DIRECTIONAL SECOND ORDER DERIVATIVES : APPLICATION TO EDGE AND CORNER DETECTION

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ABSTRACT

In this paper, we propose an adaptive scheme to design directional second order derivatives orthogonally and tangentially to the local edge. The principle lies on the definition of two adaptive filter masks which estimate the two derivatives along the normal (n) and tangential (t) directions. Both filter masks are controlled by an adaptive mask whose coefficients are tuned in accordance with the local grey level distribution. The two new filters are then applied respectively to edge and corner detection : edge detection is achieved by detecting the zero-crossing of the derivative along n, and corner detection is obtained by thresholding the amplitude of the derivative along t. Results of these detections are provided on synthetise and real-world images, and swow the robustness of the new proposed approach.

1 Introduction

Image structure analysis often requires the computation of local spatial derivatives of the intensity distribution. For instance, the problem of edge detection is classically solved by using first and/or second order derivatives [3]. An edge point is characterized by a high gradient value or a zero-crossing of the second order derivative in the normal direction of the local edge instead of the generally used laplacian operator. Edges are then obtained by thresholding the gradient or by detecting the zero-crossing of the second order derivative. In this paper we are interested with the second approach. Thus if $\mathbf{n}$ represents the orthogonal vector to the edge (figure 1) and $f(x,y)$ the input image, we should estimate the following derivative :

$$\frac{\partial^2 f(x,y)}{\partial n^2} = \frac{\partial^2 f(x,y)}{\partial x^2} cos^2(\phi) + \frac{\partial^2 f(x,y)}{\partial y^2} sin^2(\phi) + 2\frac{\partial^2 f(x,y)}{\partial x \partial y} cos(\phi)\ sin(\phi) \qquad (1)$$

with $\phi = (Ox, \mathbf{n})$.

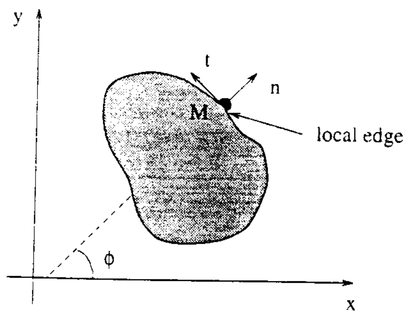


Figure 1: Normal to the local edge

As the angle ϕ is generally unknown, equation (1) is usually replaced by the laplacian $\Delta f(x,y)$:

$$\begin{aligned}\Delta f(x,y) &= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2} \\ &= \frac{\partial^2 f(x,y)}{\partial n^2} + \frac{\partial^2 f(x,y)}{\partial t^2} \qquad (2)\end{aligned}$$

with $\mathbf{t}$ the local tangent vector to the edge. The second term in (2) (derivative along $\mathbf{t}$) represents the local curvature of the edge, and zero-crossing of the laplacian does not coincide with the expected edge except if the edge is locally linear [3] : the zero-crossing of the laplacian is biaised by the morphological local curvature.
In this paper, we propose to estimate the two directional derivatives involved in (2). The principle lies on the definition of two adaptive filter masks which estimate the two derivatives along the directions $\mathbf{n}$ and $\mathbf{t}$. Each derivative is then used to detect respectively edge position and curvature.

2 Adaptive filter masks

Let us define the support S as the set of pixels likely to contribute in the filtering of the current pixel (x,y). Without loss of generality, we consider that S is a rectangular window of size $K_1 \times K_2$:

$$S = \{(i,j) \in [-\frac{K_1-1}{2}; \frac{K_1-1}{2}] \times [-\frac{K_2-1}{2}; \frac{K_2-1}{2}]\} \qquad (3)$$

