthesis filter, respectively. Let $H(\mathbf{z})$ be a 2-D filter whose d-th dimension order is L_d. If $H(\mathbf{z})$ satisfies the condition that

$$H(\mathbf{z})=\gamma_d\mathbf{z}^{-2\mathbf{c}_{\mathrm{h}}^{\{d\}}}H_k(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d\}}}),\ d\in\{0,1\} \tag{1}$$

then the impulse response of $H(\mathbf{z})$ has axial-symmetry, where $\gamma_d=\pm1$, $\mathbf{c}_{\mathrm{h}}^{\{0\}}=\frac{1}{2}\left(L_0,\ 0\right)^T$, $\mathbf{c}_{\mathrm{h}}^{\{1\}}=\frac{1}{2}\left(0,\ L_1\right)^T$, $\mathbf{z}=\left(z_0,\ z_1\right)^T$, $\mathbf{z}^{-\hat{\mathbf{I}}^{\{0\}}}=\left(z_0^{-1},\ z_1\right)^T$ and $\mathbf{z}^{-\hat{\mathbf{I}}^{\{1\}}}=\left(z_0,\ z_1^{-1}\right)^T$.

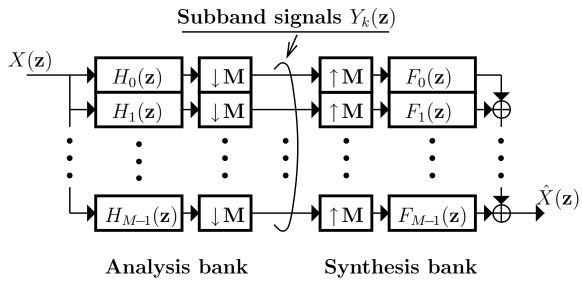

Figure 1: Parallel structure of filter banks

All of analysis and synthesis filters in axial-symmetric filter banks satisfy the condition in Eq. (1). In this work, we deal with 2-D AS filer banks of the following decimation factor:

$$\mathbf{M} = \begin{pmatrix} M_0 & 0 \\ 0 & M_1 \end{pmatrix}, \tag{2}$$

where M_0 and M_1 are even. In the followings, M denotes the number of channels, where $M = |\det\mathbf{M}| = M_0M_1$.

Let $\mathbf{E}(\mathbf{z})$ be a type-I polyphase matrix of an analysis bank and N_d be the d-th dimension order of $\mathbf{E}(\mathbf{z})$ [1]. If $\mathbf{E}(\mathbf{z})$ satisfies the condition that

$$\mathbf{E}(\mathbf{z}) = \mathbf{z}^{-2\mathbf{c}_{\Xi}^{\{d\}}}\boldsymbol{\Gamma}^{\{d\}}\mathbf{E}(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d\}}})\mathbf{P}^{\{d\}},\ d \in \{0, 1\}, \tag{3}$$

then the analysis bank consists of only AS filters, where $\mathbf{c}_{\Xi}^{\{0\}} = \frac{1}{2}(N_0,\ 0)^T$, $\mathbf{c}_{\Xi}^{\{1\}} = \frac{1}{2}(0,\ N_1)^T$ [15]. $\boldsymbol{\Gamma}^{\{d\}}$ and $\mathbf{P}^{\{d\}}$ denote the $M \times M$ diagonal matrix with ±1 elements and permutation matrix defined by

$$\boldsymbol{\Gamma}^{\{d\}} = \begin{cases} \mathbf{I}_{\frac{M}{2}} \oplus \left(-\mathbf{I}_{\frac{M}{2}}\right) & d = 0 \\ \boldsymbol{\Gamma}_M\boldsymbol{\Gamma}^{\{0\}} & d = 1 \end{cases}, \tag{4}$$

$$\mathbf{P}^{\{d\}} = \begin{cases} \bigoplus\sum_{i=0}^{M_1-1}\mathbf{J}_{M_0} & d = 0 \\ \mathbf{J}_M\mathbf{P}^{\{0\}} & d = 1 \end{cases}, \tag{5}$$

respectively, where $\oplus$ denotes the direct sum of matrices [17]. It can be easily verified that the above diagonal and permutation matrices satisfy the condition shown in [15]. Note that the polyphase matrix $\mathbf{E}(\mathbf{z})$ is defined as the transpose of the one defined in the article.

The numbers of symmetric and anti-symmetric filters with respect to the axis-wise symmetry should be the same as each other for each dimension, as well as those with respect to the point-wise symmetry [4]. In Eq. (3), this requirement is taken into account.

