Blind Single-Channel Interference Rejection Using Godard's Criterion

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Abstract

It is well known that the performance of the Constant Modulus Algorithm (CMA) for interference cancellation is limited by a so-called *notch compromise*. This paper presents a new recursive structure based on Godard's Criterion for blind interference suppression which overcomes this drawback. This interference-rejection structure is based on the linear prediction of the interference. The effectiveness of the new structure is studied in the presence of Co-Channel Interference (CCI) and additive Gaussian noise. It is shown that this structure can cancel predictable CCI.

1. Introduction

Interference rejection is an important domain of research since it allows a more efficient utilisation of the available spectrum. This paper is concerned with the problem of the reception of a constant modulus signal, such as Frequency Modulation (FM) or Quadrature Amplitude Modulation (QAM), and in the presence of strong Co-Channel Interference (CCI). The interference is assumed to be narrowband. In this case, constant modulus blind equalizers can compensate partly the CCI [1-2]. The family of constant modulus blind equalizers was presented in 1980 by Godard, and in 1983 by Treichler [1] for FM signals (Constant Modulus Algorithm, CMA). The CMA uses a classical linear equalization scheme : the channel output is filtered by a Finite Impulse Response (FIR) filter whose coefficients are calibrated to minimise a cost function of the form :

$$J_{p,q} = E[(|v(n)|^p - 1)^q] \qquad (1)$$

where $E[\cdot]$ denotes statistical expectation, p and q are positive integers, $v(n)$ is the equalizer output given by $v(n) = \mathbf{X}^{\mathbf{t}}(n)\mathbf{H}(n)$, where **t** indicates transpose, $\mathbf{H}^{\mathbf{t}}(n) = \left(h_0(n), \ldots, h_{M\text{-}1}(n)\right)$ is the equalizer tap weight vector and $\mathbf{X}^{\mathbf{t}}(n) = \left(x(n), \ldots, x(n\text{-}M\text{+}1)\right)$ is the equalizer input data vector. The classical versions of the CMA are the CMA(2-2) and the CMA(1-2) for which $p = 2 = 2$ and $p = 1$, $q = 2$ respectively.

Using a stochastic gradient algorithm as an updating rule, the equalizer coefficients are adapted by :

$$\mathbf{H}(n+1) = \mathbf{H}(n) - \mu \{v(n) - T[v(n)]\}\mathbf{X}^*(n) \qquad (2)$$

where $T[v(n)]$ is a non-linear function depending on the cost function. For example, the non-linearity which corresponds to the CMA(1-2) is given by :

$$T[v(n)] = \frac{v(n)}{|v(n)|} \qquad (3)$$

Since the original proposition of the CMA, there have been extensive studies on the CMA. Treichler and Larimore have examined a problem that arises when using CMA to suppress narrow-band interference [2]. If both the interferer and the signal of interest have constant envelopes, then it is possible to find two different filter solutions, one which suppresses the interferer and another which *captures* the interferer and suppresses the desired signal.

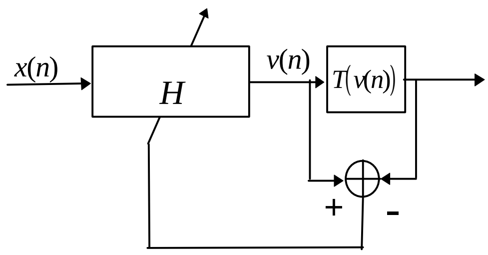

Figure 1 : CMA adaptive filter

In order to avoid the problem of capture, Ferrara [4] proposes to minimise a modified cost function :

$$J'_{1,2} = E[(|v(n)| - b)^2] \qquad (4)$$

where b is the amplitude of the target signal, given that it is approximately known or measurable. To prevent the CMA from locking to an interferer and to have a linear phase characteristic, the tap weights of the filter are constrained to be symmetric with respect to the center weight whose value is constrained to unity.

Even with those constraints, an interference-rejection filter based on the Godard's criterion is not able to notch out efficiently a CCI. This phenomenon can be explained by the *Notch Compromise* [1]. In notching the interference, the equalizer tends to distort the desired signal. Hence, the equalizer tends to a compromise, which results in poor performance in the presence of CCI.

2. A predictive approach

Decision Feedback (DF) filter is a well known structure for interference suppression [3]. The main idea of DF is very simple. The output of the receiver can be viewed as an estimate of the target signal. So, one can subtract this estimation from the received signal to isolate the interference. The output of this subtractor can be used as the input of a linear predictor. So, the DF filter attempts to whiten the interference and the noise only. The desired signal does not pass through the suppression filter, and then remains undistorded.