The pixels included in the mask may belong to different regions. The principle of adaptive filtering [1, 2] then consists in adjusting the contribution of each pixel of S according to their similarity with the current pixel. For this purpose, we define M as a set of coefficients $m_{i,j}$ (in the range $[0, 1]$) associated with the support S :

$$M = \{m_{i,j} \in [0, 1]/(i,j) \in S\} \quad (4)$$

Each coefficient $m_{i,j}$ is viewed as a level of confidence for the pixel $(x+i, y+j)$ to belong to the same region as the current pixel (x, y), i.e. $m_{i,j} \to 1$ for pixels similar to the current pixel, respectively $m_{i,j} \to 0$ for the others. When only based on the gray levels of pixels, we propose in [2] the following similarity estimation :

$$m_{i,j} = \exp\left(-\frac{(f(x+i, y+j) - f(x,y))^2}{2\triangle^2}\right) \quad (5)$$

where $\triangle$ is a parameter controling the width of the similarity function. We show in [2] that the parameter $\triangle$ must satisfy the following condition :

$$2\sigma < \triangle < H_{min} \quad (6)$$

where σ represents the standard deviation of the noise and H_{min} the minimum height of the edge to be detected.
The two directional derivatives are then obtained by combining the coefficients of M in order to design two masks D_{2_n} and D_{2_t} respectively associated to the normal and the tangential second order derivative.

2.1 Normal second order derivative : edge detection

The second order normal derivative has been introduced in [2] in order to provide a robust edge enhancement. The corresponding weighted mask D_{2_n} is defined by :

$$D_{2_n} = \{w_{i,j}^{(n)} = m_{i,j} - \overline{m} \;/(i,j) \in S\} \quad (7)$$

with $\overline{m}$ the mean of the set M :

$$\overline{m} = \frac{1}{K_1 K_2} \sum_{(i,j)\in S} m_{i,j} \quad (8)$$

the output $f_n^{(2)}(x, y)$, which estimates the second order derivative along n $(\partial^2 f/\partial n^2)$ is then given by :

$$\begin{aligned} f_n^{(2)}(x,y) &= \sum_{(i,j)\in S} w_{i,j}^{(n)} f(x+i, y+j) \\ &= \sum_{(i,j)\in S} (m_{i,j} - \overline{m}) f(x+i, y+j) \quad (9) \end{aligned}$$

We demonstrate in [2] that this filter behaves like a second order derivative orthogonal to the local edge.

In this paper, we propose to perform edge detection by applying a zero-crossing method [3, 4], in which a pixel position (x, y) is labelled as edge point if one of the following conditions is fulfilled :

$$\begin{cases} f_n^{(2)}(x-1, y).f_n^{(2)}(x,y) \leq 0 \\ and \\ |f_n^{(2)}(x-1, y) - f_n^{(2)}(x,y)| > T \end{cases} \quad (10)$$

or

$$\begin{cases} f_n^{(2)}(x, y-1).f_n^{(2)}(x,y) \leq 0 \\ and \\ |f_n^{(2)}(x, y-1) - f_n^{(2)}(x,y)| > T \end{cases} \quad (11)$$

where T is a suitable threshold used to reject spurious edges. Indeed, in homogeneous areas the derivative $f_n^{(2)}(x, y)$ is theorically zero, but in the presence of noise it is a zero mean function with small variation, and spurious zero crossing can be detected. The threshold T then allows to reject these false detections.

2.2 Tangential second order derivative : curvature detection

The determination of the second derivative is based on the following assumption : as the two second order derivatives are associated to two orthogonal directions, the masks D_{2_n} and D_{2_t} must be themselves mutually orthogonal. So according to the choosen rotation $(+\frac{\pi}{2}$ or $-\frac{\pi}{2})$ two orthogonal masks to D_{2_n} can be computed. We propose here to combine the two possible masks by taking their mean. The mask D_{2_t} is then given by :

$$D_{2_t} = \{w_{i,j}^{(t)}/(i,j) \in S\} \quad (12)$$

with the coefficients $w_{i,j}^{(t)}$ defined by :

$$w_{i,j}^{(t)} = \frac{1}{2}(w_{j,-i}^{(n)} + w_{-j,i}^{(n)}) = \frac{1}{2}(m_{j,-i} + m_{-j,i}) - \overline{m} \quad (13)$$

