IDENTIFICATION OF LINEAR TIME-VARIANT SYSTEMS BY SPECTRAL CORRELATION MEASUREMENTS

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ABSTRACT

A nonparametric algorithm for the identification of linear time-variant systems is proposed. The class of systems considered maps almost-periodic inputs into almost-periodic outputs and includes as a special case the linear almost-periodically time-variant systems and, hence, the linear time-invariant systems. The proposed identification algorithm is based on spectral cross-correlation measurements between the input and output signals. It provides, under mild conditions, at least in principle, an arbitrarily accurate estimate of the system transmission function, provided that a sufficiently long collect time is considered.

1 INTRODUCTION

The problem of identifying a system on the basis of noisy measurements of the input and output signals occurs in several fields including radar, sonar, seismology, ocean acoustics, communications and others. For example, the use of digital communication systems characterized by higher and higher bit rates requires appropriate channel modeling to efficiently counteract the degrading effects of intersymbol interference and selective fading arising from the presence of Doppler-shifted and distorted replicas of the transmitted signal.

Several identification algorithms have been proposed in the literature with reference to linear time-invariant (LTI) channels. Moreover, the recent development of signal processing algorithms based on the cyclostationarity properties of the input signal has determined a growing interest in the linear almost-periodically time-variant (LAPTV) systems [1], [2].

Very recently, the systems have been classified as deterministic and random in the fraction-of-time probability framework [3], [4]. Specifically, deterministic systems (called "stationary" in [5]) are those that transform almost-periodic inputs into almost-periodic outputs. In particular, the output of deterministic linear time-variant (LTV) systems is constituted by frequency compressed or stretched and filtered versions of the input. Deterministic LTV systems include, as special cases, LAPTV systems (and, hence, LTI systems), systems performing time scale changing, and Doppler multipath channels.

In the present paper a nonparametric algorithm for the identification of a deterministic LTV system is proposed. It exploits measurements of the spectral cross-correlation between the (possibly noisy) input and output signals. Specifically, under the assumption that the input and output noise signals are uncorrelated, it is shown that the spectral cross-correlation function between the input and the output signals turns out to be proportional to the transmission function that characterizes the LTV system. Moreover, it is pointed out that, even if the input signal is wide-sense stationary or almost cyclostationary, the input and output signals, in general, are neither jointly stationary nor almost cyclostationary and, hence, the spectral cross-correlation function cannot be estimated by the time-smoothed and frequency-smoothed cyclic periodogram methods. Therefore, an appropriate estimator is introduced by considering the discrete-time counterpart of the cyclic demodulation method presented in [6] with reference to cyclostationary signals. Such an estimator leads, at least in principle, to an arbitrarily accurate estimate of the system transmission function, provided that a sufficiently long collect time is available. Finally, simulation results are reported in order to corroborate the effectiveness of the proposed identification algorithm.

2 DETERMINISTIC LTV SYSTEMS

In the fraction-of-time probability context, a deterministic system is a possibly complex (and not necessarily linear) system that for every deterministic (i.e., constant, periodic, or polyperiodic) input time-series delivers a deterministic output time-series. Therefore, for a system input time-series

$$x(t) = e^{j2\pi\lambda t}, \tag{1}$$

the system output time-series $y(t)$ can be expressed as

$$y(t) = \sum_{\sigma\in\Omega} G_\sigma(\lambda) e^{j2\pi\varphi_\sigma(\lambda)t}, \tag{2}$$

where Ω is a finite or denumerable set and $G_\sigma(\cdot)$ and $\varphi_\sigma(\cdot)$ are complex functions and monotonic real functions (respectively) that characterize the system.

