CURVATURE VARIATION OF PROJECTED CROSS-SECTIONS FROM STRAIGHT UNIFORM GENERALIZED CYLINDERS

William Puech[1] *and Jean-Marc Chassery*[2]
1 Modelisation and Signal Laboratory, University of Toulon
Av. G. Pompidou BP 56, 83162 La Valette du Var Cedex, FRANCE,
2 TIMC-IMAG laboratory, Grenoble University
Domaine de la Merci, 38706 LA TRONCHE Cedex, FRANCE,
e-mail: puech@univ-tln.fr, jean-Marc.Chasssery@imag.fr

1 INTRODUCTION

In this paper, we present a new approach for reconstructing Straight Uniform Generalized Cylinder (SUGC) with a scene mapped on its surface. In monocular vision, by using a priori knowledge about the support surface of pictures and projected cross-sections we reconstruct the surface in order to backproject the image on the surface. To reconstruct the surface we have to analyze the curvature of several projected cross-sections. An overview on this domain is found in the papers [Eggert 93, Huang 95, Chen 96]. The curvature variation analyses of surfaces [Kåsa 96] and projected curves [Faugeras 95, Bruce 96] show the complexity of this problem.

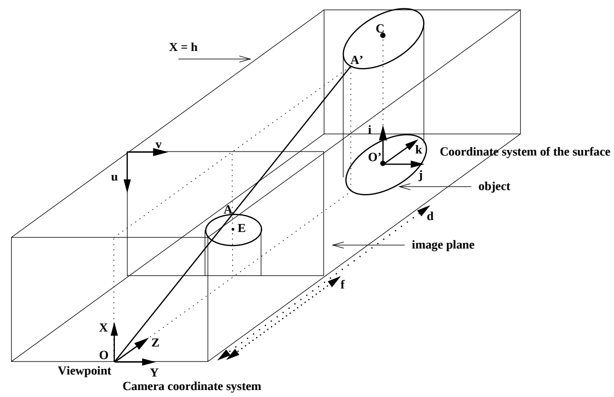

Figure 1: *A circular crosssection from a SUGC projected on the image plane*

2 OBTENTION OF AN ELLIPSE FROM A CYLINDER

In this Section we show how to obtain an ellipse from a cross-section of SUGC projected on the image plane. When circular cross-sections from a SUGC are projected on the image plane, ellipses with different parameters are obtained. We want to analyse the curvature of points belonging to the meridian the nearest from the view-point. In the camera coordinate system shown on Figure 1, a cylinder is given by:

$$\left\{ \begin{array}{c} (z - z_c)^2 + y^2 = R^2 \\ -H \leq x \leq H, \end{array} \right. \qquad (1)$$

where R is the radius of the cylinder, $2H$ the height of the cylinder and $z_c = d + R$, with d the minimal distance between the view-point and the cylinder. These distances are shown on the Figure 2.

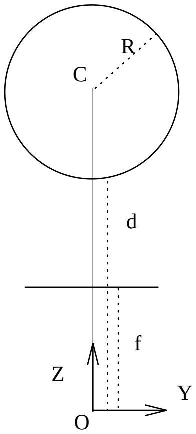

Figure 2: *Distances between the viewpoint O and the cylinder in the plane yOz*

If we take a value h for x, we obtain a circle $\mathcal{C}$, as shown on the Figure 1, belonging to the cylinder defined by (1). The equation of this circle in the camera coordinate system is :

$$\begin{cases} (z-z_c)^2+y^2=R^2 \\ x=h. \end{cases} \quad (2)$$

We construct the cone passing throught $\mathcal{C}$ and the view-point O. The equation of this cone is :

$$(\frac{zh}{x}-z_c)^2+\frac{y^2h^2}{x^2}=R^2. \quad (3)$$

We have now to calculate in the camera coordinate system, the intersection of the surface, defined by the equation (3), with the image plane $z=f$, where f is the focal distance. This intersection, corresponding to the projection of the circle $\mathcal{C}$ on the image plane, is :

$$(fh-xz_c)^2+h^2y^2=R^2x^2. \quad (4)$$

After some modifications, we obtain :

$$(x^2-\frac{fhz_c}{z_c^2-R^2})^2+\frac{h^2y^2}{z_c^2-R^2}+\frac{f^2h^2}{z_c^2-R^2}-\frac{f^2h^2z_c^2}{z_c^2-R^2}=0. \quad (5)$$

