COMPLEXITY CONSTRAINTS—A WAY TOWARD SIMPLER STRUCTURING SYSTEMS

Pertti Koivisto[1] and *Pauli Kuosmanen*[2]

Tampere University of Technology, Signal Processing Laboratory,
P.O.Box 553, FIN-33101 Tampere, Finland
e-mail: [1]mapeko@uta.fi, [2]pqo@cs.tut.fi

ABSTRACT

A method of incorporating implementation aspects in the algorithm-level design of nonlinear filters is proposed. As a case study, the trade-off between the visual properties and the complexity of soft morphological filters is studied using training-based optimization methods. Specifically, it is shown that the use of the complexity constraints can provide the filter designer valuable information on to what extent it is reasonable to increase the complexity of the filter structure.

1 INTRODUCTION

Nonlinear filter optimization techniques typically pay no attention to the implementation stage but concentrate only on noise attenuation and detail preservation capabilities. The goal of this paper is to present a novel method of also incorporating different resources of the implementation architecture, like arithmetic operations and their control, data storage, external communication, already in the algorithm-level design of the nonlinear filters. This can provide a compromise between the visual properties (the noise removal and detail preservation capabilities) and the complexity of nonlinear filters. As a case study, soft morphological filters are used.

A basic fact is that a complex filter can usually remove noise and preserve details more effectively than a small and simple filter. On the other hand, the more complex the system is, the heavier its implementation typically is for practical situations. Thus, a key question is whether the improvement in the quality of the results is large enough to justify the increase in the complexity.

In this paper, we study the trade-off between the noise removal and detail preservation ability of the filter on one hand and the ease of implementation of the filter on the other hand utilizing training-based optimization of the soft morphological filters [1]. In our study we use a simple complexity measure, but the proposed method generalizes to any numerical complexity measure.

2 SOFT MORPHOLOGICAL FILTERS

Soft morphological filters [2] form a class of stack filters and were introduced to improve the behavior of standard flat morphological filters in noisy conditions. They have many desirable properties, e.g., detail preservation [3]. The two basic soft morphological operations are *soft erosion* and *soft dilation*. Based on these operations, two compound operations, *soft opening* and *soft closing*, can be defined in the usual way.

The detail preservation ability, as well as the noise removal ability, of a soft morphological filter depend on the size and shape of its structuring set and on the value of its order index.

Definition 1 The *structuring system* $[B, A, r]$ consists of three parameters, finite sets A and $B, A \subseteq B \neq \emptyset$, in $\mathbf{Z}^m$ (where $m \in \mathbf{Z}_+$ denotes the dimensionality of the signal space) and an integer r satisfying $1 \leq r \leq \max\{1, |B \setminus A|\}$. The set B is called the *structuring set*, A its (hard) *center*, $B \setminus A$ its (soft) *boundary*, and r the *order index* of its center or the *repetition parameter*.

The *translated set* T_x, where the set T is translated by x, $x \in \mathbf{Z}^m$, is defined by $T_x = \{x + t : t \in T\}$. A *multiset* is a collection of objects, where the repetition of objects is allowed. For example, $\{1, 1, 1, 2, 3, 3\} = \{3\diamondsuit 1, 2, 2\diamondsuit 3\}$ is a multiset.

Soft morphological operations transform a signal $f\colon \mathbf{Z}^m \to \mathbf{R}$ to another signal by the following rules.

Definition 2 *Soft erosion (soft dilation)* of f by the structuring system $[B, A, r]$ is denoted by $f \ominus [B, A, r]$ $(f \oplus [B, A, r])$, where $f \ominus [B, A, r](t)$ $(f \oplus [B, A, r](t))$ is the rth smallest (largest) value of the multiset $\{r\diamondsuit f(a) : a \in A_t\} \cup \{f(b) : b \in (B \setminus A)_t\}$.

If $r = 1$ there is no difference (in regard to the result of the filtering) whether an element belongs to the hard center or to the soft boundary. Thus, for simplicity, we can suppose in the following that if $r = 1$ then all elements in the structuring set belong to the soft boundary, that is, $A = \emptyset$.