3 PROPOSED LATTICE STRUCTURE

In addition to the AS property, we consider imposing filter banks to be PU. The condition for the PU property of $\mathbf{E}(\mathbf{z})$ is expressed by $\tilde{\mathbf{E}}(\mathbf{z})\mathbf{E}(\mathbf{z}) = \mathbf{I}_M$, where the tilde notation over a matrix denotes the paraconjugation [1]. If the analysis bank holds the PU property, the counterpart synthesis bank yielding perfect reconstruction is simply obtained [1]. Thus, only analysis bank is discussed below.

In order to construct a lattice structure of 2-D AS-PUFBs, we consider formulating the order increasing process of the polyphase matrix $\mathbf{E}(\mathbf{z})$, while keeping both of the PU and AS properties.

Let $\mathbf{E}_m(\mathbf{z})$ be a polyphase matrix, whose d-th dimension order is m. We consider increasing the d-th dimension order m to $m+1$ as follows:

$$\mathbf{E}_{m+1}(\mathbf{z}) = \mathbf{S}^{\{d\}T}\mathbf{R}_{m+1}^{\{d\}}\mathbf{Q}^{\{d\}}(\mathbf{z})\mathbf{S}^{\{d\}}\mathbf{E}_m(\mathbf{z}), \tag{6}$$

where $\mathbf{R}_n^{\{d\}}$, $\mathbf{Q}^{\{d\}}(\mathbf{z})$ and $\mathbf{S}^{\{d\}}$ are the $M \times M$ paraunitary matrices defined by

$$\mathbf{R}_n^{\{d\}} = \left(\mathbf{F}_{\frac{M}{2}}^T \oplus \mathbf{F}_{\frac{M}{2}}^T\right)\left(\bigoplus\sum_{i=0}^{3}\mathbf{U}_{n,i}^{\{d\}}\right)\left(\mathbf{F}_{\frac{M}{2}} \oplus \mathbf{F}_{\frac{M}{2}}\right) \tag{7}$$

$$\mathbf{Q}^{\{d\}}(\mathbf{z}) = \frac{1}{2}\mathbf{B}_M\boldsymbol{\Lambda}^{\{d\}}(\mathbf{z})\mathbf{B}_M, \tag{8}$$

and

$$\mathbf{S}^{\{d\}} = \begin{cases} \mathbf{I}_{\frac{M}{2}} \oplus \mathbf{J}_{\frac{M}{2}}, & d = 0 \\ \mathbf{F}_M^T\left(\mathbf{F}_{\frac{M}{2}} \oplus \mathbf{F}_{\frac{M}{2}}\right), & d = 1 \end{cases}, \tag{9}$$

where

$$\mathbf{B}_M = \begin{pmatrix} \mathbf{I}_{\frac{M}{2}} & \mathbf{I}_{\frac{M}{2}} \\ \mathbf{I}_{\frac{M}{2}} & -\mathbf{I}_{\frac{M}{2}} \end{pmatrix}, \tag{10}$$

$$\boldsymbol{\Lambda}^{\{d\}}(\mathbf{z}) = \mathbf{I}_{\frac{M}{2}} \oplus \left(\mathbf{z}^{-\mathbf{1}^{\{d\}}}\mathbf{I}_{\frac{M}{2}}\right) = \mathbf{I}_{\frac{M}{2}} \oplus \left(z_d^{-1}\mathbf{I}_{\frac{M}{2}}\right), \tag{11}$$

$$\mathbf{F}_M = \begin{pmatrix} 1 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ 0 & 1 & 0 & 0 & \cdots & 0 \end{pmatrix}. \tag{12}$$

The matrices $\mathbf{U}_{n,i}^{\{d\}}$ are arbitrary $M/4 \times M/4$ orthonormal matrices.

The PU property of $\mathbf{E}(\mathbf{z})$ results in that of $\mathbf{E}_{m+1}(\mathbf{z})$, since all of $\mathbf{S}^{\{d\}}$, $\mathbf{R}_n^{\{d\}}$ and $\mathbf{Q}^{\{d\}}(\mathbf{z})$ are PU. In addition, the AS property of $\mathbf{E}_m(\mathbf{z})$ as in Eq. (1) propagates to $\mathbf{E}_{m+1}(\mathbf{z})$. Let us verify this fact.