The intersection of the cone with the image plane $z=f$, gives us the equation of an ellipse :

$$\begin{cases} \frac{(x-x_e)^2}{a^2}+\frac{y^2}{b^2}=1 \\ z=f, \end{cases} \quad (6)$$

where the three parameters a, b and x_e, are :

$$\begin{cases} a & = & \frac{Rfh}{d^2+2dR} \\ b & = & \frac{Rf}{\sqrt{d^2+2dR}} \\ x_e & = & \frac{fh(d+R)}{d^2+2dR}, \end{cases} \quad (7)$$

with f, the focal distance of the camera.

3 CURVATURE VARIATION

In this Section, we analyse the curvature variation of a SUGC with circular cross-sections, when it is projected on the image plane.

3.1 Curvature of an ellipse

In this Section, we calculate the curvature of an ellipse in function of its parameters.

Ellipses detection is a problem in image processing [Pilu 96]. Methods using the Hough transform [Bulot 96, Chatzis 96] are developped for ellipses detection. We define in the image plane an ellipse of center $E(x_e, y_e, f)$. The equation of this ellipse in the camera coordinate system is :

$$F(x,y)=\frac{(x-x_e)^2}{a^2}+\frac{(y-y_e)^2}{b^2}-1 \quad (8)$$

The derivatives of the equation (8) are :

$$\begin{cases} \frac{\delta F(x,y)}{\delta x} & = & \frac{2(x-x_e)}{a^2} \\ \frac{\delta^2 F(x,y)}{\delta x^2} & = & \frac{2}{a^2} \\ \frac{\delta^2 F(x,y)}{\delta xy} & = & 0 \\ \frac{\delta F(x,y)}{\delta y} & = & \frac{2(y-y_e}{b^2} \\ \frac{\delta^2 F(x,y)}{\delta y^2} & = & \frac{2}{b^2} \\ \frac{\delta^2 F(x,y)}{\delta yx} & = & 0 \end{cases} \quad (9)$$

The equation of the curvature K to a point of the ellipse is :

$$K=\frac{\begin{vmatrix} F_{xx} & F_{xy} & F_x \\ F_{yx} & F_{yy} & F_y \\ F_x & F_y & 0 \end{vmatrix}}{(F_x^2+F_y^2)^{\frac{3}{2}}}, \quad (10)$$

with $F_i=\frac{\delta F(i,j)}{\delta i}$ et $F_{ij}=\frac{\delta^2 F(i,j)}{\delta^2 ij}$.

With the equations (9) and (10), we obtain for the curvature of the ellipse this equation :

$$K=\frac{\left(\frac{2(x-x_e)}{a^2}\right)^2\frac{2}{b^2}+\left(\frac{2(y-y_e)}{b^2}\right)^2\frac{2}{a^2}}{\left[\left(\frac{2(x-x_e)}{a^2}\right)^2+\left(\frac{2(y-y_e)}{b^2}\right)^2\right]^{\frac{3}{2}}}. \quad (11)$$

In the particular case of the equation (6), the center of the ellipse is $y_c=0$, because we have any rotation between the image coordinate system and the coordinate system of the cylinder. The coordinate of the particular points of this ellipse, belonging to the projection of the meridian the nearest of the view-point are $x=x_e\pm a$ and $y=0$. There are two particular points with $A(x_a,0,f)$, where $x_a=x_e+a$ (Figure 1). The value of the curvature K of these two points is :

$$K=\frac{a}{b^2} \quad (12)$$

3.2 Variation of the curvature of the ellipse

From the equation (12) we are able to prove than the curvature K of the point A depends linearly of the altitude h of the circle $\mathcal{C}$, cross-section of the cylinder. With the equations (7) and (12), we obtain :

$$K = \frac{h}{fR}. \qquad (13)$$

The Figure 3 shows the visible parts of ellipses, projections of circles from a cylinder with the radius $R = 100\ mm$, the focal distance $f = 60\ mm$, $d = 500\ mm$ and $H = 250\ mm$. We have selected 11 circles to represente these projections.

