Bearings-only Target Motion Analysis by estimation of densities

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ABSTRACT

The aim of this paper is to compare, in the domain of the Bearings-only Target Motion Analysis (BTMA) with two observers, three approaches in terms of tracking performances : the Interacting Multiple Models (IMM) based on the Extended Kalman Filtering, the Hidden Markov Models (HMM) and the approximated densities filtering based on the maximum of entropy.

1. Introduction

The Bearings-only Target Motion Analysis (BTMA) problem with two observers equipped with passive sensors consists to estimate the position and velocity of a maneuvering vehicle, in our case a plane, from bearings-only measurements corrupted by noise. This is a non-linear problem of estimation.

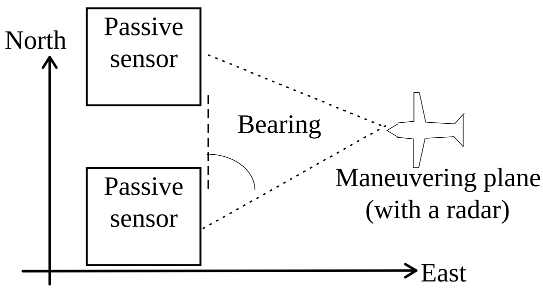

Figure 1.a. : Bearings-only Target Motion Analysis (BTMA) with two observers

If we note $X_N = \{x_1,...x_N\}$ the state vector sequence to estimate, which are position and speed in BTMA, and $Z_N = \{z_1,...z_N\}$ the observation sequence at time $\{t_1,...t_N\}$, which are bearings in BTMA, then the goal of estimation is to determine the conditional probability density function (pdf), $p(X_N/Z_N)$, which contains all the information on X_N.

In many cases, the assumption of a Gaussian pdf is done which needs only two parameters (mean and variance) to be completely determined.

The Kalman filtering is applied in the case of linear systems with Gaussian initialisation. The Gaussian property is preserved along the linear transformations and the Bayesian corrections. The Kalman filtering gives an estimation of the mean and the variance of the state given the observations, which completely determined the Gaussian pdf. A classical approach to extend the Kalman filtering to the non-linear case is to expand the equations in Taylor series about the nominal values. This lead to the Extended Kalman Filter (EKF). This extension is an approximation and the performances can be poor in the case of high non-linearity or non-Gaussian problems.
An other point that is important to see is that the problem of estimating a pdf is compatible with the evolution of many sensors for example in the BTMA. Indeed more than more parameters are measured (bearings, time difference of arrival, Doppler,...) and this measured are translated in pdf on the digitalized state space, not necessarily in a Gaussian way because the behaviour of the sensors is more than more well-known and this knowledge as to be taken in account in order to enhance the tracking. On the other hand, the estimation of a pdf is multimodal (i.e. multitargets) when the Kalman filtering is monomodal.
For the tracking, we need to follow the evolution of the pdf. In this paper we examine two methods : the Hidden Markov Model (HMM) and the approximated densities filtering based on the maximum of entropy. In a general point of view, we know the transition law $p(x_K/x_{K-1})$ and the observation law $p(z_K/x_K)$. Our aim is to estimate $p(x_K/z_K)$ recursively in time. At time k-1, we suppose to know $p(x_{K-1}/z_{K-1})$. Due to the Markov assumption for x, we can compute the prediction $p(x_K / z_{K-1}) = \int p(x_K / x_{K-1}).\, p(x_{K-1} / z_{K-1})$. Then the measurement z_K is used to do a correction in order to obtain $p(x_K/z_K)$ with Baye's formula :

$$p(x_K / z_K) = \frac{p(z_K / x_K).\, p(x_K / z_{K-1})}{p(z_K / z_{K-1})}$$

Correction   Prédiction

The knowledge of $p(x_K/z_K)$ gives the possibility to give an estimate $\hat{x}_K = I(x_K)$ of the state. For example :

$I(x_K) = \int_{x_K} x_K\, p(x_K / z_K)$ or $I(x_K) = \max_{x_K}(p(x_K/z_K))$.

The aim of the methods is to propagate the pdf and estimate $I(x_K)$.

2. The Interacting Multiple Models

The Interacting Multiple Models (IMM) in the case of two passive-only sensors has been developed in [1]. This algorithm is based on M Extended Kalman Filter (EKF) in parallels, each of them corresponding to a particular maneuvering hypothesis. Knowledge is introduced by modelling the change of hypothesis with a Markov chain. The complete equations are developed in [1].

