Adaptive Separation of Sources and Estimation of Their Directions Of Arrival

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ABSTRACT

We consider array processing when the multiple sources are statistically independent narrowband signals, as in CDMA radio communications with a multisensor receiver. For a uniform linear array, a simple source separation-based criterion is proposed which jointly recovers each source on one output channel. The corresponding system has two stages whose respective parameters are the estimates of the source directions of arrival (DOA) and certain gains which constrain the output powers. The criterion involves a coupling parameter α so that all outputs recover different sources. When all the sources have negative kurtosis, the criterion minima correspond to perfect estimation of the DOA. Adaptive implementation is possible at a very low computational price. After initialization, the coupling parameter α can be cancelled, which greatly simplifies implementation. These results have been confirmed through many computer simulations.

1 Introduction

The problem of designing an array that is capable of finding the Directions Of Arrival (DOA) of several (say N) coherent narrowband signals is a difficult one. This problem can be viewed as the design of N beamformers which are automatically steered in the N signal directions. The first idea is to independently steer several beamformers which all extract one coherent signal. In this family is the work [1], where optimization is adaptive and based on fourth order properties of the extracted signal, through the Godard (or "Constant Modulus") algorithm. Recently the problem of jointly recovering the N coherent signals with their DOA has been tackled with the new approach of unsupervised (blind) "source separation". Source separation takes advantage of the statistical independence between the N source signals to jointly recover all of them. In the previous contributions of this kind, e.g. [2], [3], the DOA are estimated at the output of two cascaded subsystems. The first subsystem is a general structure matrix whose task is source separation. It is optimized thanks to any independence criterion and thus generates a pure source on each of its N output channels. Then the DOA (and possibly the range) of each source signal is estimated independently on each channel. This procedure is necessarily suboptimal, because the (first) task of source separation does not take into account all the available information concerning the nature of sources and the propagation model.

In this paper, we take advantage of the model for narrowband space-localized source signals measured by a linear array, to avoid the first subsystem. There is indeed a first stage which recovers one pure source on each output thanks to a novel independence criterion, but the parameters of this first stage are directly the estimated DOA. The second stage simply involves N decoupled gains, one for each recovered source. It is an auxiliary stage useful for the separation method of the first stage which requires suitable power constraints.

Optimization of our source separation criterion can be conducted by an adaptive (stochastic gradient) algorithm. It has a much lower computational cost than previous methods and it ensures the capability of tracking moving sources.

2 Structure of the system

2.1 The signal model

Let $\mathbf{x} = (x_1, \ldots, x_N)^T$ be the vector of signals measured by a uniform linear array of N equidistant sensors. If $\mathbf{s} = (s_1, \ldots, s_N)^T$ denotes the vector of the N independent narrowband sources s_i, each having θ_i as respective DOA, then

$$\mathbf{x} = A\Gamma^{1/2}\mathbf{s} + \mathbf{w} \tag{1}$$

where the noise vector $\mathbf{w}$ is assumed Gaussian, zero-mean and independent of $\mathbf{s}$; where the matrix $\Gamma = \mathrm{diag}(\gamma_1, \ldots, \gamma_N)$ contains the (positive) attenuations of the sources ; $A = (\mathbf{a}(\varphi_1), \ldots, \mathbf{a}(\varphi_N))$ is the $N \times N$ matrix of steering vectors, i.e., $\mathbf{a}(\varphi) = (1, e^{j\varphi}, \ldots, e^{j(N-1)\varphi})^T$ where the angle φ_i is related to the DOA θ_i of s_i through $\varphi_i = K \sin\theta_i$. The constant K depends on the frequency of the narrowband signals and is worth π for $\lambda/2$-spaced sensors.

2.2 The first stage

If good estimates ϕ_j of the true angles φ_i are available, the mixing effect of the steering matrix A can be corrected by a first stage that calculates the vector

$$\mathbf{z} = H(\phi_1, \ldots, \phi_N)\mathbf{x} \tag{2}$$

in such a way that

$$H(\phi_1, \ldots, \phi_N)A = P\Lambda \tag{3}$$

where P is an arbitrary $N \times N$ permutation matrix (i.e. P has one and only one nonzero entry per row and per column and this entry is worth 1) and Λ is any regular diagonal matrix. Then

$$z_i = \lambda_{p(i)}\sqrt{\gamma_{p(i)}}s_{p(i)} + w'_i \tag{4}$$

Here $p(i)$ denotes the column where the entry 1 stands in the i-th row of P. Thus $\{p(1), \ldots, p(N)\}$ is a permutation of $\{1, \ldots, N\}$. The noise vector in (4) is $\mathbf{w}' = H(\phi_1, \ldots, \phi_N)\mathbf{w}$. It is Gaussian and zero mean.