The above assumption, together with the restrictions for the values of the order index in Definition 1, guarantees that each structuring system defines a unique soft morphological erosion (dilation). Thus, we can in this paper speak about the *complexity of the filter* (i.e., soft erosion/dilation) although, to be exact, we should speak about the *complexity of the structuring system*.

[1]On leave from Department of Mathematical Sciences, University of Tampere.

3 COMPLEXITY MEASURE

Any soft morphological filter is completely determined by its structuring system. When considering the complexity of the structuring system, we can see that the most important point is, of course, the size of the structuring set. The meaning of the order index is usually relative to the size of the structuring set. However, when r has one of the extreme values, e.g., $r = 1$ or $r = |B \setminus A|$, the implementation complexity of the filter may usually be highly reduced.

Moreover, the concrete implementation of the filter depends highly on the "situation at hand". That is, the same filter can be implemented in many ways, depending on the technology used and on what points (the size of the memory needed, the speed of the filtering, etc.) are considered to be important. Thus, there are also many reasonable interpretations of the complexity of the structuring system.

However, the proposed method suits any interpretation of the complexity if one has a way to describe the interpretation numerically. Thus, without any loss of generality, we will consider in this case study the following simplified measure for the *complexity* $\mathcal{C}$ of the 2-D structuring system:

$$\mathcal{C}([B, A, r]) = |B| + |(B \setminus A)_{(0,1)} \setminus (B \setminus A)_{(0,0)}|,$$

where $|\cdot|$ is the cardinality of the set in question.

The motivation for this definition of the complexity measure is the following. It is known that in situations with small and moderate amounts of one-sided impulsive noise, the size of the hard center of a good structuring system is small. In fact, usually in this kind of situation the size of the hard center of the optimal structuring system is one [1]. Hence, the size of the hard center has now only a minor (although not zero) effect on the complexity of the filter. Thus, the key question concerning the time complexity of the implemented filter is the finding of the rth smallest (or largest) element from the signal values inside the soft boundary. For this, there are several methods.

For example, if the size of the soft boundary is small some straightforward selection method (or even sorting method) may be efficient. Then, the size of the structuring set is the most important factor in the complexity. On the other hand, if the size of the soft boundary is large it is often reasonable to use, e.g., some histogram-based method, in which case one has the already processed signal samples in the previous window position and only few new samples are entering the window. Thus, more weight should be given to those elements that are "new" in each single point t. If the usual raster scan (i.e., the filter advances from left to right on every row proceeding from top to bottom) is used this means that more weight should be given to those elements that are not common to $(B \setminus A)_{t+(0,1)}$ and $(B \setminus A)_t$. As a compromise, the presented complexity measure is used.

Example 1 Let us suppose $A = \{(0,0)\}$, $B_1 = \{(-2,0), (-1,0), (0,0), (1,0), (2,0)\}$, $B_2 = \{(-1,0), (0,1), (0,0), (0,-1), (1,0)\}$, and $2 \leq r \leq 4$. Then

$$\mathcal{C}([B_1, A, r]) = |B_1| + |\{(0,0), (3,0)\}| = 7$$

and

$$\mathcal{C}([B_2, A, r]) = |B_2| + |\{(0,0), (1,1), (1,-1), (2,0)\}| = 9.$$

Example 2 Clearly, the complexity of the identity filter is two. Moreover, the assumptions and restrictions for the order index r imply that always $\mathcal{C}([B, A, r]) \geq 2$.

One should also remember that the complexity used here is just a simple example of one possible definition of the complexity of the structuring system, and the purpose is not to produce any standard. Moreover, the definition of the complexity of the structuring system is now made having the test case of the paper in mind.

4 OPTIMIZATION

Although there are no analytical criteria for deciding which soft morphological operation (and with which parameters) is the best for some situation, a suitable operation and its parameters can be found using supervised learning methods, e.g., simulated annealing and genetic algorithms [1]. Of course, some training set, for which the desired output is known, is needed.