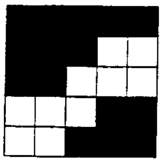
The estimation of the second order derivative along t $(\partial^2 f/\partial t^2)$ $f_t^{(2)}(x, y)$ is then expressed by :

$$\begin{aligned} f_t^{(2)}(x,y) &= \sum_{(i,j)\in S} w_{i,j}^{(t)} f(x+i, y+j) \\ &= \sum_{(i,j)\in S} [\frac{1}{2}(m_{j,-i} + m_{-j,i}) - \overline{m}] f(x+i, y+j) \end{aligned} \quad (14)$$

Finally the local curvature $c(x, y)$ is taken as the absolute value of the output $f_t^{(2)}(x, y)$:

$$c(x,y) = |f_t^{(2)}(x,y)| \quad (15)$$

On figure 2, we illustrate how the different filter masks (M, D_{2_n} and D_{2_t}) are adapted to the local structure.



support 5*5

$$M = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$D_{2n} = \frac{1}{25}\begin{bmatrix} -2 & -2 & -2 & -2 & -2 \\ -2 & -2 & -2 & 3 & 3 \\ -2 & -2 & 3 & 3 & 3 \\ 3 & 3 & 3 & -2 & -2 \\ 3 & 3 & -2 & -2 & -2 \end{bmatrix}$$

$$D_{2t} = \frac{1}{50}\begin{bmatrix} 1 & 6 & 1 & -4 & -4 \\ 1 & 6 & 1 & -4 & -4 \\ -4 & 1 & 6 & 1 & -4 \\ -4 & -4 & 1 & 6 & 1 \\ -4 & -4 & 1 & 6 & 1 \end{bmatrix}$$

Figure 2: Exemple of adaptive masks

3 Applications

3.1 Edge detection

In this section, we describe through a synthetise image the behavior of the new edge detector. In order to improve the noise robustness the image is first processed by an adaptive lowpass filter using the coefficients $m_{i,j}$ [1, 2] :

$$f_1(x, y) = \frac{\sum_{(i,j)\in S} m_{i,j}\ f(x+i, y+j)}{\sum_{(i,j)\in S} m_{i,j}} \qquad (16)$$

The synthetise image is made up of a set of dark triangles on a bright area (figure 3). These triangles are increasingly smoothed along the horizontal direction, and different amounts of noise have been added along the vertical direction (gaussian noise of variance $\sigma =$ 0, 30, 60, 90, 120). The image is processed with a succession of four lowpass filtering (size 5×5, $\Delta = 140$). Then the derivative (D_{2_n}) is applied (size 9×9, $\Delta = 140$) followed by a zero-crossing (threshold $T = 30$). The result is presented in figure 4. One can observe that edges are detected even in the noisy and smoothed part of the image.

In the second application, we show the edge detection of the image Boat (figure 5). The image is processed with a 5×5 filter mask, a parameter $\Delta = 30$ and a threshold $T = 3$. The result, presented in figure 6, shows the robust edge detection.

3.2 Curvature detection

We illustrate the curvature detection through the synthetise image composed of a square, a circle and an ellipse (figure 7). The image is processed with a filter mask D_{2_t} of size 3×3 with $\Delta = 30$. One can observes on figure 8 that all points having large curvature have been detected.