Starting with $H^1 = 1$, we define the matrix H iteratively:

$$\begin{aligned} H^q(\phi_1, \ldots, \phi_q) &= \begin{bmatrix} e^{-j\phi_q}H^{q-1}(\phi_1, \ldots, \phi_{q-1}) & 0 \\ e^{-j\phi_{q-1}}\mathbf{h}^{q-1}(\phi_1, \ldots, \phi_{q-1})^T & 0 \end{bmatrix} \\ &+ \begin{bmatrix} 0 & H^{q-1}(\phi_1, \ldots, \phi_{q-1}) \\ 0 & \mathbf{h}^{q-1}(\phi_1, \ldots, \phi_{q-1})^T \end{bmatrix} \end{aligned} \tag{5}$$

with $H(\phi_1, \ldots, \phi_N) = H^N(\phi_1, \ldots, \phi_N)$; $\mathbf{h}^{qT}$ is the last (q-th) row of the $q \times q$ matrix $H^q(\phi_1, \ldots, \phi_q)$. For instance

$$\begin{aligned} H^2 &= \begin{pmatrix} -e^{j\phi_2} & 1 \\ -e^{j\phi_1} & 1 \end{pmatrix} \\ H^3 &= \begin{pmatrix} e^{j(\phi_2+\phi_3)} & -(e^{j\phi_2} + e^{j\phi_3}) & 1 \\ e^{j(\phi_1+\phi_3)} & -(e^{j\phi_1} + e^{j\phi_3}) & 1 \\ e^{j(\phi_1+\phi_2)} & -(e^{j\phi_1} + e^{j\phi_2}) & 1 \end{pmatrix} \end{aligned} \tag{6}$$

More generally, the explicit expression for the entries of $H(\phi_1, \ldots, \phi_N)$ is

$$\begin{aligned} h_{il} &= (-1)^{N-l} \textstyle\sum_{i_1 < \ldots < i_{N-l}, i_p \neq i}^{N} \\ & \exp j(\phi_{i_1} + \ldots + \phi_{i_{N-l}}) \quad l < N \end{aligned} \tag{7}$$

$$h_{iN} = 1 \tag{8}$$

Note that z_i is independent of ϕ_i. Moreover it is easy to show that

$$\det H(\phi_1, \ldots, \phi_N) = \Pi_{i=2}^{N}\Pi_{k=i}^{N}(e^{j\phi_{i-1}} - e^{j\phi_k}) \tag{9}$$

The reason for this definition of $H(\phi_1, \ldots, \phi_N)$ is provided by the following lemma and corollary

Lemma 1: The matrix $C(\phi_1, \ldots, \phi_N) \triangleq H(\phi_1, \ldots, \phi_N)A$ has entries

$$c_{il} = \Pi_{k=1\ k\neq i}^{N}(e^{j\phi_l} - e^{j\phi_k}) \tag{10}$$

Corollary: If all the ϕ_i are distinct, the output z achieves source separation (up to the noise) if and only if

$$\phi_i = \varphi_{p(i)} \qquad i = 1, \ldots, N \tag{11}$$

for some permutation $p(1), \ldots, p(N)$ of $1, \ldots, N$.

In other words, correct angular localisation is equivalent to source separation: relation (3) is valid. Hence, the φ_i can be estimated thanks to a source separation criterion.

2.3 Necessity of a second stage

According to (10) and (11), when the ϕ_i are good, the entries of Λ are

$$\lambda_{p(i)} = \Pi_{l\neq p(i)}(e^{j\varphi_{p(i)}} - e^{j\varphi_l}) \tag{12}$$

According to (4), in the noiseless case, the power of z_i is

$$E(|z_i|^2) = \gamma_{p(i)}\Pi_{l\neq p(i)}|e^{j\varphi_{p(i)}} - e^{j\varphi_l}|^2 E(|s_{p(i)}|^2) \tag{13}$$

which varies in an unknown fashion with the true angles φ_j and attenuations γ_j. Unfortunately, to our best knowledge, there is not yet available a source separation criterion that is completely free of any constraint on the powers of the outputs z_i. For instance, the works based on Comon's approach assume that all the z_i have unit power. Similarly for the criteria introduced in [6], using the cross-cumulants of the z_i. Hence a second stage which calculates the vector

$$\mathbf{y} = \mathrm{diag}\ (\sqrt{d_1}, \ldots, \sqrt{d_N})\mathbf{z} \tag{14}$$

where the (positive) parameter d_i controls the power of z_i.

