Multidimensional Multirate Filter without Checkerboard Effects

Yasuhiro Harada *Shogo Muramatsu* *Hitoshi Kiya*
Graduate Course of Electronics Eng., Graduate School of Eng.,
Tokyo Metropolitan University, Hachioji-shi,192-03 Japan.
Tel: +81 426 772745; fax: +81 426 772756
e-mail: kiya@eei.metro-u.ac.jp

ABSTRACT

The checkerboard effect is caused by the periodic time-variant property of interpolators which compose a multirate system. Although the conditions for some one-dimensional(1D) multirate systems to avoid the checkerboard effect have been shown, the checkerboard effect in multidimensional(MD) multirate systems has not been considered. In this paper, the conditions for MD multirate filter and filter bank to avoid the checkerboard effect are considered. Besides, the properties of the MD filters with no checkerboard effect are given. Simulation example shows that the checkerboard effect can be avoided by using the proposed condition.

1 Introduction

Multirate signal processing is widely used in subband coding, and adaptive signal processing, etc [6]. Recently Multidimensional(MD) multirate signal processing is expected in sampling format conversion, and in many other applications of digital video processing.

The checkerboard effect is caused by the periodic time-variant property of the multirate filters which consist of up-samplers and digital filters. The conditions for some one-dimensional(1D) multirate systems to avoid the checkerboard effect have been shown[1, 2, 3, 4, 5]. In [1], the checkerboard effect for 1D multirate filter banks was pointed out. In [2, 4], the checkerboard effect was considered and some theorems about the conditions to avoid the checkerboard effect were derived. In [3, 5], the conditions for 1D multirate filter banks were considered and some design methods were given. However, the conditions for MD multirate systems without checkerboard effect have not been considered.

In this paper, some theorems about the conditions for MD multirate filters and filter banks to avoid the checkerboard effect are derived. Simulation examples show that the checkerboard effect can be avoided by using the proposed conditions.

All through this work, we use the following notations for D-dimensional systems and signals[6].

$\mathbf{z}$: the $\mathbf{z}$ denotes the $D \times 1$vector $[z_0 z_1 \cdots z_{D-1}]^T$. The subscript T on the vector denotes the transposition. $\mathbf{z}^{(\mathbf{M})}$: the $\mathbf{z}^{(\mathbf{M})}$ is the $D \times 1$vector whose kth component is obtained as $(\mathbf{z}^{(\mathbf{M})})_k = z_0^{M_{0,k}} z_1^{M_{1,k}} \cdots z_{D-1}^{M_{D-1,k}}$ where $\mathbf{M}$ is a $D \times D$ nonsingular integer matrix, and $M_{k,l}$ denotes the k,lth element of $\mathbf{M}$. $LAT(\mathbf{M})$: $LAT(\mathbf{M})$ is the set of integer vectors $\mathbf{p}$ defined by $\mathbf{p} = \mathbf{M}\mathbf{n}, \mathbf{n} \in \mathcal{N}$, where $\mathcal{N}$ is the set of $D \times 1$ integer vectors and $\mathbf{n}$ is a $D \times 1$ integer vector $[n_0, n_1, \cdots, n_{D-1}]^T$. $FPD(\mathbf{M})$: $FPD(\mathbf{M})$ is the set of vectors defined by $FPD(\mathbf{M}) = \{\mathbf{M}\mathbf{x} | \mathbf{x} \in [0,1)^D\}$, where $[0,1)$ denotes the set of $D \times 1$ vectors $\mathbf{x}$ so that all elements of $\mathbf{x}$ satisfy $0 \le x_i < 1, i = 0, \cdots, D-1$. $\mathcal{N}(\mathbf{M})$: $\mathcal{N}(\mathbf{M})$ is the set of integer vectors in $FPD(\mathbf{M})$.

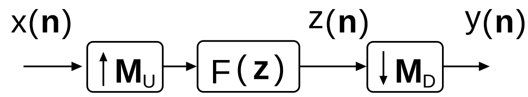


Figure 1: MD Multirate Filter

2 MD Multirate Filter Without Checker board Effect

In this section, some theorems about the conditions for MD multirate filters to avoid the checkerboard effect are shown.