3. The Hidden Markov Models

Elements on Hidden Markov Models (HMM) can be found in [2]. Applications of the HMM to the problem of tracking are developed in [3] and [4].
In the HMM with the Viterbi algorithm, the pdf is estimated on a fixed grid of the state vector x and the recursion is built following the equation $\delta(x_K) = p(z_K/x_K).\max_{x_{K-1}}(p(x_K/x_{K-1}).\delta(x_{K-1}))$.

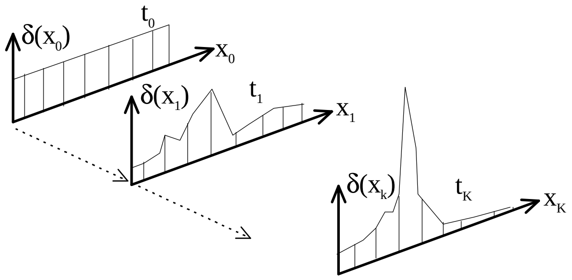

Figure 3.a. : HMM and Viterbi algorithm

One particularity of the HMM is the fact that it doesn't introduce a formal model of evolution of the target as in the IMM. A simple hypothesis of a maximal acceleration for example is sufficient. One the other hand, if the user wants it, it's also possible and easy to introduce complex prior information on the motion of the target.

We give here the mains equations :

Notations :

N the number of states in the model,

O_t : Observation at time t,

B : observation probability distribution in the state j

$B = \{b_j(O_t)\}$, $b_j(O_t) = pr[O_t/\text{state j at t}]$, $1 \le j \le N$

A : state transition probability distribution

$A = \{a_{ij}\}$, $a_{ij} = pr[\text{state j at t+1/state i at t}]$, $1 \le i,j \le N$

π : initial state distribution

$\pi = \{\pi_i\}$, $\pi_i = pr[\text{state i}]$, $1 \le j \le N$

T : number of observations.

Viterbi Algorithm:

Initialization

$$\delta_1(i) = \pi_i b_i(O_1), \quad 1 \le i \le N$$

Récursion

$$\delta_t(j) = \max_{1 \le i \le N}[\delta_{t-1}(i) a_{ij}] b_j(O_t)$$

$$\Psi_t(j) = \arg\max_{1 \le i \le N}[\delta_{t-1}(i) a_{ij}]$$

$$2 \le t \le T,\ 1 \le j \le N$$

Termination

$$q_T = \arg\max_{1 \le i \le N}[\delta_T(i)]$$

We get the state from the *backtracking* :

$$q_t = \Psi_{t+1}(q_{t+1}),\ t = T-1, T-2, \ldots 1$$

B is given by the model of sensor and can be on a analytic form (uniform, Gaussian,...) or can be given in a numeric form, more or less complex, in the state space. In the example of section 5, we have taken a Gaussian distribution.

A can be fixed with many criterion. In our case, we suppose that at time t the target can have a maximal acceleration (for example ±3g). This give the states possible at time t+1, each of them having the same probability. As we see, this hypothesis is very simple and could be easily improved in order to enhance the performances.

If we know nothing about the target, except some large bounds on position and speed, π can be an uniform distribution on the state space. On a other hand, it's possible to introduce prior information to reduce computation. One way to introduce this information is to apply an algorithm as least squares or IMM during a few seconds and then to switch quickly on HMM using the results of the first algorithm.

In our case, the state space is in 4 dimensions and the targets are highly maneuvering. The hypothesis chosen for A are very simple and the number of states possible can be very high. In order to reduce the computation, we have chosen a *sub-optimal* version of the HMM which consists on keeping the N_{max} states the more probable at time t.

4. Approximated densities filtering

In the approximated densities filtering ([5], [6]), the goal is to find the pdf of a system at time t in such a way that it maximises the entropy of the system at this time. This research is done under constraints γ_i with $\gamma_i = E[\varphi_i(x)]$ where φ_i are functions that fit the problem. It can be shown that the solution is $p(x) = \exp\left(\sum_i \lambda_i . \varphi_i(x)\right)$

where λ_i are Lagrange's multipliers. As indicated in the introduction, there is a prediction followed by an evolution, and then the update $p(x_K/z_K)$ is obtained with Baye's rule.