3 Separation criterion

To separate independent zero-mean, circular sources, we have proposed [5] the criterion

$$\begin{aligned} \mathcal{J} &= \textstyle\sum_{i=1}^{N} E(|y_i|^4 - 2\sigma|y_i|^2) - 2\alpha\ln(|\det DH|^2) \\ & \alpha > 0,\ \sigma > 0 \end{aligned} \tag{15}$$

where the matrix DH corresponds to the overall separation matrix for the two cascaded stages. The first term in (15) corresponds to source "extraction" and is minimum when each output y_i extracts a pure source (up to a multiplicative factor). Clearly this term tends to render $|y_i|$ close to $\sqrt{\sigma}$, hence an implicit power control for the y_i.

The second term in (15) enforces the recovered sources to be different. This term couples the channels y_i to one another. According to (9) and (15)

$$\begin{aligned}\mathcal{J} &= \sum_{i=1}^{N}(d_i^2 \mathrm{E}(|z_i|^4) - 2\sigma d_i \mathrm{E}(|z_i|^2) - 2\alpha \ln d_i) \\ &-2\alpha \sum_{i=2}^{N}\sum_{k=i}^{N} \ln |e^{j\phi_{i-1}} - e^{j\phi_k}|^2\end{aligned} \tag{16}$$

For given values of the ϕ_j, the quantities $\mathrm{E}(|z_i|^2)$ and $\mathrm{E}(|z_i|^4)$ are fixed. So the optimum gains d_i are obvious functions of $\mathrm{E}(|z_i|^2)$ and $\mathrm{E}(|z_i|^4)$. It is shown in [4] that the $N!$ true angle configurations (11) are stationary points of $\mathcal{J}$. The associated gains d_i satisfy

$$d_i \mathrm{E}(|z_i|^2) = (2m_{p(i)})^{-1}(\sigma + \sqrt{\sigma^2 + 4\alpha m_{p(i)}}) \tag{17}$$

with the help of the index

$$m \triangleq \mathrm{E}(|s|^4)/\mathrm{E}(|s|^2)^2 \tag{18}$$

which is the source fourth-order normalized moment (SFONM). The result (17) means that the optimal second stage is made of N independent Automatic Gain Controls (AGC) which respectively regulate the N output powers $\mathrm{E}(|y_i|^2)$ at fixed levels, depending only on σ, α and the corresponding SFONM. It then follows from (13) that the unknown attenuations γ_j are such that (for $\alpha = 0$)

$$\begin{aligned}\gamma_{p(i)} \mathrm{E}(|s_{p(i)}|^2) &= \frac{2^{2-2N}}{d_i} \\ \sigma &\left(\Pi_{l \neq p(i)} |\sin \frac{\varphi_{p(i)} - \varphi_l}{2}|^2 m_{p(i)} \right)^{-1}\end{aligned} \tag{19}$$

It is also shown in [4] that for $N = 2$, the separating configurations ($\phi_1 = \varphi_1$, $\phi_2 = \varphi_2$) and ($\phi_1 = \varphi_2$, $\phi_2 = \varphi_1$) are the only stable minima of $\mathcal{J}$, if and only if m_1 and m_2 are less than 2, i.e., the two circular sources have negative kurtosis. This is the case for communication applications.

4 Adaptive implementation

Note that the criterion $\mathcal{J}$ is the expected value of a stochastic criterion. Hence adaptive minimization of $\mathcal{J}$ is possible with the stochastic gradient procedure.

