IMAGE DECOMPOSITION CAPABILITIES OF THE JOINT WAVELET AND RADON TRANSFORM

*G. Olmo, L. Lo Presti, E. Magli**
Dipartimento di Elettronica, Politecnico di Torino
Corso Duca degli Abruzzi 24 - 10129 Torino - Italy
Ph. +39-11-5644033; fax: +39-11-5644099
e-mail: authorname@polito.it

ABSTRACT

It is known that the detection of segments in digital pictures can be effectively performed by means of the Radon Transform (RT). We describe a post-processing method based on wavelets, which provides information to be used in the recognition task. We show that the RT can also be successfully applied to the detection of rectangles, and is able to provide information on their width. A further generalization is described in the case of the detection of circular and square spots, along with an application to the ship and wake detection in aerial images of sea regions.

1 INTRODUCTION

In this paper we discuss the topic of pattern detection and classification in digital images. In particular, we want to focus our attention first on linear features, i.e. segments and rectangles, and then on more general shapes, such as circular and square spots. When the pictures are taken from aerial vehicles and represent land or sea regions, these features can be thought of as man-made objects; pattern detection and recognition capabilities can then be exploited for surveillance purposes. In particular, we will refer to the problem of inferring the presence of sailing ships in aerial images through the search for their segment-like wakes; an example of application will be described in Sect. 6.

An image processing tool which is well suited for this kind of application is the Radon transform (RT). However, the RT does not provide sufficient information for both detection and recognition purposes; it is then necessary to perform a further analysis in order to be able to associate the proper shape to each of the detected patterns. Therefore we propose a new technique, based on the Continuous Wavelet Transform (CWT) that can be employed in this classification task.

2 THE RADON TRANSFORM

Let $f(x,y)$ be an image. Its continuous RT is defined in [1] as:

$$\widehat{f}(\rho,\theta)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\delta(x\cos\theta+y\sin\theta-\rho)dxdy$$

with $\theta \in [0,\pi]$ and $\rho \in [-\infty,\infty]$. The $\rho-\theta$ space will be referred to as the *Radon space* (RS).

The energy concentration capability of this transform gives rise to two major properties:

1. First, thin segments in the image give rise to narrow peaks in the RS.
2. Second, an image which is non-zero in a single point has an RT which is non-zero along a sinusoidal curve.

The first property can be advantageously exploited for the detection of segments; in fact, a thresholding operation can be performed, which selects the highest peaks in the RS. However, due to the noise and background clutter, a further analysis is generally required in order to discriminate a peak due to a segment from a noise peak. It must be pointed into evidence that the analysis can be performed by using a digital version of the RT, based on the fast, FFT-based algorithm described in [3].

3 WAVELET-BASED PROCESSING

A post-processing technique for the peak analysis is now described, which makes information available, to be exploited in both detection and recognition; the method is based on the *Continuous Wavelet Transform* [2].

The algorithm stems from the consideration that, in the RS, the peaks which are due to segments always exhibit the same wave shape, except for a *scale* factor; therefore each peak shape can be compared to an ideal one, in order to see if they match. For both computational and performance considerations, it is convenient to take into account only a one-dimensional section of the peak shape along the ρ direction, as in that case, due to the computation algorithm, the wave shape is much more definite.

*The authors are with the *Signal Analysis and Simulation* group at *Politecnico di Torino*

The comparison between the ideal and real peak shapes can be effectively performed by means of a peculiar kind of adaptive filtering, which is described in the following.

The *Continuous Wavelet Transform* (CWT) of a signal $s(x)$ is defined [2] as:

$$CWT_s(a,b) = \int_{-\infty}^{+\infty} s(x)\psi^*_{a,b}(x)dx$$

with:

$$\psi_{a,b}(x) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{x-b}{a}\right)$$

It can be noticed that the CWT can be regarded as the convolution between the signal itself and a set of filters, whose impulse responses are scaled versions of a single prototype, namely the wavelet function. This involves that the filtering is adapted not only to the wave shape of the signal (i.e. the peak shape), but also to its scale factor; clearly, the wavelet function must possibly match the peak shape.

Among all the possible choices, the *Gaussian*[1] and *Mexican Hat*[2] wavelets have yield the best results. In Fig. 1 they are compared with the typical 1-D peak shape. It can be noticed that, in the case of a *segment*, the peak shape has narrow support and fast decay.