2.1 MD Multirate Filter

Fig.1 shows an MD multirate filter, where $\mathbf{M}_U$ and $\mathbf{M}_D$ denote an up-sampler with the factor $\mathbf{M}_U$ and a down-sampler with the factor $\mathbf{M}_D$ respectively. $\mathbf{M}_U$ and $\mathbf{M}_D$ are $D \times D$ non singular integer matrices and they are mutually prime matrices which mean that $LAT(\mathbf{M}_U) \subseteq LAT(\mathbf{M}_D)$ and $LAT(\mathbf{M}_D) \subseteq LAT(\mathbf{M}_U)$ are not satisfied[7]. $F(\mathbf{z})$ is the transfer function of an MD FIR filter. where $f(\mathbf{n})$ denotes the impulse response of $F(\mathbf{z})$.

The MD multirate filter in Fig.1 can be equivalently represented as shown in Fig.2, where $R_{\mathbf{k}_l}(\mathbf{z})$ is a type-II polyphase filter of the filter $F(\mathbf{z})$ and defined as

$$F(\mathbf{z}) = \sum_{l=0}^{M-1} \mathbf{z}^{(\mathbf{k}_l)} R_{\mathbf{k}_l}(\mathbf{z}^{(\mathbf{M}_U)}), \tag{1}$$

where $\mathbf{k}_l$ is a $D \times 1$ integer vector defined as $\mathcal{N}(\mathbf{M}_U)$[6] and M is the absolute determinant of $\mathbf{M}_U$.

In 1D multirate systems, it is known that the checkerboard effect is caused by the periodic time-variant property of multirate filters which consist of up-samplers and digital filters[1, 2, 3, 4, 5]. In the following, we consider the checkerboard effect in the MD multirate filter by using the above expression.

2.2 Periodicity of Step Response

In Fig.1, when the input signal $x(\mathbf{n})$ is the unit step signal $(u(\mathbf{n}) = 1, \mathbf{n} \in [0,\infty)^D)$ and the components $n_0, n_1, \cdots, n_{D-1}$ of the vector $\mathbf{n}$ are enough large, the signal $z(\mathbf{n})$ becomes the steady state value $s_{\mathbf{k}}(\mathbf{n})$ as

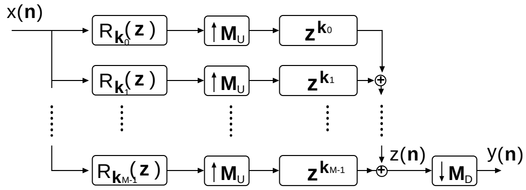


Figure 2: Polyphase structure

$$s_{\mathbf{k}}(\mathbf{n}) = \begin{cases} R_{\mathbf{k}_0}(\mathbf{1}) & \mathbf{n} = \mathbf{M}_U\mathbf{t} - \mathbf{k}_0 \\ \vdots & \\ R_{\mathbf{k}_{M-1}}(\mathbf{1}) & \mathbf{n} = \mathbf{M}_U\mathbf{t} - \mathbf{k}_{M-1} \end{cases} \quad (2)$$

where $R_{\mathbf{k}_l}(\mathbf{1})$ is given as

$$R_{\mathbf{k}_l}(\mathbf{1}) = R_{\mathbf{k}_l}(e^{j\omega_0}, \cdots, e^{j\omega_{D-1}})|_{\omega_0,\cdots,\omega_{D-1}=0}, \quad (3)$$

and denotes the DC gain of the polyphase filter $R_{\mathbf{k}_l}(\mathbf{z})$. Thus the output signal $y(\mathbf{n})$ is reduced to $ss(\mathbf{n})$ as

$$ss(\mathbf{n}) = s_{\mathbf{k}}(\mathbf{M}_D\mathbf{n}), \quad (4)$$

From Eq.(2), $s_{\mathbf{k}}(\mathbf{n})$ is not constant and has the period $\mathbf{M}_U$. As a result, $ss(\mathbf{n})$ also has the period $\mathbf{M}_U$. Note that $\mathbf{M}_D$ does not cause the checkerboard effect. Thus, we can see that the multirate systems which include up-samplers and digital filters have the periodic step response. The periodic artifact caused by this periodic step response is called the checkerboard effect.