4.1 Adaptation of the angles $(\phi_1, \dots, \phi_n)$

It is easily seen that

$$\frac{1}{2} \cdot \frac{\partial \mathcal{J}_s}{\partial \phi_k} = \sum_{i=1}^{N} (|y_i|^2 - \sigma) \frac{\partial |y_i|^2}{\partial \phi_k} - \alpha \sum_{i \neq k}^{N} \operatorname{cotan} \frac{\phi_k - \phi_i}{2} \tag{20}$$

Moreover, with the Kronecker symbol δ_{ik}, and using the auxiliary signal

$$\begin{aligned}{}^k z_i &\triangleq \sum_{l=1}^{N-1} (-1)^{N-l-1} x_l \sum_{i_1 < \dots < i_{N-l-1}, i_p \neq i,k}^{N} \\ &\exp \mathrm{j}(\phi_{i_1} + \dots + \phi_{i_{N-l-1}})\end{aligned} \tag{21}$$

$$\frac{\partial |y_i|^2}{\partial \phi_k} = 2\delta_{ik} d_i \mathrm{Im}\{z_i^{\star\, k} z_i e^{j\phi_k}\} \tag{22}$$

This yields the angular increments

$$\begin{aligned}\Delta\phi_k &= \mu_\phi \sum_{i \neq k} [d_i(\sigma - |y_i|^2) \mathrm{Im}\{z_i^{\star\, k} z_i e^{j\phi_k}\} + \\ &\frac{\alpha}{2} \operatorname{cotan} (\frac{\phi_k - \phi_i}{2})]\end{aligned} \tag{23}$$

where μ_ϕ is the positive step-size. It can be remarked that, with the truncated vector $\mathbf{x}' \triangleq (x_1, \dots, x_{N-1})^T$ as input, the auxiliary vector ${}^k\mathbf{z} = ({}^k z_i, \dots, {}^k z_n)^T$ is the output of an $(N-1)$-th order subsystem which does not involves the angle ϕ_k:

$${}^k\mathbf{z} = H^{N-1}(\phi_1, \dots, \phi_{k-1}, \phi_{k+1}, \dots, \phi_N)\mathbf{x}' \tag{24}$$

4.2 Adaptation of the gains $(d_1, \dots, d_N)$

Two strategies are possible

Strategy 1: Joint Optimization (JO). Computation of the opposite gradient of $\mathcal{J}_s$ versus d_i provides

$$\Delta d_i = \mu_d [|z_i|^2 (\sigma - |y_i|^2) + \alpha / d_i] \tag{25}$$

where μ_d is the positive step-size.

Strategy 2: Automatic Gain Control (AGC). Thanks to the result (17) we adapt d_i by regulating $\mathrm{E}(|y_i|^2)$. For instance, this is feasible with

$$\Delta d_i = \mu_d \left(\frac{\sigma + \sqrt{\sigma^2 + 4\alpha m_{p(i)}}}{2m_{p(i)}} - |y_i|^2 \right), \ \mu_d > 0 \tag{26}$$

5 Computer Simulations

There are $N = 2$ independent noiseless sources with the DOA $\theta_1 = 9.6°$ and $\theta_2 = 41.8°$ and corresponding to complex QAM modulation.

Well-conditioned case. The attenuations $\gamma_1 = 1$ and $\gamma_2 = 0.8$ are in the same range ; s_1 is 4-QAM ($m_1 = 1$) and s_2 is 16-QAM ($m_2 = 1.32$).

Ill-conditioned case. The attenuations $\gamma_1 = 1$ and $\gamma_2 = 0.1$ are quite different. Both s_1 and s_2 correspond to 4-QAM ($m_1 = m_2 = 1$).

For the well-conditioned case, full lines illustrate the JO strategy, dotted lines correspond to AGC. Fig. 1 and 2 compare the results of JO and AGC for estimating respectively the angles and the attenuations. Fig. 3 compares the separation performance indices. It is zero iff the separation property (3) is achieved. Clearly both strategies successfully restore the two sources and correctly estimate the angles and attenuations. The separation index reaches values as low as -45 dB for JO and -30 dB for AGC. Unsurprisingly the JO strategy has better achievements (faster speed and lower steady-state fluctuations) than the AGC. On the other hand the AGC strategy has lower computational complexity than

JO (no division). Moreover in the case where all the SFONM are different it permits to associate each angle θ_i with the source attenuation γ_i in the corresponding direction. For instance we can choose the identity permutation $P = I$ and then the γ_i follow from (19) and from the steady-state values of the ϕ_i and d_i.
The cancellation of α is possible in steady-state once the ϕ_j have all reached distinct values.
For the ill-conditioned case, Fig. 4 displays the separation performance index. It is below -20 dB which remains a satisfactory level, although it is less good than for the well-conditioned case.

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