In this paper, we propose to use the same principle with the more general *bussgang* criterion [5]. Indeed, the output of a bussgang non-linearity can also be viewed as an estimate of the target signal. Particularly, the Gordard's criterion can be used in a DF structure in order to suppress narrowband interference. The principle of this « Predictive » CMA is depicted in Figure 2.

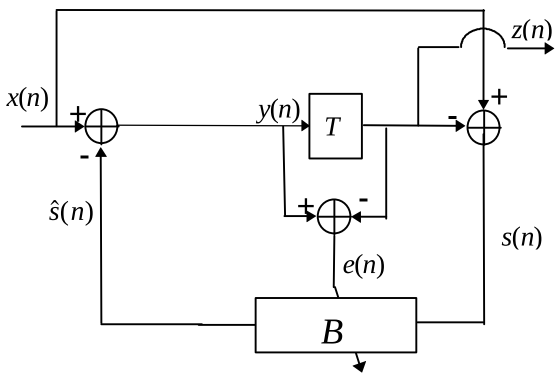

Figure 2 : principle of the Predictive CMA

To illustrate this idea, the case of a digital signal reception corrupted by a finite sum of Continuous Waves (CW) plus Additive White Gaussian Noise (AWGN) is considered.

The received signal $x(n)$ is given by the sum of three independent stationary random signals :

$$x(n) = b\ d(n) + I(n) + w(n) \qquad (5)$$

where $d(n)$ is a unit power, independent sequence of 4-QAM data symbols, $w(n)$ is AWGN of variance σ_w^2 and :

$$I(n) = \sum_{k=1}^{L} \alpha_k \exp j(\omega_k n + \theta_k) \qquad (6)$$

where α_k are constant amplitudes, θ_k are independent random phases uniformly distributed in $[0,2\pi]$, and ω_k are the normalised pulsation of the interference. In this paper, we also assume that b is known. So, we choose to minimise :

$$J'_{1,2} = E[(|y(n)| - b)^2\,] \qquad (7)$$

The CMA non-linearity is then

$$T[y(n)] = b\frac{y(n)}{|y(n)|} = z(n) \qquad (8)$$

By introducing the phase in the CMA cost function, one obtains :

$$J'_{1,2} = E\left[\left|y(n) - b\frac{y(n)}{|y(n)|}\right|^2\right] = E[\,|y(n) - z(n)|^2\,] \qquad (9)$$

with $y(n) = x(n) - \hat{s}(n)$ and $z(n) = x(n) - s(n)$.

So, we have

$$J'_{1,2} = E[\,|s(n) - \hat{s}(n)|^2\,] \qquad (10)$$

with $\hat{s}(n) = \mathbf{B}^{\mathbf{t}}(n)\mathbf{S}(n-1)$ where $\mathbf{S}^{\mathbf{t}}(n\text{-}1) = (s(n\text{-}1), ..., s(n\text{-}N))$ and $\mathbf{B}^{\mathbf{t}}(n) = (b_1(n),.., b_N(n))$ is the linear predictor.

It should be noted that the signal $s(n)$ is a non-linear function of its previous estimation :

$$s(n) = x(n) - T[x(n) - \hat{s}(n)\,] \qquad (11)$$

The theoretical performance analysis of this scheme is complicated by this non-linear dependence. However, for such algorithms, a usual approach is to commute between a CMA to a Decision Directed (DD) adaptation. The transition can be hard or soft. Far from the convergence point, the filter is adapted with the CMA(1,2). In this mode, if the non-linear function is sufficiently hard, one can assume that $s(n)$ does not depend on $\hat{s}(n)$ too strongly. Near convergence, the filter is adapted through a DD algorithm. In this mode, a standard approach is to assume that the decision is correct :

$$T[y(n)] = b\ d(n) \qquad (12)$$

If this assumption holds,

$$s(n) = I(n) + w(n) \qquad (13)$$

and the problem of reducing interference is equivalent to the linear prediction of interference plus noise. Using (10) and (13), the optimum tap weight vector is given by the well known Wiener-Hopf equation :

$$\mathbf{B}_{opt} = \mathbf{R}^{-1}\mathbf{P} \qquad (14)$$

where $\mathbf{R} = E[\mathbf{S}(n\text{-}1)\mathbf{S}^{\mathbf{H}}(n\text{-}1)]$ is the covariance matrix, $\mathbf{P} = E[s(n)\mathbf{S}^*(n\text{-}1)]$, and the superscript $^{\mathbf{H}}$ is the conjugate transpose.

