ROBUST SECOND-ORDER BLIND EQUALIZATION OF POLYPHASE CHANNELS

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ABSTRACT

This work deals with the problem of linear polyphase blind equalization (BE), i.e. we are interested in equalizing the output of a single-input-multiple-output (SIMO) channel, without observing its input. A recent result by Liu and Dong [1] showed that if the sub-channel polynomials are co-prime, the equalizer output whiteness suffices for the equalization of a white input. Based on this observation, we propose a simple decorrelation criterion for second-order based blind equalization. Due to its second-order nature, this criterion is insensitive to the distance of the input from Gaussianity, hence it performs BE even for Gaussian or non-Gaussian inputs. Moreover, unlike other second-order techniques, our approach bypasses channel estimation and computes directly the equalizer. By doing so, it avoids the problem of ill-conditioning due to channel order mismatch which is crucial to other techniques. Combined to its good convergence properties, these characteristics make the proposed technique an attractive option for robust polyphase BE, as evidenced by both our analysis and computer simulation results.

1 INTRODUCTION

Blind equalization of polyphase (SIMO) channels is a field that has been receiving increased attention in the recent years. Applications include fractionally-spaced signal processing at the receiver, and/or reception through an array of sensors. Despite the early introduction of polyphase receivers more than two decades ago [2], [3], a number of important advantages they offer became apparent only recently. We outline some of these important results:

1. In [4], Tong, Xu, and Kailath showed that when the sub-channels corresponding to the different sampling phases of the channel response are co-prime, channel identification is possible based solely on the received signal's second-order statistics (SOS). This result initiated a considerable amount of research in the area of SOS-based BE which has lead to many techniques that achieve BE without the use of higher-order statistics (HOS).
2. In [5], Slock showed that under the same conditions mentioned in [4], perfect (zero forcing (ZF)) equalization of linear channels is possible in the absence of noise with finite-length filters. This has allowed for a better understanding of the behavior of polyphase receivers. Moreover, it has lead to new results regarding the behavior of adaptive equalization algorithms when used with polyphase receivers.
3. One of the most notable results which made use of 2 above is linked to the behavior of the popular constant modulus algorithm (CMA) [6]: it was shown in [7], [8], [9] that when fractionally-spaced, the CMA is globally convergent in the absence of noise (again under the co-prime assumption). This result shows how the long-standing problem of local minima of the CMA can be avoided if fractional spacing and/or antenna arrays are used at the receiver.

Together with the above results, a number of problems in the proposed techniques remain. Namely, the structure-based approaches that soon followed [4], such as [5], [10], [11] suffer from lack of robustness with respect to the estimated channel order: order mismatch results in wrong dimensions of the signal or noise subspace, which may lead to severe performance degradation. As a result, alternatives to structure-based methods such as [12], [13] have been proposed. These techniques rely on SOS statistical criteria (as opposed to the structure of the covariance matrix) in order to estimate the channel response. Even though they are more robust to the problem of channel order mismatch, a drawback is that they need to estimate the channel first in order to compute the equalizer. This may affect performance if not done judiciously, and adds extra complexity to the equalizer design, as opposed to *direct* BE algorithms (such as the CMA) that compute directly the equalizer.

An interesting twist in the BE literature came recently when Liu and Dong showed in [1] that in the case of a white input, the *equalizer output whiteness* is a necessary and sufficient condition for BE. Based on this theorem, *direct* SOS BE algorithms were proposed in [14],

[15]. These algorithms offer the double benefit of being insensitive to both the input distance from Gaussianity (unlike HOS-based algorithms such as the CMA) and to channel order mismatch (unlike most SOS-based methods), as they do not require channel estimation. In this paper we focus on the adaptive algorithm proposed in [15] and study its performance in terms of convergence behavior.

2 A WHITENING APPROACH FOR POLYPHASE BLIND EQUALIZATION

The sampled polyphase output of a SIMO (1-input-m-output) channel can be written in vector notation as follows

$$\mathbf{x}(n) = \sum_{k=0}^{M-1} s(n-k)\mathbf{h}(k) + \mathbf{b}(n) \tag{1}$$

where the vectors $\mathbf{x}$, $\mathbf{h}$, $\mathbf{b}$ are all $m \times 1$ and denote the polyphase received signal, channel, and additive noise, respectively. Each of the elements of these vectors corresponds to a different phase (sampling instant and/ or antenna element). The channel $\mathbf{h}$ is assumed to be time-invariant and FIR of length M, whereas the input $\{s(n)\}$ is assumed to be an i.i.d. symbol sequence, and $\mathbf{b}(n)$ denotes the i.i.d. noise sequence. Stacking K successive samples in $X(n) = [\mathbf{x}^T(n), \mathbf{x}^T(n-1), ..., \mathbf{x}^T(n-K+1)]^T$, $B(n) = [\mathbf{b}^T(n), \mathbf{b}^T(n-1), ..., \mathbf{b}^T(n-K+1)]^T$ yields the well-known linear model, where both channel and symbols are unknown quantities:

