A CONSTANT MODULUS APPROACH TO MULTIPLE ACCESS INTERFERENCE REJECTION

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ABSTRACT

This paper addresses the problem of blind Multiple Access Interference (MAI) suppression in Direct Sequence (DS) Code Division Multiple Access (CDMA) systems. The Constant Modulus (CM) criterion is used to devise a blind adaptive DS CDMA receiver that achieves the same performance as the Minimum Mean Square Error (MMSE) receiver for high values of the Signal to Noise Ratio (SNR). The main limitation of the CM receiver is that an interferent user may be extracted instead of the desired one. Two approaches are investigated that practically overcome this problem when an estimate of the desired user code is available. The first one is an adequate choice of the adaptive receiver initial conditions and the second one is the incorporation of linear constraints.

1 INTRODUCTION

Multiple Access Interference (MAI) caused by code non-orthogonality constitutes the main limitation of Direct Sequence Code Division Multiple Access (DS CDMA) systems. Different techniques have been proposed to adaptively suppress MAI (see [1] and references therein for an overview). Minimum Mean Square Error (MMSE) receivers [2] can be used but its adaptive implementation requires the transmission of training sequences. Alternative blind implementations based on the Linearly Constrained Minimum Variance (LCMV) criterion have been proposed [3] but they are extremely sensitive to inaccuracies in the acquisition of the desired user timing and spreading code.

In this paper we investigate the use of the Constant Modulus (CM) criterion for the blind suppression of MAI in DS CDMA systems. It can be shown that, when applying the CM criterion to CDMA receivers, interferences can be optimally cancelled relying only on its statistical independence property. No *a priori* knowledge of the desired user spreading code is initially required. The main disadvantage of CM receivers is that they may capture an interferent signal instead of the desired one [4]. In order to overcome this limitation we propose two approaches that assume the availability of a rough estimate of the desired user code and timing at the receiver. First, we show that an adequate initialization of the adaptive algorithm using this estimate considerably reduces the capture problem. However, a further reduction in the capture probability can be achieved if we incorporate the same linear constraint as in the LCMV receiver. The resulting modified CM receiver will be termed Linearly Constrained Constant Modulus (LCCM). Both CM and LCCM receivers turn out to be very robust to code estimation innacuracies.

The paper is organized as follows. Section 2 introduces the signal model. In section 3 we demostrate that the CM receiver performs the same as the MMSE receiver for high values of the Signal to Noise Ratio (SNR). Sections 4 and 5 introduce the proposed initialization strategy and the LCCM receiver, respectively. Section 6 is devoted to the conclusions.

*This work has been supported by CICYT, grant TIC96-0500-C10-02, and Xunta de Galicia, grant XUGA 10502A96.

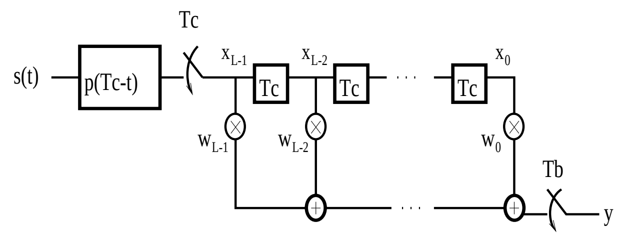

Figure 1: Block diagram of a DS CDMA linear receiver for the demodulation of a single user.

2 SIGNAL MODEL

Let us consider a synchronous baseband DS CDMA system with N users. Each user i is assigned a unique code sequence $c_i[j], \quad j = 0, \cdots, (L-1)$. The received signal is

$$r(t) = \sum_{i=1}^{N} A_i b_i \sum_{j=0}^{L-1} c_i[j] p(t - jT_c) + n(t), \quad 0 \le t \le T_b \quad (1)$$

where b_i and A_i are the i-th user transmitted symbol and received amplitude, $p(t)$ is the chip pulse waveform, T_c is the chip period, $T_b = LT_c$ is the symbol period and $n(t)$ is the Additive White Gaussian Noise (AWGN).