To describe the principle, we consider the non-linear model :

$$\begin{cases} x(t+1) = g(x(t), w(t)) \\ z(t) = x(t) + v(t) \end{cases}$$

where w and v are the state and observation noises.
The goal is to determine at each time t the pdf, p(x/z) with an exponential density which maximizes the entropy of the state under constraints.

$$p(x) = \exp(\sum_i \lambda_i . \varphi_i(x)) \text{ avec } \lambda_i \in \Re$$

Prediction.

The goal is to compute $p(x_t/z_{t-1})$. At this point, we are at time t-1 and we know $p(x_{t-1}/z_{t-1})$.

Constraints $\gamma_i(t)$

We have, at time t :

$$\gamma_i(t) \triangleq E[\varphi_i(x(t))] \triangleq \int \varphi_i(x).\, p(x_t / z_{t-1}).\, dx$$

And also:

$$\gamma_i(t) = E[\varphi_i(g(x(t-1), w(t-1)))] =$$
$$\iint \varphi_i(g(x,w)).p(x_{t-1} / z_{t-1}).\, f_w(w).\, dx.\, dw \quad \textit{(1)}$$

Knowing $\varphi_i(g(x(t-1), w(t-1)))$, $p(x_{t-1}/z_{1->t-1})$ and f_w, it's possible to compute the constraints γ_i.

Lagrange's parameters $\lambda_i(t)$:

At this point, we have computed the constraints $\gamma_i(t)$.
We know that

$$\gamma_i(t) \triangleq E[\varphi_i(x(t))] \triangleq \int \varphi_i(x).\, p(x_t / z_{t-1}).\, dx \text{ and}$$

$$p(x_t / z_{t-1}) = \exp\left(\sum_{j=0}^{k} \lambda_j \varphi_j(x) \right)$$

In order to calculate the Lagrange parameter's $\lambda_i(t)$, we have to solve the following system of (k+1) equations :

$$\gamma_i(t) = \int \varphi_i(x).\exp\left(\sum_{j=0}^{k} \lambda_j \varphi_j(x) \right) dx \qquad \text{with } i \in [0,k]$$

After that, we can compute :

$$p(x_t / z_{t-1}) = \exp\left(\sum_i \lambda_i (t / t-1).\, \varphi_i(x) \right)$$

Update

The goal is to compute $p(x_t/z_t)$. At this point we are at time t, we have computed $p(x_t/z_{t-1})$ and have an observation z_t.

With the equation z(t) = x(t) + v(t), the predicted law improves with the observation z_t.

In writing :

$$\begin{cases} x = x \triangleq h_1(x,z) \\ v = z - x \triangleq h_2(x,z) \end{cases}$$

One can easily obtain :

$$p_{t/t}(x_t / z_t) = \frac{p_X(x).\, f_V(z-x)}{f_Z(z)} = \exp(m(x))$$

$$\text{with } f_Z(z) = \int_{-\infty}^{+\infty} f_{X,Z}(x,z)dx = \int_{-\infty}^{+\infty} f_X(x) f_V(z-x)dx$$

and m(x) a function.

Choice of functions φ_i in BTMA

In many cases, the integral in *(1)* is not computable analytically. One method to approximate this integral is to choose functions φ_i as part of a grid of the space (position, speed) and then to approximate the prediction by Monte-Carlo methods. This principle is also used in [7].

The figure 4.a shows the principle of the method : the prediction is done by Monte-Carlo's runs, and the evolution by Bayes'rule.

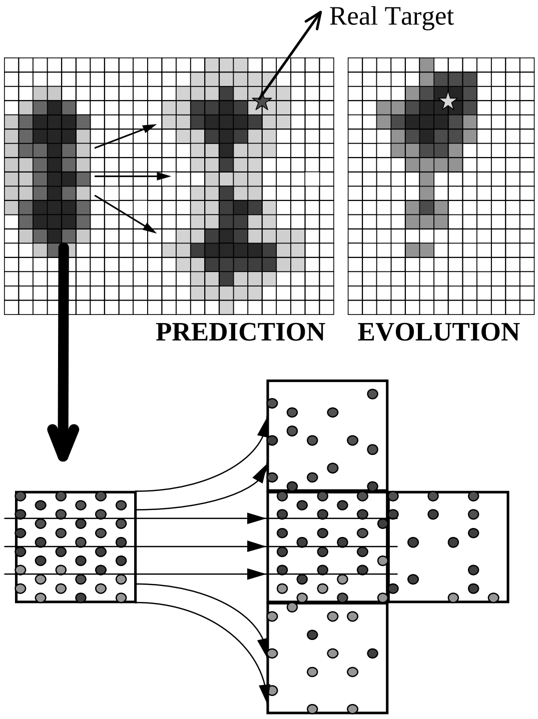

Figure 4.a. Monte Carlo's method.