Let us consider increasing the d'-th dimension order from m to $m+1$. Now, Eq. (6) can be rewritten as follows:

$$\mathbf{E}_m(\mathbf{z}) = \mathbf{S}^{\{d'\}T}\mathbf{Q}^{\{d'\}}(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d'\}}})\mathbf{R}_{m+1}^{\{d'\}T}\mathbf{S}^{\{d'\}}\mathbf{E}_{m+1}(\mathbf{z}), \tag{13}$$

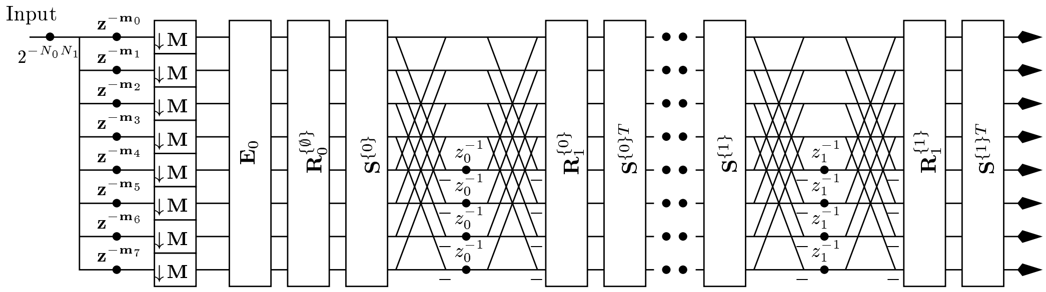

Figure 2: An example of the proposed lattice structure of 2-D ASPUFBs, where the number of channel $|\det(\mathbf{M})|$ is assumed to be 8 as an example. $\mathbf{z}^{-\mathbf{m}_i}$ denotes the 2-D delay element determined by the factor $\mathbf{M}$.

By substituting the above equation into the AS condition of $\mathbf{E}_m(\mathbf{z})$, we have

$$\mathbf{E}_{m+1}(\mathbf{z}) = \mathbf{z}^{-2\mathbf{c}^{\{d\}}_{\Xi_m}}\mathbf{V}^{\{d'\}\{d\}}_{m+1}(\mathbf{z})\mathbf{E}_{m+1}(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d\}}})\mathbf{P}^{\{d\}}, \quad d \in \{0,1\} \tag{14}$$

where $\mathbf{c}^{\{d\}}_{\Xi_m}$ is a vector whose d'-th element is $m/2$, and

$$\mathbf{V}^{\{d'\}\{d\}}_n(\mathbf{z}) = \mathbf{S}^{\{d'\}T}\mathbf{R}^{\{d'\}}_n\mathbf{Q}^{\{d'\}}(\mathbf{z})\times \mathbf{S}^{\{d'\}}\mathbf{\Gamma}^{\{d'\}}\mathbf{S}^{\{d'\}T}\mathbf{Q}^{\{d'\}}(\mathbf{z}^{\hat{\mathbf{I}}^{\{d'\}}\hat{\mathbf{I}}^{\{d\}}})\mathbf{R}^{\{d'\}T}_n\mathbf{S}^{\{d'\}}. \tag{15}$$

Let $\mathbf{1}^{\{0\}} = (1,0)^T$ and $\mathbf{1}^{\{1\}} = (0,1)^T$. From the fact that

$$\mathbf{V}^{\{d'\}\{d\}}_n(\mathbf{z}) = \begin{cases} \mathbf{z}^{-\mathbf{1}^{\{d\}}}\mathbf{\Gamma}^{\{d\}} & d = d' \\ \mathbf{\Gamma}^{\{d\}} & d \neq d' \end{cases}, \tag{16}$$

Eq. (14) is reduced to

$$\mathbf{E}_{m+1}(\mathbf{z}) = \begin{cases} \mathbf{z}^{-2\mathbf{c}^{\{d\}}_{\Xi_{m+1}}}\mathbf{\Gamma}^{\{d\}}\mathbf{E}_{m+1}(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d\}}})\mathbf{P}^{\{d\}}, & d{=}d' \\ \mathbf{z}^{-2\mathbf{c}^{\{d\}}_{\Xi_m}}\mathbf{\Gamma}^{\{d\}}\mathbf{E}_{m+1}(\mathbf{z}^{-\hat{\mathbf{I}}^{\{d\}}})\mathbf{P}^{\{d\}}, & d{\neq}d' \end{cases} \tag{17}$$

$$d \in \{0,1\} \tag{18}$$

where $\mathbf{c}^{\{d\}}_{\Xi_{m+1}}$ is a vector whose d'-th element is $(m+1)/2$. The last result implies that $\mathbf{E}_{m+1}(\mathbf{z})$ sufficiently satisfies the AS condition, and the only d'-th dimension order is increased.