The optimization methods presented in [1] allow one to handle the complexity of the structuring system in several ways. First, it is possible to use the complexity as an error criterion, perhaps together with some constraints such as at least 80 % of the original noise must be removed. However, this generally leads to an ambiguous set of solutions from which it is difficult to choose the best one in some other sense.

Another possibility (the one used in this paper) is to use the complexity as a constraint, i.e., to search the optimal filter, e.g., under the MAE, provided that the complexity of the filter is under a beforehand fixed level. A third possibility is to make a new error criterion by simply combining, e.g., the MAE and the complexity value. The easiest way to do this is simply to multiply the MAE and the complexity value by some weights and then to add these two results together. The problem in this method is that it is not necessarily easy to select suitable weights for the parts.

5 TEST CASE

The experimental tests in this paper are based on the following 2-D test case. The training image is a 128×128 part of the 512×512 gray-level image "Harbour", with $(60, 190)$ as the coordinates of the upper left corner. The comparison images are the entire image "Harbour" and the 512×512 gray-level image "Lena". For test purposes, all images were corrupted by positive impulsive noise with noise probabilities $0.01, 0.05, 0.1$, and 0.2. Here, a positive impulse means the largest possible pixel value. In each case the optimal soft erosions with complexity less than or equal to $2, 3, 4, \ldots, 20$ were found. The error criterion used in the experiments reported in this paper was the MAE. The same experiments were also carried out using the MSE as the criterion. The results were similar to those presented here, but to save space they are omitted.

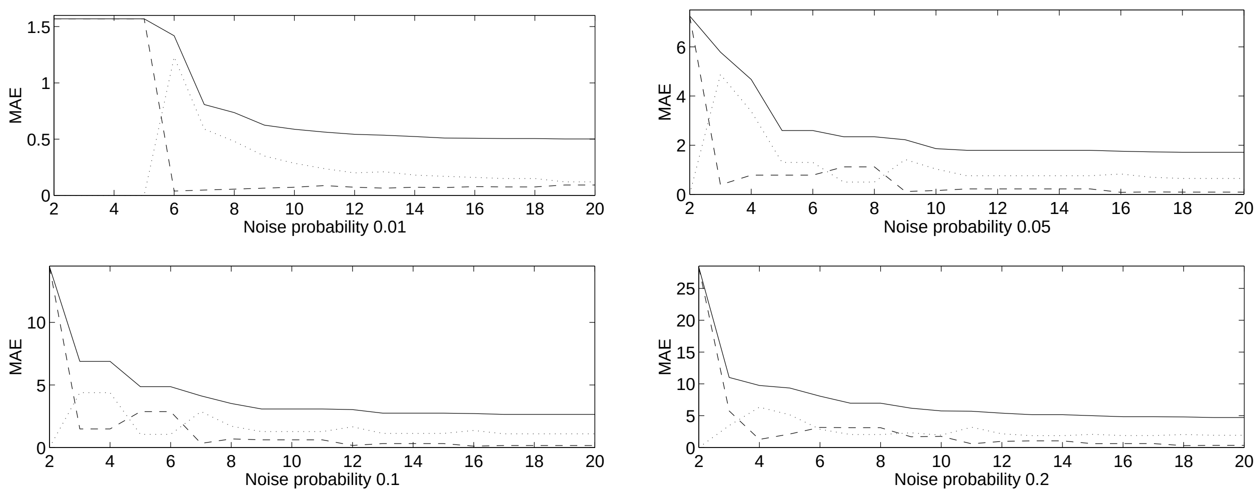

Figure 1: The MAEs between the original training image and the filtered noisy training images as a function of the maximal complexity of the structuring system, solid: the total MAE, dashed: the MAE caused by the impulses not removed, dotted: the MAE caused by the filtering of the noncorrupted pixels.

6 EXPERIMENTAL RESULTS

Figure 1 shows some results of our experiments, in which we studied the performance and the properties of the soft morphological filters optimal under the complexity constraints in our test cases. Because the identity filter is now the only reasonable filter whose complexity is two the leftmost error values also express the original error between the noisy and the noise-free images.