5. Simulations

We have done a simulation with a maneuvering plane being at 50 km form the two observers. The probability of detection is 0.8 and one of station has no measurements during a few seconds (target hidden by a hill). The observations are refreshed with a period of 1second.

We show on figure 5.a the estimation of density by the algorithm described in section 4. The estimator chosen is the maximum of the density at each time, the real trajectory is also drawn. We have tested the three algorithms IMM, HMM and approximated densities on the same scenario. On figure 5.d., we represent the error in distance. One can see that HMM and approximated density have good performances, particularly when there is an observer alone. This kind of behaviour has been verified with Monte-Carlo test on many simulations and many cases. Note that a few simulations have been done with the "optimal" HMM and the results are equivalents to those obtained with the approximated density.

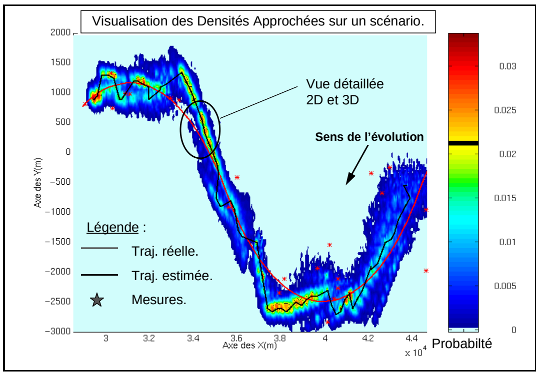

Figure 5.a. Approximated density.

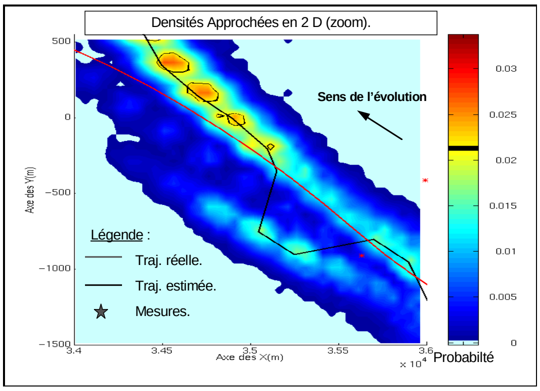

Figure 5.b. Zoom 2D on the approximated density.

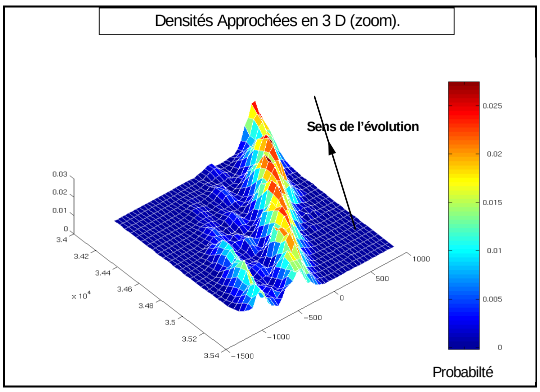

Figure 5.c. Zoom 3Don the approximated density.

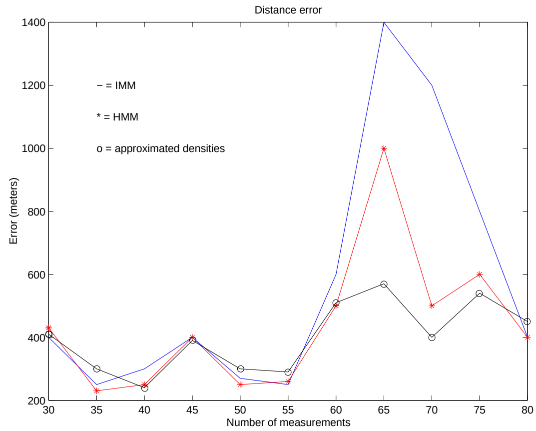

Figure 5.d. Distance error for IMM, HMM and approximated density.

6. Conclusion

We have seen in this paper the application of three algorithms to the BTMA problem with two observers. It's seems that the HMM and the approximated densities have good performances comparing it to the Extended Kalman Filtering and it could provide an enhancement of the performances in a complete association-fusion-tracking chain. In terms of computations the EKF needs less calculations than the two others algorithms. The next step that we are studying is to simplify the HMM and the approximated density algorithm in order to have a good compromise computations-performances.

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