Therefore, the following product form of the polyphase matrix provides us an ASPUFB of order (N_0, N_1) which holds both of the PU and AS (Eq. (3)) properties.

$$\mathbf{E}(\mathbf{z}) = \left\{\prod_{d=0}^{1}\prod_{\substack{n=1 \\ N_d\neq 0}}^{N_d}\mathbf{S}^{\{d\}T}\mathbf{R}^{\{d\}}_n\mathbf{Q}^{\{d\}}(\mathbf{z})\mathbf{S}^{\{d\}}\right\}\mathbf{R}^{\{\emptyset\}}_0\mathbf{E}_0, \tag{19}$$

where $\mathbf{E}_0$ is an arbitrary $M \times M$ orthonormal matrix which satisfies the AS condition that $\mathbf{E}_0 = \mathbf{\Gamma}^{\{d\}}\mathbf{E}_0\mathbf{P}^{\{d\}}$ for $d \in \{0,1\}$. The polyphase matrix of the type-II 2-D DCT is a good candidate for the matrix $\mathbf{E}_0$. $\mathbf{E}_0$ can be fixed during the design phase.

Table 1: Coding gain G of several MD-LPPUFBs with rectangular decimation for the isotropic acf model with $\rho = 0.95$. $\mathbf{M}$ and $(N_0, N_1)^T$ denote the decimation matrix and the order of polyphase matrix, respectively.

$\mathbf{M}$	$\begin{pmatrix} N_0 \\ N_1 \end{pmatrix}$	G [dB]		
		Sep.	Gen.	Prop.
$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$	$(0,0)$	10.75	10.78	10.78
	$(1,1)$	11.20	11.28	11.21
	$(2,2)$	11.42	11.55	11.43

According to the product form in Eq. (19), we can obtain a lattice structure of ASPUFBs as shown in Fig. 2. Let us here summarize the properties of the proposed structure.

- By controlling the orthonormal matrices $\mathbf{U}^{\{d\}}_{n,i}$, the lattice structure can be characterized, and then an ASPUFB can be designed.
- The system is causal and minimal. and the size of all filters results in $M_0(N_0+1) \times M_1(N_1+1)$.

In order to control the orthonormal matrices $\mathbf{U}^{\{d\}}_{n,i}$, we can use the Givens factorization technique [1]. Since the AS and PU properties are guaranteed during the design phase, ASPUFBs can be designed by means of an unconstrained non-linear optimization process.

4 DESIGN EXAMPLE

In order to verify the significance of our proposed method, we show a design example, where the object function of the optimization is chosen as the maximum coding gain [1] for the isotropic autocorrelation function (acf) model with the correlation coefficient $\rho = 0.95$ [18].

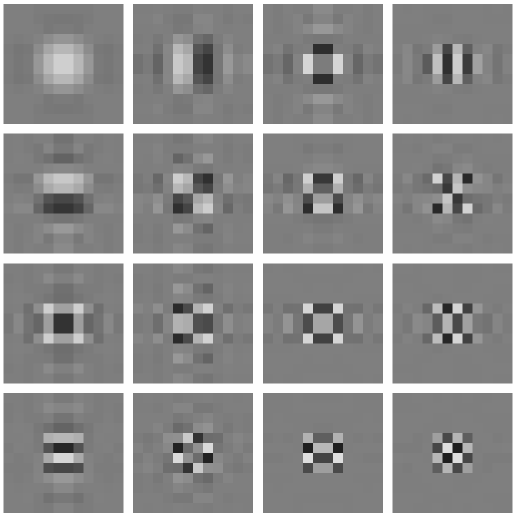

Figure 3: Basis images of a design example of an ASPUFB, where $M_0 = M_1 = 4$ and $N_0 = N_1 = 2$.

Figure 3 shows the resulting basis images. The coding gain results in 11.432 [dB], whereas that of the corresponding separable structure with the 1-D LPPUFB [7] is 11.418 [dB]. Table 1 compares the coding gain (denoted by PROP.) with those of the separable one (denoted by SEP.). For reference, the coding gain of the corresponding non-separable one of general M-D LPPUFBs proposed in [13] is also shown (denoted by GEN.).

As a result, it can be verified that our proposed structure possesses capability to take higher coding gain than that of separable one, holding the AS and PU properties.

5 CONCLUSIONS

In this summary, we have proposed a design method of ASPUFBs with a lattice structure. The AS and PU properties are guaranteed during the design phase. Thus, an unconstrained non-linear optimization process can be used to design it. By showing some design examples, the significance of our proposed structure was verified.

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