The error between the filtered and the noise-free images originates in three sources, namely, in the error caused by the impulses not removed, in the error caused by the removed impulses, and in the error caused by the filtering of the noncorrupted pixels. Of course, the more impulses are removed, the more the error caused by the removed impulses increases. The relation between the other two noise sources, on the other hand, is the key concept in the selection of applicable filters.

Let us first consider our test case with the noise probability 0.01. In this case the amount of the impulses, and thus the possible improvement in the original error, is so small that the error caused by the filtering of the noncorrupted pixels easily becomes larger than the error between the orinal noisy and noise-free images. Hence, for example, the identity filter is the optimal soft erosion when the maximal complexity of the structuring system is less than or equal to five.

In fact, the complexity of the simplest soft erosion $[B, A, r]$ that results in a decreased total error is now six. Moreover, the significant filter parameters are: $A = \{(0, 0)\}$, $|B \setminus A| = 3$, and $r = 3$. Unfortunately, also this soft erosion is too powerful in the sense that it alters too many pixels that are not corrupted. Thus, the error caused by the filtering of the noncorrupted pixels is in this case quite large, as can be seen, e.g., from the dotted curve in the leftmost upper image in Figure 1. The impulse removal properties of this soft erosion, on the other hand, are excellent. Thus, the filtering problem with the noise probability 0.01, as with other low noise probabilities, is not how to remove the impulses with efficiency but how at the same time not to change too many pixels that are not corrupted.

When the maximal complexity of the soft erosion increases, the optimal soft erosion changes in such a way that the conditions $A = \{(0, 0)\}$ and $r = |B \setminus A|$ also hold for the new structuring system. Thus, changes may happen only in the size and the shape of the soft boundary $B \setminus A$. In practice, however, the size of the soft boundary never decreases when the complexity of the soft erosion increases. In contrast, the size of $B \setminus A$ increases steadily together with the complexity of the soft erosion.

The effect of this increase is, naturally, that the number of the changes in the pixel values during the filtering process decreases. Thus, also the number of the falsely altered pixels decreases. At the same time also less impulses are removed, but the amount of such impulses is, however, so small that the total error decreases steadily. Of course, there is some limit after which the error caused by the remaining impulses starts to dominate, and the total error starts to increase. As can be seen from the leftmost upper image in Figure 1, this limit is probably already encountered.

Let us next consider the test cases with larger noise probabilities than 0.01. From the total MAE between the original training image and the filtered noisy training images (solid curves in Figure 1) it can be noticed that when the possible maximal complexity increases the error decreases steadily. For small complexity values the decrease is very rapid but when the complexity is larger than 5, the decrease becomes slower, and if the complexity is larger than 10, the decrease in the error is no longer significant compared to the increase in the complexity.

The dashed curves in Figure 1 show that all optimal filters, excluding the identity filter, remove impulses satisfactorily. Thus, a good impulse removal ability can already be obtained by soft morphological filters with low complexity.