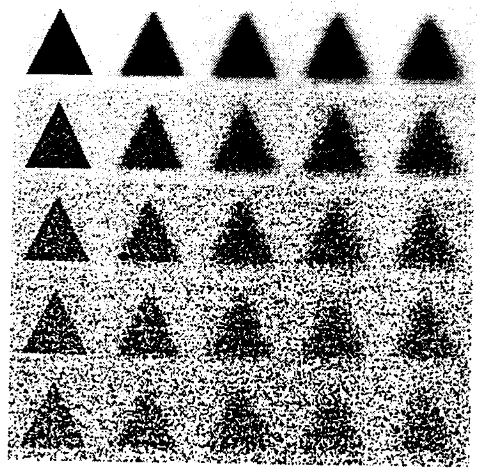


Figure 3: Synthetise image

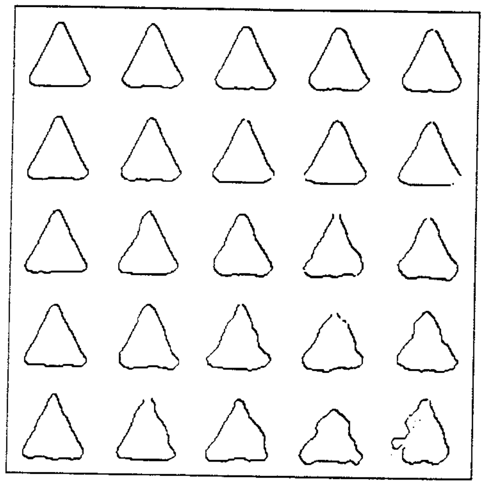


Figure 4: Edge Detection ($K_1 = K_2 = 9, \Delta = 140, T = 30$)

Figure 5: Boat image

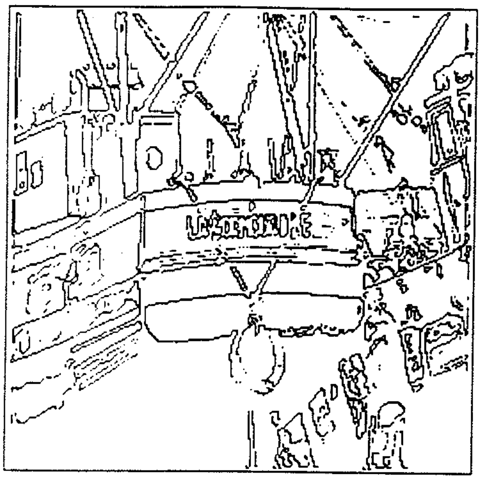


Figure 6: Detection ($K_1 = K_2 = 5, \triangle = 30, T = 3$)

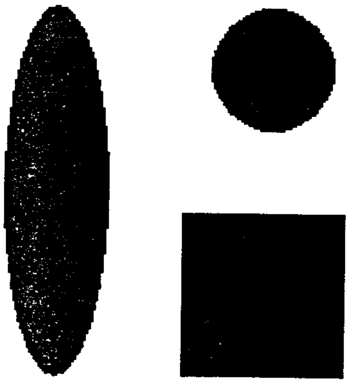


Figure 7: Synthetise image

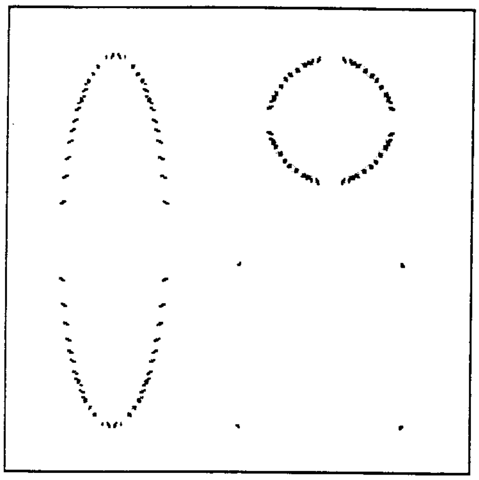


Figure 8: Curvature detection ($K_1 = K_2 = 3, \triangle = 30$)

4 Conclusion

The new adaptive operator achieves unbiased second order derivatives in order to improve edge and morphological curvature detection. It can be noted that no gradient estimation is needed to define the local edge orientation. Furthermore results provided on synthetise and real-world images show that edges proximity has no influence on their detection, even when a lowpass filter is applied.

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