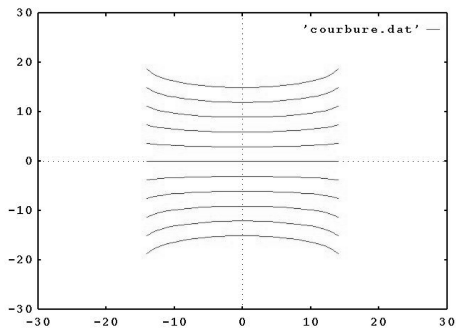

Figure 3: *Evolution of the curvature K with $R = 100mm$ $f = 60mm$ $d = 500mm$ and $H = 250mm$ for $x = \frac{-5H}{5}, \frac{-4H}{5}, ..., \frac{4H}{5}, \frac{5H}{5}$*

4 SIMULATION RESULTS

In this section we illustrate the efficiency of the reconstruction method on a real example. This example consists of a painting on a vault in a church shown in Figure 4.

Figure 4: *View of an arch of a Byzantine church*

Figure 5 illustrates the result of an edge detection on the image shown Figure 4. The projection of the axe and the curve are shown in the Figure 6. The reconstructed surface is shown in Figure 7.

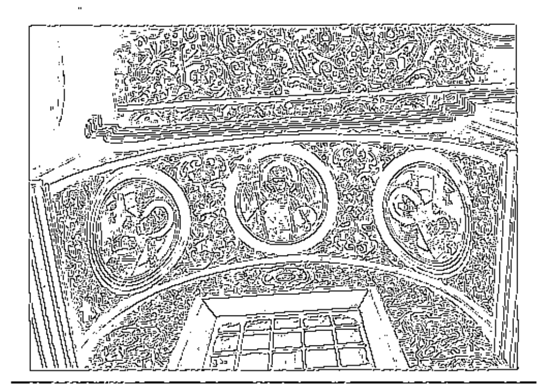

Figure 5: *Edge detection of the Figure 4*

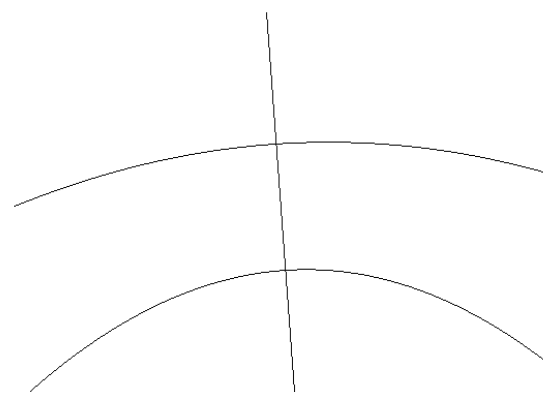

Figure 6: *Curves and axe from the Figure 5*

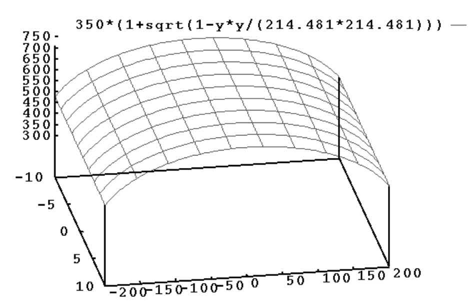

Figure 7: *Reconstruction of the straight uniform generalized cylinder*

We are the able to reconstruct different views of the painting as depicted in Figures 8 and 9.

Figure 8: *A synthetic visualization*

Figure 9: *An other synthetic visualization*

5 CONCLUSION

So, for an image, the focal distance f and the radius R of the cylinder are fixed, we have shown the curvature K of the point A, belonging to the meridian the nearest from the view-point, changes linearly with the altitude h of the plane of this cross-section.

In this paper we show the possibility to analyse the curvature and to reconstruct a generalized cylinder. We have developed also the problem of rotations between the camera coordinate system and the coordinate system of the Generalized Cylinder. This work is applied for the visualization of painting on columns or vaults [Puech 97]. In this case, we simulate a camera to obtain new images from different viewpoints.

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