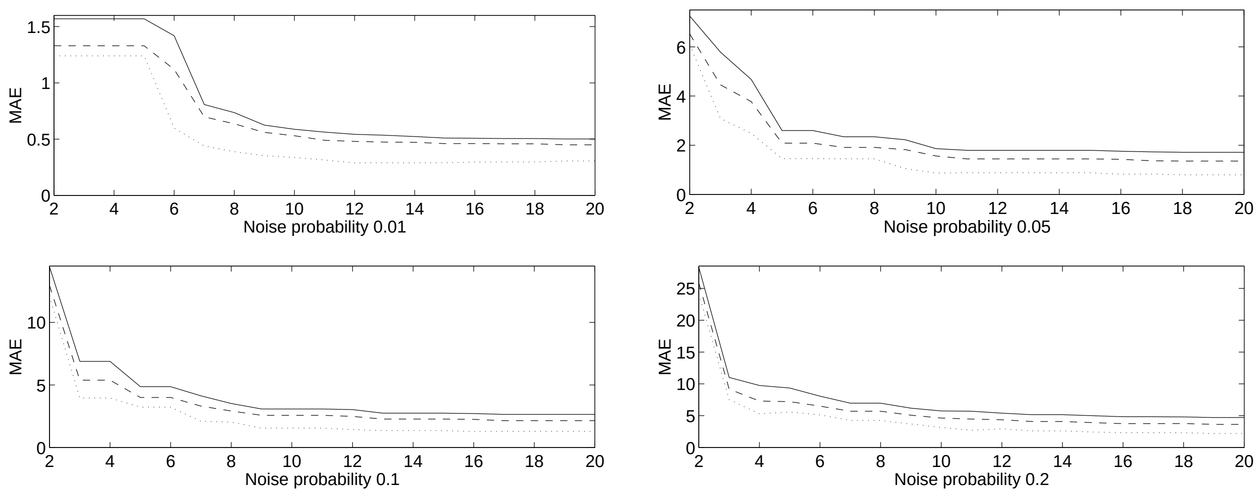

Figure 2: The MAEs between the original and the filtered (noisy) images, as a function of the maximal complexity of the structuring system, solid: the training image, dashed: "Harbour", dotted: "Lena".

When the complexity of the soft erosion is very small (three of four) the filtering of the noncorrupted pixels causes quite a lot of error (cf. the dotted curves in Figure 1). The reason for this is that the filters in question (the standard erosions of size two or three and the three-point median) cause heavy blurring. In these cases also most of the total error derives its origin from the filtering of the noncorrupted pixels.

With smallish complexities (from five to ten) Figure 1 illustrates quite well the struggle between the impulse removal (error caused by the remaining impulses) and the detail preservation (error caused by the lost details) abilities of the optimal filters. With larger complexities the optimal soft erosions are already able to remove almost all of the impulses, and the decrease in the total error is now based solely on the fact that a more complex structuring system is able to handle those pixels better that are not corrupted.

Figure 2 illustrates the performance of the found optimal soft erosions in the comparison images. As can be seen, in all cases the results are similar to those in the training image. Hence, the conclusions concerning the effect of the complexity of the structuring system are not limited to the training image, but they hold in other images as well. The lower overall level of the errors is now explained by the fact that the comparison images have fewer details than the training image, and thus the noise removal in them is easier.

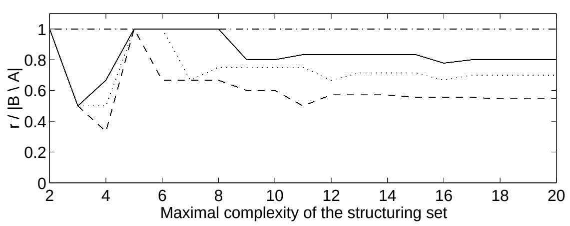

Figure 3: The ratios $r/|B\setminus A|$ of the found optimal structuring systems correponding to the impulse probabilities 0.01 (dash-dotted), 0.05 (solid), 0.1 (dotted), and 0.2 (dashed).

Figure 3 gives another view on the properties of the found optimal soft erosions. As can be seen, excluding the small complexities, the ratio $r/|B \setminus A|$ is quite invariant with respect to the maximal complexity of the optimal structuring system. That is, the ratio $r/|B\setminus A|$ of the optimal soft erosion has about the same value regardless of the desired maximal complexity of the structuring system. This, once again, confirms the crucial role of this ratio in the selection of applicable soft morphological filters (cf. [1]).

7 CONCLUDING REMARKS

In this paper, we introduced a general and simple method of integrating some implementation aspects already in the nonlinear filter parameter optimization. Concerning the soft morphological filters and the complexity criterion used, the following rules of thumb can be used when applicable filters are sought. First, a good soft morphological filter should be complex enough not to cause heavy blurring. Second, if the filter is already complex enough to remove almost all of the noise, a more complex structuring system is hardly ever reasonable, except with very low noise probabilities. Between these borderlines lies the set of the practical filters. The exact size of the borders depends, of course, on the application at hand.

References

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