The minimum mean-square error is given by :

$$J'_{opt} = \sum_{k=1}^{L} \alpha_k^2 + \sigma_w^2 - \mathbf{P}^{\mathbf{H}}\mathbf{B}_{\mathbf{opt}} \qquad (15)$$

If we introduce the power spectral density $D(\omega)$ of the interference $I(n)$, it can be deduced from (10) and (13) that :

$$J'_{1,2} = \frac{1}{2\pi}\int_{-\pi}^{\pi} |A(\omega)|^2 D(\omega)d\omega + \sigma_w^2\left(1 + \mathbf{B}^{\mathbf{H}}\mathbf{B}\right) \quad (16)$$

with $A(\omega) = 1 - B(\omega)$, where $B(\omega)$ denotes the frequency response of the filter B.

This expression shows that the interference can be totally eliminated in the extreme case of $\sigma_w^2 = 0$ and as soon as $N \geq L$. In comparison with the Transversal CMA, the performance is only bounded by the noise, and not by the notch compromise.

3. Computer Simulations

In order to verify the convergence behaviour of the proposed scheme, we consider the presence of one interferer, i.e. $L = 1$ in (6). Hence, the received signal is :

$$x(n) = b\ d(n) + \alpha_1 \exp j(\omega_1 n + \phi_1) + w(n) \quad (17)$$

With the proposed structure, it is clear that only one complex coefficient is needed ($N = 1$) to compensate perfectly an CW interference in the absence of noise. It is straightforward from (14) and (15) that

$$b_{1opt} = \frac{\alpha_1^2}{\alpha_1^2 + \sigma_w^2}\exp(j\omega_1) \qquad (18)$$

and $$J'_{opt} = \alpha_1^2 + \sigma_w^2 - \frac{\alpha_1^4}{\alpha_1^2 + \sigma_w^2} \qquad (19)$$

(18) and (19) show the influence of the noise on the steady state. It should be noted that b_1 lies on the unit circle when the noise vanishes.

A computer simulation is realised with α_1 = 1.0, σ_w^2 = 0.01 and ω_1 = 0.2 π. The filter B is adapted through a stochastic gradient algorithm :

$$\mathbf{B}(n+1) = \mathbf{B}(n) + \delta e(n)\mathbf{S}^*(n-1) \qquad (20)$$

where δ is a positive step size, $e(n) = y(n) - \frac{y(n)}{|y(n)|}$ far from convergence and $e(n) = y(n) - b\,d(n)$ near the convergence.

The performance index $J'_{1,2}$ is estimated using :

$$J'_{1,2}(n) = \lambda\ J'_{1,2}(n-1) + (1-\lambda)\left|y(n) - b\frac{y(n)}{|y(n)|}\right|^2$$

with $J'_{1,2}(0) = 1$, λ = 0.98, while the switching threshold is 0.1 (–10 dB).

In Figure 3, we have plotted two learning curves, one for the real part and one for the imaginary part of b_1. We observe that both curves converge in average to the theoretical value given by (18).

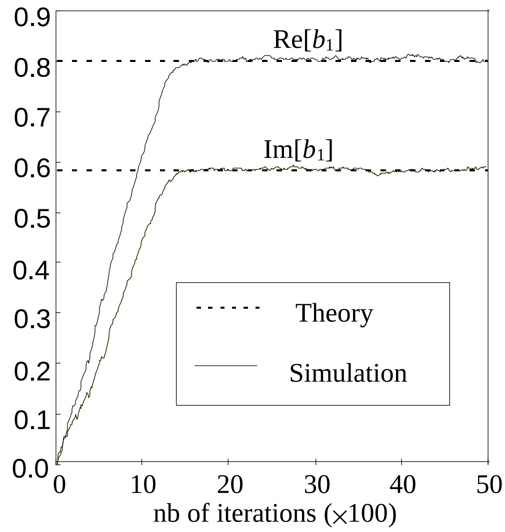

Figure 3 : real and imaginary part of b_1

In Figure 4, the convergence behaviour of the Predictive CMA is shown. Here we also observe a good agreement between theory and experiment.