$$X(n) = \mathcal{H}[s(n), .., s(n-K-M+2)]^T + B(n) \tag{2}$$

where

$$\mathcal{H} = \begin{pmatrix} \mathbf{h}(0) & \cdots & \mathbf{h}(M-1) & \mathbf{0} & \cdots & \mathbf{0} \\ & & & \vdots & & \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{h}(0) & \cdots & \mathbf{h}(M-1) \end{pmatrix} \tag{3}$$

is the Sylvester-like channel matrix of size $mK \times (M+K-1)$. The output of a linear multichannel receiver with mK taps (here denoted W) is defined by:

$$y(n) \stackrel{\text{def}}{=} W^H X(n)$$

In the above ^{T} and ^{H} denote transpose, and Hermitian transpose, respectively.

The following theorem regarding the blind equalizability of SIMO channels appeared recently in [1] for the case of a white input $s(n)$:

Theorem 1 *(Whitening Theorem) Assuming that the input $s(n)$ is white, that the channel matrix $\mathcal{H}$ has full column rank (classical length and zero condition [5]), and that there is no additive noise, the equalizer output $y(n)$ will be white if and only if zero forcing equalization has been achieved, up to some delay d, gain ρ, and unknown phase rotation ϕ, i.e.*

$$y(n) = \rho e^{j\phi} s(n-d) \tag{4}$$

It is well known that this result does not hold in the traditional single channel case. The reason is that perfect equalizability (hence output whiteness) in the single channel case requires an IIR receiver structure. Thus output whiteness can indeed be achieved, however the IIR structure allows for an arbitrary all-pass filtering ambiguity.

The key issue here is that multichannel (polyphase) equalizability can be achieved with a *finite length* (FIR) equalizer, for example W can be drawn from the pseudo-inverse of $\mathcal{H}$. Enforcing the FIR property of the combined channel-equalizer removes the all-pass ambiguity, since non trivial all-pass filters cannot be FIR! In the sequel we will see how this important Theorem can lead to robust SOS-based methods for direct adaptive blind equalization.

3 A DECORRELATION ALGORITHM FOR POLYPHASE BLIND EQUALIZATION

Based on the above theorem, a direct BE criterion can be formulated, so as to force the equalizer output to be white, as follows (see [15]):

$$\min_W J(W) = (r_0 - 1)^2 + \lambda \sum_{l=1}^{N-1} |r_l|^2 \tag{5}$$

where r_l is the autocorrelation of the equalizer output defined as $E(y(k)y^*(k-l))$, and λ is a weighting scalar. N is the maximum achievable delay l for which r_l is non-zero (in the absence of noise). Denoting the channel-equalizer cascade as $G(z) = \sum_{i=1}^m H_i(z)W_i(z)$, where $H_i(z)$, $W_i(z)$ denote the polynomials corresponding to the i-th phase of channel and equalizer, respectively, the equalizer output can be written in the absence of noise in the z-domain as $Y(z) = G(z)S(z)$, or in the time domain as $y(n) = G^T S(n)$. Then $G = [g_1 \ \cdots \ g_N]^T$ and $S(n) = [s(n) \ \cdots \ s(n-N+1)]^T$ will have N coefficients each.

The corresponding stochastic gradient adaptive algorithm that implements the criterion (5) is given by

$$\begin{aligned} W(k+1) = W(k) - \mu[(r(0)-1)y^*(k)X(k) + \\ \lambda \sum_{l=1}^{N-1} r^*(l)y^*(k-l)X(k)] \end{aligned} \tag{6}$$

In order to implement (6) in practice, we need to estimate the correlation terms r_l, for example with simple rectangular-window averaging: $\hat{r}_l = \sum_{i=1}^{\Delta} y(i)y^*(i-l)$. Notice that despite the second-order nature of the criterion (5), if we choose $\lambda = 0$ and $\Delta = 1$ (instantaneous averaging), (6) is identical to the CMA 2-2 algorithm. However for $\Delta > 1$ and/or $\lambda > 0$ the algorithm (6) is different from the CMA which is memoryless and uses instantaneous HOS of the equalizer output to penalize non-Gaussianity. This will be evidenced in section 5 by the ability of (6) to equalize super-Gaussian signals.