Figure 1 plots the linear receiver for the demodulation of a single user in DS CDMA systems. The received signal is passed through a chip-matched filter and a T_c-tapped delay line FIR filter with coefficients $\mathbf{w} = [w_0, \cdots, w_{L-1}]^T$ followed by a bit rate sampler. The output of the chip-matched filter is

$$x_j = \int_{jT_c}^{(j+1)T_c} s(t) p(t - jT_c) dt = \sum_{i=1}^{N} A_i b_i c_i[j] + n_j \quad (2)$$

where $n_j = \int_0^{T_c} n(t)p(t)dt$. Rewriting (2) in vector form

$$\mathbf{x} = \sum_{i=1}^{N} A_i b_i \mathbf{c}_i + \mathbf{n} = \mathbf{CAb} + \mathbf{n} \quad (3)$$

where $\mathbf{x} = [x_0, \cdots, x_{L-1}]^T$, $\mathbf{c}_i = [c_i[0], \cdots, c_i[L-1]]^T$, $\mathbf{n} = [n_0, \cdots, n_{L-1}]^T$, $\mathbf{b} = [b_1, \cdots, b_N]^T$, $\mathbf{C} = [\mathbf{c}_1, \cdots, \mathbf{c}_N]$ and

$$\mathbf{A} = \begin{bmatrix} A_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & A_N \end{bmatrix} \quad (4)$$

Correspondingly, the receiver output is

$$y = \mathbf{w}^T \mathbf{x} = \mathbf{w}^T \mathbf{CAb} + \mathbf{w}^T \mathbf{n}. \quad (5)$$

and should provide an estimate of the symbols transmitted by the desired user.

3 THE CONSTANT MODULUS RECEIVER

In the CM receiver the filter coefficients $\mathbf{w}$ are selected according to the following optimization problem

$$\min_{\mathbf{w}} \quad J(\mathbf{w}) = E[(y^2 - 1)]. \quad (6)$$

Optimum weights can be computed using the stochastic gradient algorithm

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu(y^2(n) - 1)y(n)\mathbf{x}(n) \quad (7)$$

where μ is the step size parameter. Notice that the cost function $J(\mathbf{w})$ is not a quadratic form of $\mathbf{w}$ and, therefore, it contains multiple stationary points that may impair the convergence of the adaptive algorithm (7).

Let us analyze the stationary points in $J(\mathbf{w})$ for a noiseless environment with $N \leq L$ statistically independent users. The receiver output can be written as $y = \mathbf{g}^T \mathbf{b}$ where $\mathbf{g} = \mathbf{AC}^T \mathbf{w} = [g_1, \cdots, g_N]^T$ is the vector formed by the amplitudes of the different user symbols at the receiver output. Using the statistical independence property of these symbols, the cost function $J(\mathbf{w})$ can be written in terms of $\mathbf{g}$ as

$$J(\mathbf{w}) = \phi(\mathbf{g}) = (k-3)\sum_{i=1}^{N} g_i^4 + 3(\mathbf{g}^T\mathbf{g})^2 - 2(\mathbf{g}^T\mathbf{g}) + 1 \quad (8)$$

where $k = \frac{E[b_i^4]}{E^2[b_i^2]}$ is the normalized fourth order moment of the transmitted symbols and $k-3$ will be referred as the kurtosis. To obtain (8) we also assumed that $E[b_i^2] = 1$.

Computing the points where the gradient $\nabla_{\mathbf{g}}\phi$ vanishes and analyzing the Hessian matrix at these points, three groups of stationary points are identified:

- $\mathbf{g} = \mathbf{0}$. This is an undesired stationary point where all user signals are cancelled. The Hessian matrix is definite negative at this point and it is, therefore, a maximum.
- $\mathbf{g} = [0, \cdots, g_l^o, \cdots, 0]^T$. This point corresponds to the extraction of the l-th user and perfect suppression of the remaining $N-1$ users. The Hessian analysis shows that these N points are minima when the kurtosis of the modulation scheme is negative[1]. Note that this is the same solution achieved by the MMSE receiver in absence of noise.

[1]This is always the case for DSSS.

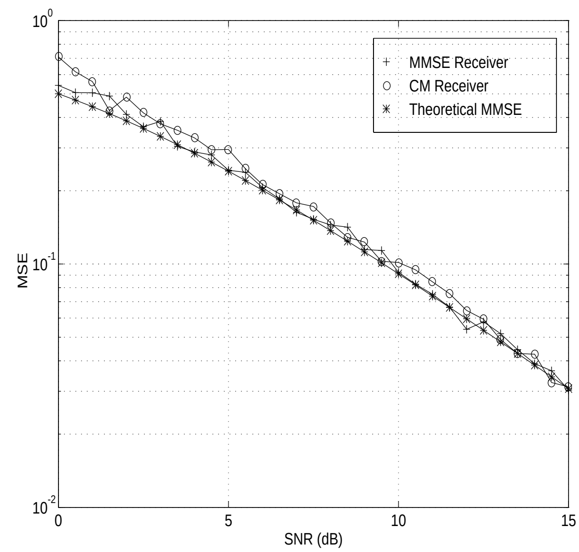

Figure 2: MSE obtained for the CM and MMSE receiver in a CDMA channel with AWGN and 16 users (using length 31 Gold codes) vs. theoretical MMSE given by (10). Plot obtained for CDMA channels with AWGN and multipath propagation (impulse response $h[n] = [1, \quad 0.5, \quad 0.1]$). All the interferent users are 6 dB stronger than the desired one.