According to (1) and (2), for a deterministic linear time-variant system the transmission function [3], [5] can be written as

$$\begin{aligned} H(f,\lambda) &= \sum_{\sigma\in\Omega} G_\sigma(\lambda)\,\delta(f-\varphi_\sigma(\lambda)) \\ &= \sum_{\sigma\in\Omega} H_\sigma(f)\,\delta(\lambda-\psi_\sigma(f)), \end{aligned} \tag{3}$$

where $\delta(\cdot)$ is Dirac's delta function, the functions $\psi_\sigma(\cdot)$, referred to as the frequency mapping functions, are the inverse functions of $\varphi_\sigma(\cdot)$, and

$$H_\sigma(f) \triangleq |\dot{\psi}_\sigma(f)|\,G_\sigma(\psi_\sigma(f)), \tag{4}$$

in which $\dot{\psi}_\sigma(\cdot)$ denotes the derivative of the function $\psi_\sigma(\cdot)$. In other words, the input/output relationship can be expressed as

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} h(t,u)\,x(u)\,\mathrm{d}u \\ &= \sum_{\sigma\in\Omega} h_\sigma(t)\otimes x_{\psi_\sigma}(t), \end{aligned} \tag{5}$$

where $h(t,u)$ is the system impulse-response function, $h_\sigma(t)$ is the inverse Fourier transform of $H_\sigma(f)$, $\otimes$ denotes convolution, and

$$x_{\psi_\sigma}(t) \triangleq \int_{-\infty}^{+\infty} X(\psi_\sigma(f))\,e^{j2\pi ft}\,\mathrm{d}f, \tag{6}$$

in which $X(f)$ is the Fourier transform of $x(t)$. Then, the output signal is constituted by frequency compressed or stretched and filtered versions of the input signal.

The class of deterministic LTV systems includes that of the LAPTV systems which, in turn, includes the LTI systems. For the LAPTV systems, the frequency mapping functions $\psi_\sigma(f)$ are linear with unitary slope, that is,

$$\psi_\sigma(f) = f-\sigma, \qquad \sigma\in\Omega, \tag{7}$$

and then the impulse-response function can be expressed as

$$h(t,u) = \sum_{\sigma\in\Omega} h_\sigma(t-u) e^{j2\pi\sigma u}. \tag{8}$$

The systems performing time scale changing belong to the class under consideration. In such a case, the impulse-response function is given by

$$h(t,u) = \delta(u-at), \tag{9}$$

where $a \neq 0$ is the scale factor, the set Ω contains just one element, and

$$\psi_\sigma(f) = \frac{f}{a}, \qquad H_\sigma(f) = \frac{1}{|a|}. \tag{10}$$

Finally, let us note that linear time-variant systems that cannot be modeled as deterministic include chirp modulators, modulators whose carrier is a pseudo-noise sequence (as in the spread-spectrum modulation), channels introducing a time-varying delay, and systems performing time windowing.

3 THE IDENTIFICATION METHOD

In this section, a nonparametric method for the identification of deterministic LTV systems is proposed. It provides an estimate of the system transmission function (3) based on the spectral cross-correlation between the (possibly noisy) input and output signals.

The spectral cross-correlation function between two finite-power signals $y(t)$ and $x(t)$ is defined as

$$S_{yx}(f_1,f_2) \triangleq \lim_{\Delta f\to 0} \left\langle \Delta f\, Y_{1/\Delta f}(t,f_1)\, X^*_{1/\Delta f}(t,f_2) \right\rangle, \tag{11}$$

where $*$ and $\langle\cdot\rangle$ denote conjugation and infinite-time average operation, respectively, and $X_{1/\Delta f}(t,f)$ and $Y_{1/\Delta f}(t,f)$ are the short-time Fourier transforms (STFTs)

$$X_{1/\Delta f}(t,f) \triangleq \int_{t-1/2\Delta f}^{t+1/2\Delta f} x(u)\,e^{-j2\pi fu}\,\mathrm{d}u \tag{12}$$

and

$$Y_{1/\Delta f}(t,f) \triangleq \int_{t-1/2\Delta f}^{t+1/2\Delta f} y(u)\,e^{-j2\pi fu}\,\mathrm{d}u. \tag{13}$$

The spectral cross-correlation function represents the temporal correlation (with zero lag) between the spectral components of $y(t)$ and $x(t)$ in the bands $(f_1-\Delta f/2, f_1+\Delta f/2)$ and $(f_2-\Delta f/2, f_2+\Delta f/2)$, respectively, normalized to $1/\Delta f$, when the bandwidth Δf approaches zero.