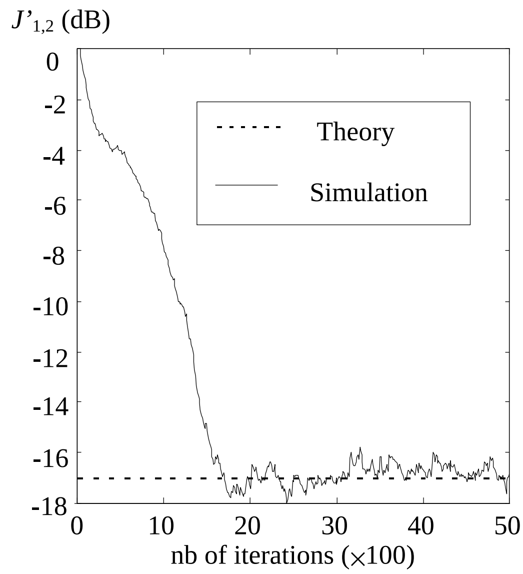

Figure 4 : convergence behaviour of the Predictive CMA

The Predictive CMA (PCMA) is also compared to its transversal counterpart. The received signal $x(n)$ is given by (5), and we have selected the following parameters : $b = 1$, $\sigma_w^2 = 0.01$ and

$$I(n) = 2.0\, e^{0.2\pi nj} + 1.8\, e^{0.4\pi nj} + 0.8\, e^{0.8\pi nj} \qquad (21)$$

The PCMA with $N = 6$ and $\delta = 0.005$ is compared to the Transversal CMA (TCMA), with $M = 65$ coefficients and $\mu = 0.00005$.

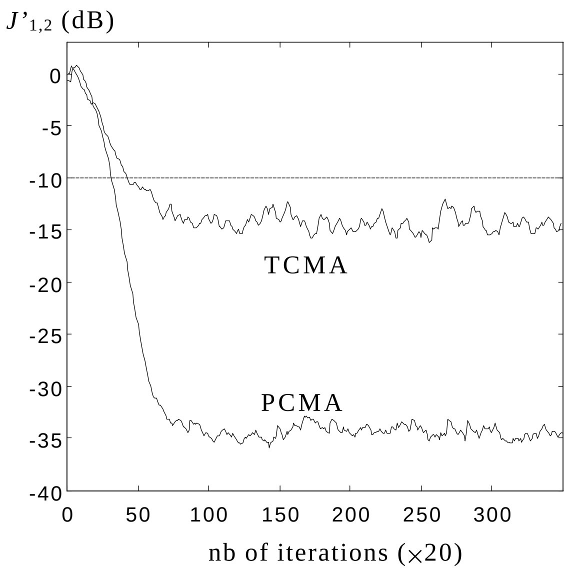

Figure 5 : Convergence behaviours of the Transversal and Predictive CMA

From the simulation (Fig. 5), it should be noted that the TCMA has high residual error due to the notch compromise, even with a great number of coefficients. This problem is illustrated on Figure 6.

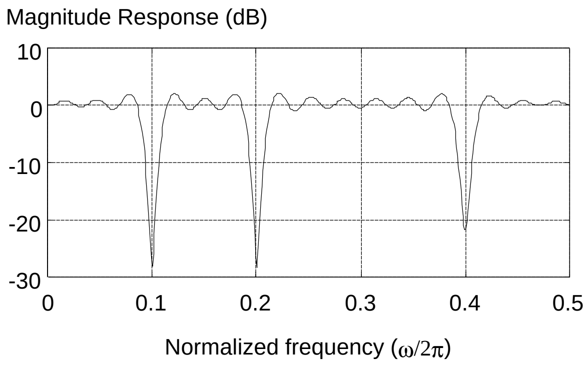

Figure 6 : Frequency response of the Transversal CMA

Moreover, the linear prediction of interference allows better performance with fewer coefficients. Figure 7 shows that the filter B converges to the classical linear predictor.

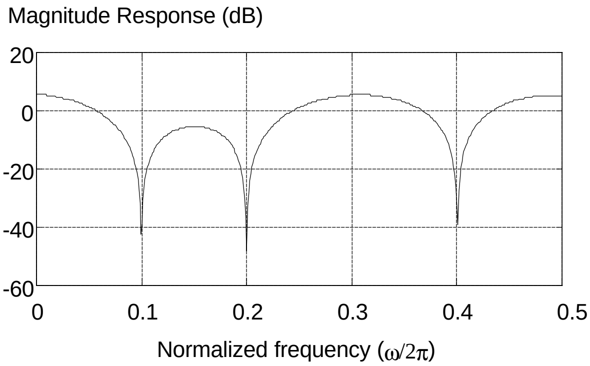

Figure 7 : Frequency response of 1-*B*(ω)

4. Conclusions

A new scheme for blind single-channel interference rejection is proposed. Assuming that the amplitude of the target signal is approximately known, the proposed scheme overcomes the notch compromise. Far from convergence the proposed Predictive CMA is adapted to minimise a CMA cost function, and near convergence, a DD approach is used. With the standard assumption that the decision is correct, the problem of reducing interference is equivalent to minimising a mean square prediction error. More generally, the proposed principle can be applied to the other Bussgang algorithms.

5. References

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