2.3 The Conditions for MD Multirate Filter without Checkerboard Effect

We derive some theorems about the conditions for MD multirate filters without checkerboard effect.

Theorem 1 *A necessary and sufficient condition for MD multirate filters to avoid the checkerboard effect is given as*

$$R_{\mathbf{k}_0}(\mathbf{1}) = \cdots = R_{\mathbf{k}_{M-1}}(\mathbf{1}). \quad (5)$$

Proof. To avoid the checkerboard effect, the steady state values of the step response must be constant. From Eq.(2), it is clear that this condition is equal to Eq.(5). $\diamond$

Theorem 2. *A necessary and sufficient condition for MD filters to avoid the checkerboard effect is given as*

$$F(e^{j\boldsymbol{w}}) = 0 \quad at \quad \boldsymbol{w} = 2\pi\mathbf{M}_U^{-T}\mathbf{k}'_i, \quad (6)$$

$$\mathbf{k}'_i \in \mathcal{N}(\mathbf{M}_U^T)\mathbf{k}'_i \neq \mathbf{0},$$

where $F(e^{j\boldsymbol{w}})$ *is the frequency response of* $F(\mathbf{z})$ *and* $\boldsymbol{w}$ *is the column vector of angular frequencies, that is* $\boldsymbol{w} = [\omega_0, \cdots, \omega_{D-1}]^T$.

Proof. First, let us show that Eq.(6) is a necessary condition. By substituting $\mathbf{z} = e^{j\boldsymbol{w}}|_{\boldsymbol{w}=2\pi\mathbf{M}_U^{-T}\mathbf{k}'_l}$ into $F(\mathbf{z})$, we have

$$F(e^{j2\pi\mathbf{M}_U{}^{-T}\mathbf{k}'_i}) = \sum_{l=0}^{M-1} R_{\mathbf{k}_l}(\mathbf{1})e^{j2\pi\mathbf{k}'_i{}^T\mathbf{M}_U{}^{-1}\mathbf{k}_l}. \quad (7)$$

Please note that $\mathbf{z}^{\mathbf{M}_U}|_{\mathbf{z}=e^{j\boldsymbol{w}},\boldsymbol{w}=2\pi\mathbf{M}_U^{-T}\mathbf{k}'_l} = \mathbf{1}$. If the checkerboard effect is not caused, all polyphase filters have the same DC gain. In that case, Eq.(7) can be rewritten as

$$F(e^{j2\pi\mathbf{M}_U{}^{-T}\mathbf{k}'_i}) = R_{\mathbf{k}_0}(\mathbf{1})\sum_{l=0}^{M-1} e^{j2\pi\mathbf{k}'_i{}^T\mathbf{M}_U{}^{-1}\mathbf{k}_l}. \quad (8)$$

From [6], the following property is satisfied.

$$\sum_{l=0}^{M-1} e^{j2\pi\mathbf{k}'_i{}^T\mathbf{M}_U{}^{-1}\mathbf{k}_l} = 0. \quad (9)$$

By substituting Eq.(9) into Eq.(8), we obtain

$$F(e^{j2\pi\mathbf{M}_U{}^{-T}\mathbf{k}'_i}) = 0. \quad (10)$$

Thus, Eq.(6) is a necessary condition.

Next, let us show that Eq.(6) is a sufficient condition. When Eq.(6) is satisfied, Eq.(7) can be rewritten as

$$\mathbf{M}_{exp} \times \begin{bmatrix} R_{\mathbf{k}_0}(\mathbf{1}) \\ R_{\mathbf{k}_1}(\mathbf{1}) \\ \vdots \\ R_{\mathbf{k}_{M-1}}(\mathbf{1}) \end{bmatrix} = \begin{bmatrix} F(\mathbf{1}) \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad (11)$$

$$\mathbf{M}_{exp} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{k(1,0)} & \cdots & e^{k(1,M-1)} \\ \vdots & \vdots & \vdots \\ e^{k(M-1,0)} & \cdots & e^{k(M-1,M-1)} \end{bmatrix}.$$