If $y(t)$ is the output of a deterministic LTV system whose input is $x(t)$, the STFT (13), accounting for (5), can be expressed as

$$Y_{1/\Delta f}(t,f) =$$

$$\sum_{\sigma\in\Omega}\int_{-\infty}^{+\infty} h_\sigma(s) X_{\psi_\sigma,1/\Delta f}(t-s,f)\, e^{-j2\pi f s}\,\mathrm{d}s\ , \quad (14)$$

where $X_{\psi_\sigma,1/\Delta f}(t,f)$ is the STFT of the time series $x_{\psi_\sigma}(t)$. Then, by substituting (14) into (11), one obtains that

$$S_{yx}(f_1,f_2) = \sum_{\sigma\in\Omega} H_\sigma(f_1) S_{xx}(\psi_\sigma(f_1), f_2)\ , \quad (15)$$

where $S_{xx}(f_1,f_2)$ is the spectral correlation function of the time series $x(t)$ (defined by (11) with $y(t) \equiv x(t)$) and the relationship

$$\lim_{\Delta f\to 0}\left\langle \Delta f\, X_{\psi_\sigma,1/\Delta f}(t,f_1)\, X^*_{1/\Delta f}(t,f_2)\right\rangle$$

$$= \lim_{\Delta f\to 0}\left\langle \Delta f\, X_{1/\Delta f}(t,\psi_\sigma(f_1))\, X^*_{1/\Delta f}(t,f_2)\right\rangle \quad (16)$$

has been accounted for.

Let us now assume that the input signal $x(t)$ is a second-order wide-sense stationary signal with zero mean and power spectral density η, i.e.,

$$S_{xx}(f_1,f_2) = \eta\, \delta_{f_2-f_1}\ , \quad (17)$$

where δ_f denotes Kronecker's delta function. Thus, (15) becomes

$$S_{yx}(f_1,f_2) = \eta \sum_{\sigma\in\Omega} H_\sigma(f_1)\, \delta_{f_2-\psi_\sigma(f_1)}\ . \quad (18)$$

Therefore, by comparing (18) with (3), it results that the estimation of the system transmission function reduces to that of the spectral cross-correlation function. Moreover, each frequency mapping function $\psi_\sigma(f_1)$ can be determined by considering the support of $S_{yx}(f_1,f_2)$ in the (f_1,f_2) plane and the corresponding $H_\sigma(f_1)$ can be obtained by evaluating the amplitude of $S_{yx}(f_1,f_2)$ on the curve $f_2=\psi_\sigma(f_1)$.

It is worthwhile to point out that the identification formula (18) is still applicable when noisy measurements of the input and output signals are available, provided that the input and output noise signals are uncorrelated with each other and with the input signal.

4 SIMULATION RESULTS

In this section, simulation results are presented to show the effectiveness of the proposed identification algorithm, which is based on the estimation of $S_{yx}(f_1,f_2)$.

Let us observe that the considered class of LTV systems is such that even if the input signal $x(t)$ is wide-sense stationary, the two signals $y(t)$ and $x(t)$, in general, are neither jointly stationary nor jointly cyclostationary. Hence, the spectral cross-correlation function cannot be estimated by the time-smoothed and frequency-smoothed cross-periodogram methods. In fact, for a given collect time T, since $T\Delta f \gg 1$ (for reliability requirements), the former exhibits a spectral resolution Δf much greater than the desired value $1/T$. The latter performs the smoothing along lines parallel to the diagonal $f_2 = f_1$ in the (f_1,f_2) plane and, hence, it works only with time series $x(t)$ and $y(t)$ that are jointly wide-sense stationary or almost cyclostationary. Moreover, the estimator of the spectral cross-correlation function proposed in [7] can be used only when the frequency mapping functions are linear. Furthermore, when these functions are not known (as in the case of the system identification), a very time-consuming search procedure must be singled out.