- $\mathbf{g} = [g_1, \cdots, g_R, 0, \cdots, 0]^T$. These stationary points correspond to the extraction of a linear combination of R different user signals. Analyzing the Hessian, it is found that these points are not minima when the kurtosis is negative.

Although the previous analysis shows that algorithm (7) will only converge to solutions where a single user is demodulated, it also makes apparent that the CM receiver can be captured by an interferent user instead of extracting the desired one. In sections 4 and 5 we will show how an estimate of the desired user code, such as the one provided by a code acquisition circuit, can be used to combat the capture problem.

Next, we focus on the effect of the AWGN in the channel. Assuming the SNR is sufficiently high, it can be shown [5] that the noisy optimum amplitude vector is just a perturbed version of the noiseless optimum

$$\tilde{\mathbf{g}} = [\gamma\triangle g_1, \quad \gamma\triangle g_2, \cdots, g_l^o + \gamma\triangle g_l, \cdots, \gamma\triangle g_N]^T \quad (9)$$

where $\gamma = SNR^{-1}$. At this point, a small amount of MAI still remains because the receiver achieves a desirable balance between interference suppression and noise enhancement. In fact, the MSE value corresponding to (9) is practically the same achieved with an MMSE receiver for a high SNR. The exact expression for the MMSE is

$$MMSE = 1 - A_1^2 \mathbf{c}_1^T \left(\mathbf{CAAC}^T + \sigma_n^2 \mathbf{I}_L\right)^{-1} \mathbf{c}_1 \quad (10)$$

where σ_n^2 is the noise variance and $\mathbf{I}_L$ is the $L \times L$ identity matrix. Figure 2 shows the validity of our approximation. It plots three curves of MSE versus SNR. Two curves correspond to values obtained through simulation of the CM and MMSE receivers, respectively. The third one plots the theoretical curve given by (10). It is clear that all three

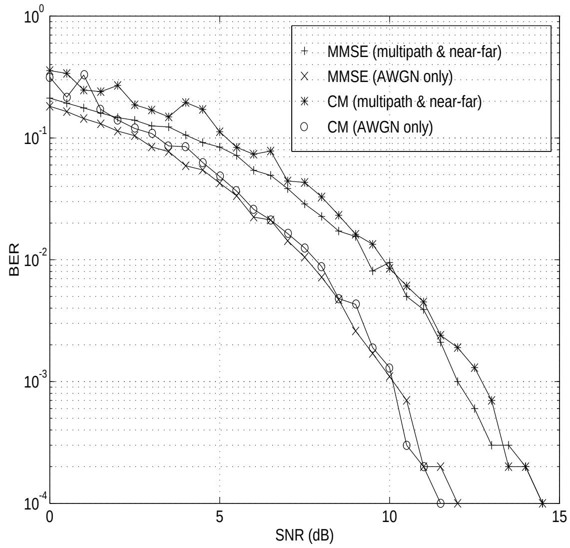

Figure 3: BER vs. SNR for the CM and the MMSE receivers in a CDMA system with Gold codes of length 31 and 16 users. Plots obtained for a CDMA channel with AWGN and CDMA multipath channel (with impulse response $h[n] = [1, \ 0.5, \ 0.1]$). All the interferent users are 6 dB stronger than the desired one.

curves are practically the same. Correspondingly, the Bit Error Rate (BER) of the CM receiver also matches quite tightly the values obtained with the MMSE receiver, as can be seen from the simulation results plotted in figure 3.

4 INITIALIZATION STRATEGY

In this section, we discuss how to use the approximate knowledge of the desired user code available at the receiver to initialize the adaptive algorithm (7) in a way that successfully reduces the capture probability of the CM receiver.

Let $\hat{\mathbf{c}}_1$ be an estimate of the desired user code, $\mathbf{c}_1$. We propose to set the algorithm initial conditions as

$$\mathbf{w}(0) = \frac{\hat{\mathbf{c}}_1}{\hat{\mathbf{c}}_1^T \hat{\mathbf{c}}_1}. \tag{11}$$

The analysis leading to this initialization strategy is explained in detail in [6] and we will only indicate its major guidelines here. The analysis is based on the study of the Ordinary Difference Equation (ODE) associated to algorithm (7),

$$\mathbf{w}(n+1) = \mathbf{w}(n) - \mu E\left[(y^2(n) - 1)y(n)\mathbf{x}(n)\right], \tag{12}$$

and its aim is to determine the relevant factors in the prediction of the extracted user. Results are presented in terms of the output user amplitudes, g_i.