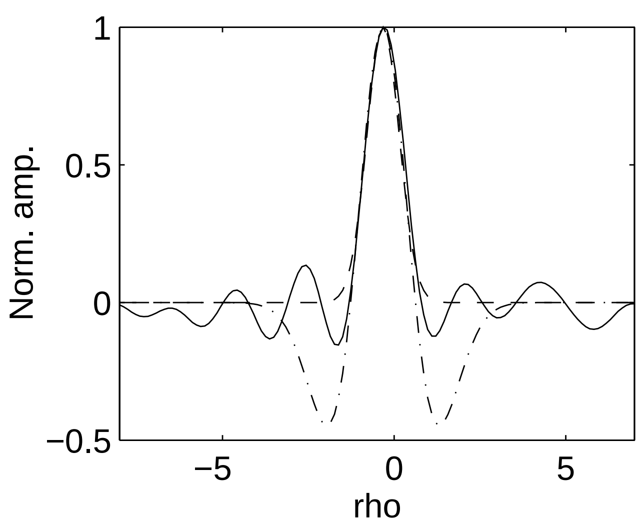

Figure 1: Comparison between the peak shape and the wavelet functions. Solid: peak shape; dashed: Gaussian wavelet; dashdot: Mexican Hat wavelet.

The complete *linear feature detection algorithm* has been detailed in [3], and is summarized here for clarity. The algorithm consists of the following steps:

[1]The Gaussian function is not actually a wavelet, as it is not a bandpass function; on the contrary it performs a lowpass filtering.

[2]This wavelet is the second derivative of a Gaussian

1. A threshold is established in the RS. Each peak of the RS which has exceeded the threshold is subject to the CWT algorithm.

2. In particular, for each of these peaks a short data vector, centered around the peak and expanding in the ρ direction, is extracted from the RS. Then the CWT of this vector is computed.

3. The maximum of the CWT occurs at the best matching between the peak shape and the wavelet function. In addition, the value of the scale variable a at which this maximum occurs is proportional to the width of the linear feature.

4. The value of this maximum is regarded as a measure of the probability that the peak is due to a linear feature. Therefore the decision about the presence of the linear feature can be made more reliable by computing the CWT maxima related to more than one wavelet; in fact, it will be shown in the following that different wavelet functions yield a different characterization of the peak shapes (and thus of the objects which have raised them).

4 GENERALIZATION TO RECTANGLES

Let us now consider an image containing a *rectangle*, that is, a segment with a non-zero width. It can be proved [4] that the RT of such an image consists of an amplitude distribution centered around a peak, as in the case of the segment. The difference is that, in the ρ direction, the peak has now a wider support, and its shape is rectangular instead of triangular. Hence it follows that, in order to detect rectangles, the same kind of CWT analysis may be performed, provided that a suitable wavelet is used (for instance the *Haar* wavelet[3]). Fig. 2 compares the peak shape of the RT of a rectangle with the Haar and the Mexican Hat wavelet. It can be seen that the Haar wavelet is well matched to the peak shape. However, when the image is corrupted by noise, the Mexican Hat wavelet is also suitable as the decay of the peak shape is less abrupt.

An example on a real images is now provided. Fig. 3 shows an image containing a river of nearly constant and rectangular shape (a bridge is also visible).

It can be seen that the peak shape related to the river, which is shown in Fig. 4, is nearly rectangular (even if not perfectly due to the background clutter). The better matching of the Haar with respect to the Mexican Hat wavelet is confirmed by the related CWT maxima (0.97 versus 0.91).

[3]It can be noticed that the Haar wavelet is not fully matched to the peak shape; this is not a problem, as during its negative, unmatched wave the peak shape is expected to be very close to zero. However, for comparison purposes, the related CWT maxima must be normalized so as to take into account this effect. A normalization coefficient of 0.77 is suitable (see [4] for details)

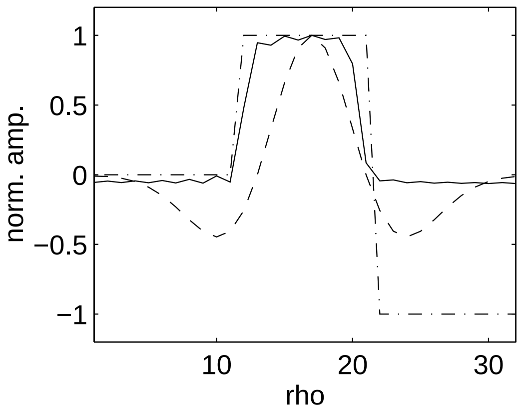

Figure 2: Comparison between the peak shape and the wavelet functions in the case of a rectangle. Solid: peak shape; dashed: Haar wavelet; dashdot: Mexican Hat wavelet