An appropriate estimator of $S_{yx}(f_1,f_2)$ is obtained here by considering the discrete-time counterpart of the cyclic demodulation method presented in [6] with reference to cyclostationary signals. Such a method leads to an estimator that is the discrete-time counterpart of definition (11) where, however, the time average is evaluated on a finite collect time T and Δf is small (but nonzero). Such an estimator exhibits a spectral resolution of the order of $1/T$ and leads, at least in principle, to arbitrarily accurate estimates of the functions $\psi_\sigma(f)$ and $H_\sigma(f)$ characterizing the system, provided that a sufficiently long collect time is available and, moreover, $T\Delta f \gg 1$.

In the simulation experiment, the estimation of the spectral cross-correlation function has been carried out by discretizing both time and frequency with sampling increments $T_s = T/K$ and $\Delta f_s = 1/T$, where K is the number of samples. The LTV system to be identified is characterized by $\Omega = \{\sigma\}$ and

$$\psi_\sigma(f) = (1+v/c)\, f\ , \quad (19)$$

$$H_\sigma(f) = (1+jf/B)^{-p}\ . \quad (20)$$

It describes a Doppler channel with bandwidth B and constant relative radial speed v between trasmitter and receiver operating in a medium whose propagation speed is c. In (19) and (20), $B = 0.125/T_s$, $p = 8$, and $v/c = 1/15$ have been assumed. The input signal $x(t)$ is white Gaussian noise whose discrete-time version has unitary variance. Both input and output signal measurements are contaminated by additive white Gaussian noises independent with each other and independent of $x(t)$. The signal-to-noise ratio at both the input and the output is 20dB.

Figure 1 shows the support in the (f_1,f_2) plane of the spectral cross-correlation function evaluated on the basis of $K = 2^{11}$ samples and assuming $\Delta f = 1/128T_s$. The curve $f_2 = \psi_\sigma(f_1)$, i.e., the line

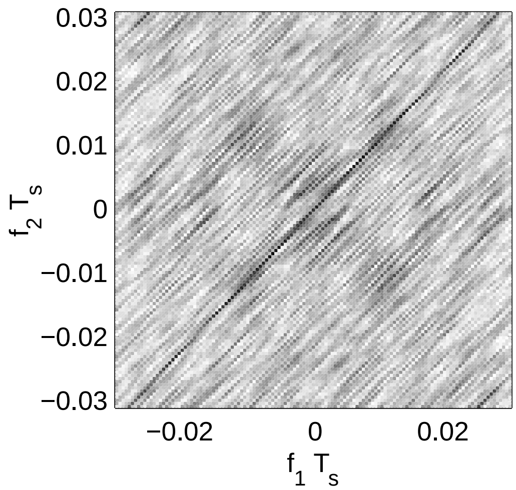

Figure 1

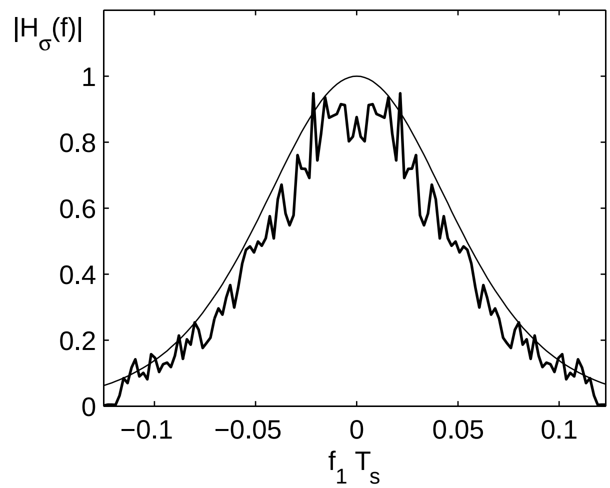

Figure 2

with slope $1 + v/c = 16/15$, is easily recognizable. In Fig. 2, the magnitude of the slice of $S_{yx}(f_1, f_2)$ with $f_2 = 16f_1/15$ evaluated by $K = 2^{13}$ samples and $\Delta f = 1/512T_s$ (thick line) is reported. For comparison purpose, the magnitude of the true $H_\sigma(f)$ (thin line) is also represented.

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