$F(\mathbf{1})$ is the DC gain of $F(\mathbf{z})$ and corresponds to the case of $\mathbf{k}'_0 = \mathbf{0}$ in Eq.(7). For the convenience, we use $e^{k(i,j)}$ instead of $e^{j2\pi\mathbf{k}'_i{}^T\mathbf{M}_U{}^{-1}\mathbf{k}_j}$. Using the Cramer's rule to Eq.(11), $R_{\mathbf{k}_j}(\mathbf{1})$ is given by

$$R_{\mathbf{k}_j}(\mathbf{1}) = \frac{(-1)^j F(\mathbf{1})}{|\mathbf{M}_{exp}|} \times$$

$$\begin{vmatrix} e^{k(1,0)} & \cdots & e^{k(1,j-1)} & e^{k(1,j+1)} & \cdots & e^{k(1,M-1)} \\ \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ e^{k(1,0)} & \cdots & e^{k(1,j-1)} & e^{k(1,j+1)} & \cdots & e^{k(1,M-1)} \end{vmatrix}, \quad (12)$$

where $|\mathbf{A}|$ is the determinant of the matrix $\mathbf{A}$. By substituting Eq.(9) into Eq.(12), Eq.(12) can be arranged as

$$R_{\mathbf{k}_j}(\mathbf{1}) = \frac{F(\mathbf{1})}{|\mathbf{M}_{exp}|} \times \begin{vmatrix} e^{k(1,1)} & \cdots & e^{k(1,M-1)} \\ \vdots & \cdots & \vdots \\ e^{k(1,1)} & \cdots & e^{k(1,M-1)} \end{vmatrix} \quad (13)$$

Since the right-hand side of Eq.(13) does not depend on the index j, all polyphase filters $R_{\mathbf{k}_j}(\mathbf{z}), j = 0, \cdots, M-1$ have the same DC gain. Therefore, Eq.(6) is a sufficient condition for MD multirate filters without checkerboard effect. $\diamond$

Theorem 3. *If* $F(\mathbf{z})$ *is represented as Eq.(14) , the checkerboard effect is not caused. Eq.(14) is a sufficient condition for MD multirate filters to avoid the checkerboard effect, although it is a necessary and sufficient condition for 1D multirate filters[3].*

$$F(\mathbf{z}) = P(\mathbf{z})\sum_{l=0}^{M-1} \mathbf{z}^{(-\mathbf{k}_l)}, \quad \mathbf{k}_l \in \mathcal{N}(\mathbf{M}_U). \quad (14)$$

In this paper, although we only show the result of the derivation for Theorem 3, it can be derived easily[8].

3 MD Multirate Filter Banks without Checkerboard Effect

In this section, we consider the conditions for MD maximally decimated filter banks.

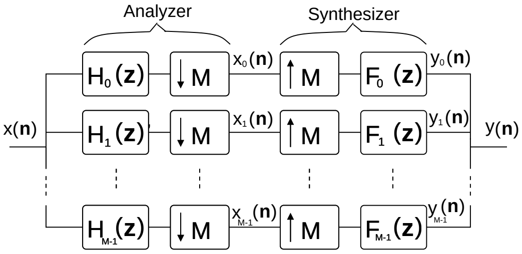


Figure 3: MD multirate filter bank

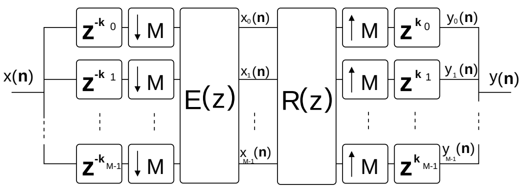


Figure 4: polyphase structure

3.1 MD Multirate Filter Banks[11]

Fig.3 shows an MD maximally decimated filter bank with a factor $\mathbf{M}$, where $\mathbf{M}$ is a $D \times D$ nonsingular integer matrix. In Fig.3, $H_m(\mathbf{z})$ and $F_m(\mathbf{z})$ denote an analysis filter and a synthesis filter respectively. M is the absolute determinant of $\mathbf{M}$ and the channel number $m = 0$ corresponds to the lowpass channel.