For a simple scenario containing two users with perfectly orthogonal spreading codes, a boundary between the correct extraction and capture regions on the $|g_1|, |g_2|$ plane can be derived. Such boundary is given by

$$\frac{A_2^2\rho_{22}|g_2(n)|(qg_2^2(n) + 3m^2g_1^2(n) - m)}{A_1^2\rho_{11}|g_1(n)|(qg_1^2(n) + 3m^2g_2^2(n) - m)} - \frac{|g_2(n)| - B}{|g_1(n)| - A} = 0, \tag{13}$$

where $\rho_{ii} = \mathbf{c}_i^T\mathbf{c}_i$ is the autocorrelation coefficient of the i-th user code, $q = E[b_i^4]$, $m = E[b_i^2]$ and (A, B) is the crosspoint of

$$\begin{aligned} f_1(g_1(n), g_2(n)) &= A_1^2\rho_{11}(qg_1^2(n) + 3m^2g_2^2(n) - m), \quad (14) \\ f_2(g_1(n), g_2(n)) &= A_2^2\rho_{22}(qg_2^2(n) + 3m^2g_1^2(n) - m). \quad (15) \end{aligned}$$

Figure 4 plots curve (13) for several values of the Signal to Interference Ratio, $SIR = 20log_{10}(A_1/A_2)$. When the initial conditions, $(|g_1(0)|, |g_2(0)|) = (|\hat{\mathbf{c}}_1^T\mathbf{w}(0)|, |\hat{\mathbf{c}}_2^T\mathbf{w}(0)|)$, lay on the right of the corresponding boundary, the algorithm leads to the extraction of the desired user (user 1) and when they lay on the left of the boundary the interferent user (user 2) is extracted. It is apparent that, when the desired user code estimate is perfect, the initial conditions yielded by (11) are $(|g_1(0)|, |g_2(0)|) = (1, 0)$ and the capture problem is eliminated.

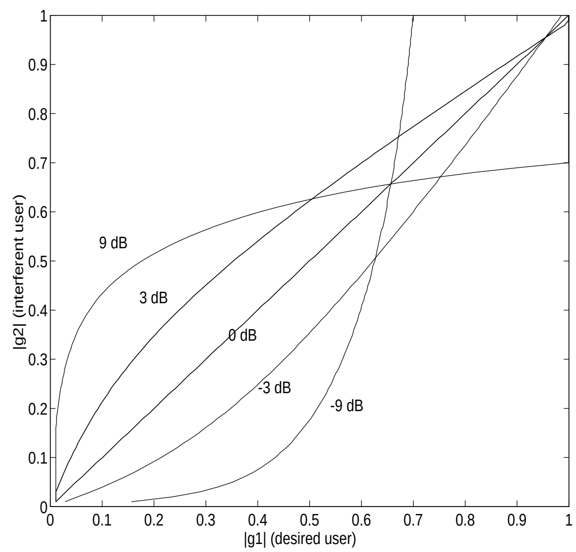

Figure 4: Boundaries between the correct extraction and capture regions for several values of the SIR.

In practice, however, the desired user code estimate is likely to be just an approximation of the received code. In order to determine the robustness of our initialization strategy, we carried out computer simulations assuming that the estimate error $\triangle\mathbf{c}_1 = \hat{\mathbf{c}}_1 - \mathbf{c}_1$ is a vector of Gaussian random variables with zero mean and autocorrelation matrix $\sigma_c^2\mathbf{I}_L$. Figure 5 plots the capture probability with respect to σ_c^2. It is clearly seen that the distortion variance σ_c^2 must be very large ($\sigma_c^2 > 0.4$) in order to obtain capture probabilities above 10^{-4}. Note, then, that an approximate knowledge of $\mathbf{c}_1$ overcomes the capture problem in CM receivers whereas is not enough to ensure an adequate performance of LCMV receivers.