4 CONVERGENCE ANALYSIS

In order to demonstrate the convergence behavior of the algorithm (6), we will examine the stationary points of the cost function in (5). As is typically done in the analysis of BE cost functions, we will analyze J in the cascade domain, hence J will be denoted as a function of the cascade vector: $J(G)$. The obtained results can be then easily translated to the equalizer domain W, as we assume that the zeros-and-length condition is satisfied (this guarantees a one-to-one mapping between the stationary points in the G and W domains, since in this case $\mathcal{H}$ is full column rank). We will also assume the absence of additive noise in our analysis. Due to its simplicity and useful insight, we will only consider the simple case of two coefficients in the global response ($N = 2$). In the following we also fix for simplicity $\lambda = 2$, and the input variance $\sigma_s^2 = E|s(n)|^2 = 1$.

4.1 Analysis

In this case $G = [g_1 \; g_2]^T$ and $y(n) = g_1 s(n) + g_2 s(n-1)$. The cost function in (5) then takes the form:

$$J(G) = \left(|g_1|^2 + |g_2|^2 - 1\right)^2 + 2|g_1|^2|g_2|^2 \tag{7}$$

and the stationary points of $J(G)$ are found from $\frac{\partial J(G)}{\partial g_i^*} = 0$ for $i = 1, 2$, which gives

$$\begin{array}{l} 2g_1(|g_1|^2 + 2|g_2|^2 - 1) = 0 \\ 2g_2(|g_2|^2 + 2|g_1|^2 - 1) = 0 \end{array} \tag{8}$$

Eq. (8) has four classes of solutions. The first two classes are $\left(|g_1|^2 = 1 \; ; \; g_2 = 0\right)$ and $\left(|g_2|^2 = 1 \; ; \; g_1 = 0\right)$ and constitute the global minima of the cost function in (7). The third "class" is the solution $(g_1 = 0 \; ; \; g_2 = 0)$ and constitutes a local maximum: denoting a general perturbation setting in the 2-D plane around $[0 \; 0]$ as $\widetilde{G} = [\pm\epsilon_1 \;\; \pm\epsilon_2]$, where ϵ_1, ϵ_2 are small positive constants, we find that $J(\widetilde{G}) \simeq 1 - 2(\epsilon_1^2 + \epsilon_2^2) < J([0 \; 0])$, $\forall$ (ϵ_1, ϵ_2). Hence the setting $G = [0 \; 0]$ is a local maximum. The fourth class of solutions is:

$$|g_1|^2 = |g_2|^2 = 1/3 \tag{9}$$

Notice that $J(G)$ evaluated at the setting (9) equals $J(G_0) = 1/3$. In order to show that the solution (9) is a saddle point, we first consider the following perturbation setting:

$$\begin{array}{l} |\widetilde{g}_1|^2 = 1/3 + \epsilon \\ |\widetilde{g}_2|^2 = 1/3 - \epsilon \end{array} \tag{10}$$

where ϵ is a small positive constant. The value of J at the setting (10) equals $1/3 - 2\epsilon^2 < 1/3 = J(G_0)$. Hence a setting of the type (9) cannot be a local minimum. Now we consider the following perturbation:

$$\begin{array}{l} |\widetilde{g}_1|^2 = 1/3 + \epsilon \\ |\widetilde{g}_2|^2 = 1/3 + \epsilon \end{array} \tag{11}$$

The value of J at the settings of the type (11) equals $1/3 + 6\epsilon^2 > J(G_0)$. Hence a setting of the type (9) cannot be a local maximum either. As a result, it can only be a saddle point. We summarize the above analysis in the following theorem:

Theorem 2 *(Convergence in the case $N = 2$) In the case where the cascade response G has only two coefficients ($N = 2$), $\lambda = 2$, $\sigma_s^2 = 1$ and no additive noise is present, the cost function (5) has no undesired local minima. Its only minima are the optimal global settings $G_1 = [e^{j\phi} \;\; 0]$ and $G_2 = [0 \; e^{j\phi} \;]$, where ϕ_1, ϕ_2 are arbitrary phases.*

4.2 Discussion

According to Theorem 2, in the case $N = 2$, the algorithm (6) will be globally convergent to its optimal settings in the absence of noise. Notice also that the criterion achieves gain identification, whereas it results in a phase ambiguity that appears typically in blind equalization algorithms. Since the zeros-and-length condition has been assumed (in order to satisfy Theorem 1), this global convergence property will reflect to the equalizer domain W as well. This result is important as the role of local minima is known to be crucial for several blind adaptive algorithms. The extension of Theorem 1 to the case $N > 2$ is the subject of current research.