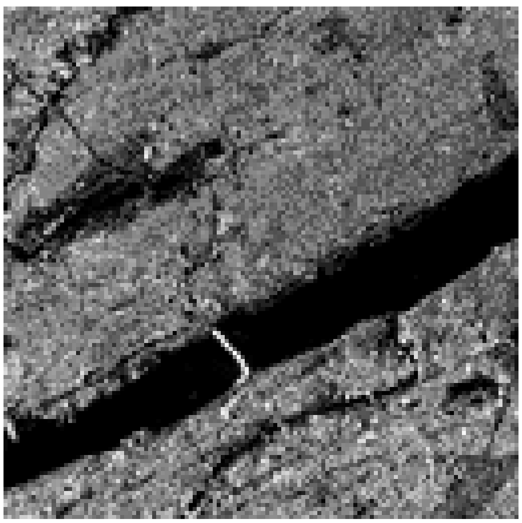

Figure 3: Image containing a river

5 GENERALIZATION TO CIRCULAR AND SQUARE SPOTS

It can be seen that the RT of a picture containing a circular spot assumes high values along a sinusoidal-shaped region of the RS, with a non-zero extent in the ρ direction. This is due to the property of the RT of mapping a point in the image onto a sinusoid in the RS. If the coordinates (x_0, y_0) of the center of the circle in the image are known, then the equation of the average sinusoidal curve in the RS is given by $\rho = x_0 \cos\theta + y_0 \sin\theta$.

The RT of an image containing a square spot can be found by superimposing two effects. The former is due to the inscribed circle, and forces a sinusoidal distribution to appear in the RS; the latter is produced by the remaining part of the square, and causes a modulation of the extent of the distribution in the ρ direction.

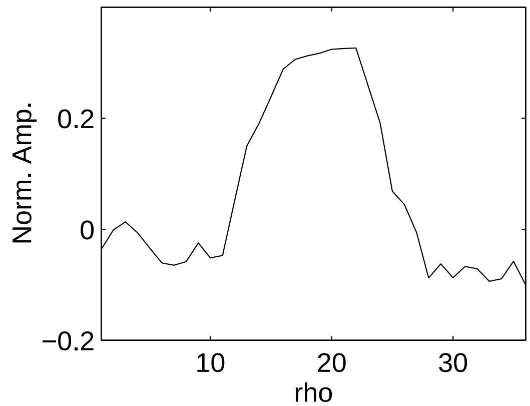

Figure 4: Wave shape of the peak related to the river

6 EXAMPLE OF APPLICATION: SHIP AND WAKE DETECTION

The previous results can be advantageously applied to the problem of the ship wake detection. Such images usually contain two "V" shaped segments (the so-called Kelvin wake), while the ship appears as a spot at the intersection of the segments. Many authors suggest using preprocessing techniques, in order to avoid that the effects of the spot superimpose to those of the segments in the RS . Instead, the analytical knowledge of the sinusoidal curve may be exploited in order to verify the presence of the ship itself in addition to the wake. Fig. 5-b shows the RT of a synthetic image containing two segments in the Kelvin wake configuration plus the spot. It is clear that the sinusoidal distribution may be extrapolated from the RT coefficients. The presence of two maxima along this curve can be exploited to detect the ship in addition to the wake; the following steps are suitable:

1. Detection of the segments and computation of the coordinates (x_0, y_0) of their intersection, i.e. the cusp of the wake where the ship possibly lies.
2. Extraction of the related sinusoidal curve of the RS
3. Analysis of the wave shape of this curve. A ship is reliably detected if:
    - Two peaks are present along the wave shape
    - A "floor", due to the effect of the spot, is also present

An example is reported in Fig. 5; the plots of Fig. 5-c and 5-d are related to the image of Fig. 5-a, respectively without and with the spot representing the ship. It can be easily seen that the effect of the spot is to raise a floor in the extracted curve.

7 CONCLUSIONS

A processing technique has been proposed, based on the wavelet analysis of the peaks in the RT of an image. It

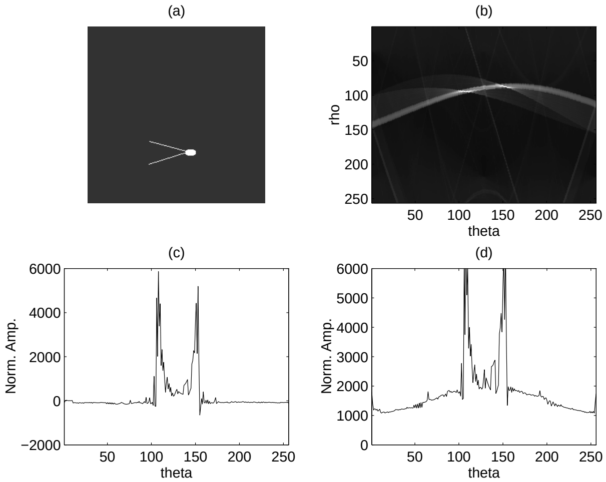

Figure 5: (a) Image of a wake plus the ship (b) Its RT (c) Extracted curve when the spot is absent (d) Extracted curve when the spot is present

has been found that the high resolution in both space and frequency of the CWT can be exploited for adaptive filtering and classification at the same time, as the *scale* at which the best matching between the peak and the wavelet occurs is proportional to the width of the detected pattern; this makes possible a generalization to the case of rectangles. Moreover, the presence of circles and squares can be inferred through the analysis of the sinusoidal-shaped distributions that they create in the RS. This allows for an increase of performance in the application of the ship wake detection.

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