5 THE LINEARLY CONSTRAINED CONSTANT MODULUS RECEIVER

The capture problem is a consequence of the existence of multiple minima in the CM cost function. An alternative to the initialization strategy proposed in section 4 is to use the desired user code estimate to modify the CM cost function in order to suppress the undesired minima. This can be done incorporating the same linear constraint as in the LCMV

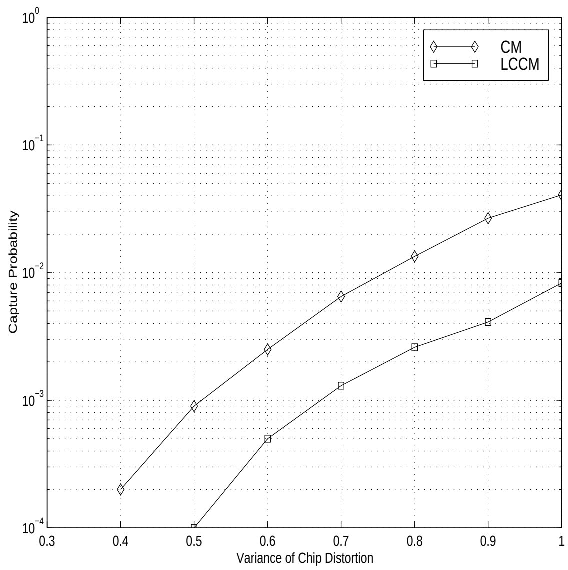

Figure 5: Capture probability of the CM (initialized with the desired user code estimate) and LCCM receivers vs. variance of chip distortion (σ_c^2). The channel contains 16 users employing Gold sequences of length 31 as signature waveforms. All the interferent users are 6 dB stronger than the desired one.

criterion. The resulting receiver will be termed Linearly Constrained Constant Modulus (LCCM).

In the LCCM receiver, the filter coefficients $\mathbf{w}$ are selected according to the optimization problem

$$\min_{\mathbf{w}} \quad J(\mathbf{w}) = E[(y^2-1)^2] \quad \text{subject to} \quad \mathbf{w}^T\mathbf{c}_1 = 1 \qquad (16)$$

where $\mathbf{c}_1$ is the desired user code. It can be shown [7] that, in the absence of noise, the solution to (16) in terms of the output user amplitudes, g_i, is unique. Furthermore, this solution is the same obtained with the MMSE and decorrelating receivers, i.e., $\mathbf{g}_o = [1, \quad 0, \cdots, 0]^T$.

In order to find an adaptive algorithm that computes the optmimum weights we will first convert problem (16) into an unconstrained form. This can be done using the Generalized Sidelobe Canceller (GSC) [8] decomposition

$$\mathbf{w} = \mathbf{w}_q - \mathbf{B}\mathbf{w}_a, \qquad (17)$$

where $\mathbf{w}_q = \frac{\mathbf{c}_1}{\mathbf{c}_1^T\mathbf{c}_1}$ is the quiescent vector, $\mathbf{B}$ is the blocking matrix whose columns span the null space of $\mathbf{c}_1$ (i.e. $\mathbf{B}$ is full rank and satisfies $\mathbf{B}^T\mathbf{c}_1 = \mathbf{0}$) and $\mathbf{w}_a$ is the $(L-1)\times 1$ unconstrained adaptive weight vector. Decomposition (17) ensures that $\mathbf{w}$ always satisfies the constraint regardless of $\mathbf{w}_a$. As a consequence, (16) is equivalent to

$$\min_{\mathbf{w}_a} \quad J(\mathbf{w}_a) = E[(y^2-1)^2]. \qquad (18)$$

A simple way to solve (18) is to use the stochastic gradient algorithm

$$\mathbf{w}_a(n+1) = \mathbf{w}_a(n) + \mu(y^2(n)-1)y(n)\mathbf{B}^T\mathbf{x}(n) \qquad (19)$$

where $\mu << 1$ is the step size parameter.

The LCCM criterion has been defined in (16) using the desired user code $\mathbf{c}_1$. As in the previous section, we must take into account that in a practical receiver only an estimate, $\hat{\mathbf{c}}_1$, will be available. In this case, the solution to (16) is not unique and the receiver may still be captured by interfering users. However, the LCCM approach turns out to be very robust to code estimation errors. Figure 5 also plots the capture probability obtained with the LCCM receiver when the initial conditions of algorithm (19) are set to $\mathbf{w}_a(0) = [0, \quad 0, \cdots, 0]^T$. It can be seen that this approach performs even better than the *initialized* CM receiver in terms of capture probability.

6 CONCLUSIONS

We have investigated the application of the CM criterion to MAI suppression in DS CDMA receivers. It has been shown that the minimization of the CM cost function leads to the demodulation of a single user with the same MSE performance as the MMSE receiver. The main drawback of CM receivers is that they may capture an interference instead of the desired user. However, an adequate initialization of the adaptive algorithm using an estimate of the desired user code is enough to succesfully reduce the capture probability. A modification of the CM receiver, termed LCCM, is also presented. This approach uses the available estimate of the desired user code to incorporate a linear constraint on the CM cost function and the resulting receiver provides a further reduction of the capture probability.

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