In order to demonstrate the shape of the saddle points predicted by the above analysis, we consider the real case, i.e. g_1, g_2, are both real coefficients. Figure 1 shows the shape of the cost function $J(G)$ around a real setting G_0 of the form (9). We have superimposed on this figure the level of the center value $J(G_0) = 1/3$. As can be seen from the figure, G_0 is clearly a saddle point of $J(G)$. Notice also that the positive slope of the saddle point can be seen in Figure 1 to be three times larger than the corresponding negative slope, as predicted by the analysis in section 4.1.

5 COMPUTER SIMULATIONS

In order to demonstrate the advantages of the algorithm (6) that stem from its second-order nature, we simulate a super-Gaussian input $\{s(k)\}$ that equals 0 with probability 3/4 and otherwise takes on equi-probably one of the four values $\pm 1 \pm j$. Notice that $s(k)$ is zero-mean and has a positive kurtosis ($K(s) = 3/4$). $s(k)$ is transmitted through a single-input-two-output polyphase channel, whose two phases have the following impulse responses: $h_1 = [1 \; 0.5]$, $h_2 = [1 \; 2]$ (notice that the polynomials $H_1(z)$ and $H_2(z)$ have no common roots, thus making SOS-based blind equalization possible). The signal is received with an SNR of 30 dB on each branch, and then passed through a two-phase fractionally spaced equalizer of 3 taps/phase. In two separate experiments, the equalizer is updated through the CMA and the decorrelation algorithm (6), respectively. In both cases the equalizer is center-spike initialized and the stepsize $\mu = 4 \times 10^{-4}$.

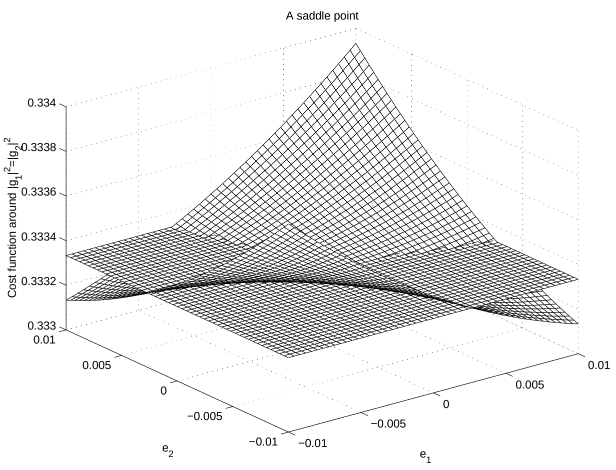

Figure 1: A saddle point ($N = 2$)

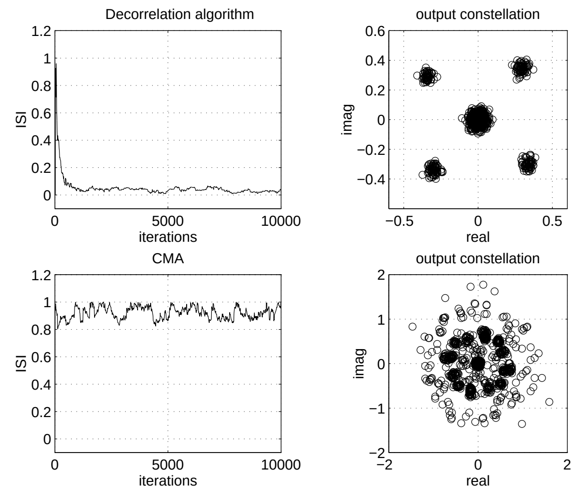

Figure 2: A computer simulation example

We also use instantaneous averaging ($\Delta = 1$) for the algorithm (6). We evaluate the algorithm performance by plotting the evolution of the closed-eye measure of the equalizer output as the algorithm adapts. As can be seen in Figure 2, the algorithm (6) manages to quickly open the channel eye and retrieve the transmitted constellation, in accordance with our expectations. On the other hand, the CMA fails to equalize the signal due to its super-Gaussian nature.

6 CONCLUSIONS

We have considered the problem of blind equalization of linear co-prime polyphase channels. Based on a recently shown whiteness theorem, we proposed a statistical criterion that relies on the SOS of the equalizer output in order to achieve BE of a white input. The optimization of this criterion can be done with a simple (decorrelation) algorithm. This approach is robust in the sense that the performance does not depend on the input distance from Gaussianity or on the estimation of the channel order. Moreover, a convergence analysis in the case of two coefficients in the combined channel/equalizer domain has shown the global convergence property of the proposed algorithm. These positive features have been corroborated by computer simulated results. We believe that these characteristics make the decorrelation approach attractive for a number of BE applications.

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