The MD filter bank in Fig.3 can be equivalently represented as shown in Fig.4, where $\mathbf{E}(\mathbf{z})$ and $\mathbf{R}(\mathbf{z})$ are the $M \times M$ type-I and type-II polyphase matrices[6] whose i, j th element are given by the polyphase filter $E_{i,j}(\mathbf{z}^{\mathbf{M}})$ and $R_{i,j}(\mathbf{z}^{\mathbf{M}})$ respectively. $E_{i,j}(\mathbf{z}^{\mathbf{M}})$ and the $R_{i,j}(\mathbf{z}^{\mathbf{M}})$ are obtained by

$$H_m(\mathbf{z}) = \sum_{l=0}^{M-1} \mathbf{z}^{(-\mathbf{k}_l)} E_{m,\mathbf{k}_l}(\mathbf{z}^{(\mathbf{M})}),\ \mathbf{k}_l \in \mathcal{N}(\mathbf{M}) \quad (15)$$

$$F_m(\mathbf{z}) = \sum_{l=0}^{M-1} \mathbf{z}^{(\mathbf{k}_l)} R_{\mathbf{k}_l,m}(\mathbf{z}^{(\mathbf{M})}),\ \mathbf{k}_l \in \mathcal{N}(\mathbf{M}), \quad (16)$$

By using the polyphase matrices, the perfect reconstruction(PR) condition for MD filter banks is given as

$$\mathbf{R}(\mathbf{z})\mathbf{E}(\mathbf{z}) = \mathbf{z}^{-\mathbf{c}}\mathbf{I}_M, \quad (17)$$

where $\mathbf{c}$ is a $1 \times D$ arbitrary integer vector and $\mathbf{I}_M$ is the $M \times M$ identity matrix. When an MD filter bank satisfies Eq.(17), it is called the MD perfect reconstruction(PR) filter bank.

3.2 Checkerboard Effect in MD Multirate Filter Banks

As shown in Fig.3, MD filter banks consist of analyzer and synthesizer. The analyzer divides the input signal $x(\mathbf{n})$ into M subband signals $x_m(\mathbf{n})$ and the synthesizer combines $x_m(\mathbf{n})$ into the output signal $y(\mathbf{n})$. Here, each channel of the synthesizer includes an up-sampler and a digital filter. Therefore, the checkerboard effect in MD filter banks is caused by the synthesizer.

Next, let us consider the steady state value of the step response as well as the case of MD multirate filters. In Fig.4, when the input signal $x(\mathbf{n})$ is the unit step signal $u(\mathbf{n})$ and the components of the vector $\mathbf{n}$ are enough large, the subband signal $x_m(\mathbf{n})$ becomes the steady state value $a_m(\mathbf{n})$

$$a_m(\mathbf{n}) = H_m(\mathbf{1}), \quad (18)$$

and the signal $y_m(\mathbf{n})$ for the mth channel in the synthesizer becomes the steady state value $s_m(\mathbf{n})$

$$s_m(\mathbf{n}) = H_m(\mathbf{1}) \times \begin{cases} R_{0,m}(\mathbf{1}) & , \mathbf{n} = \mathbf{M}\mathbf{t} - \mathbf{k}_0 \\ \vdots & \vdots \\ R_{M-1,m}(\mathbf{1}) & , \mathbf{n} = \mathbf{M}\mathbf{t} - \mathbf{k}_{M-1} \end{cases}, \quad (19)$$

where $r_{l,m}(\mathbf{n})$ denotes the impulse response of the $R_{l,m}(\mathbf{z})$.

From Eq.(19), $s_m(\mathbf{n})$ is not constant and has the period $\mathbf{M}$. If the PR condition Eq.(17) is satisfied, the periodicity of $s_m(\mathbf{n})$ is able to be canceled each other and thus the output signal $y(\mathbf{n})$ is constant. However, even if an MD filter bank is designed under the PR condition, it is broken in some practical applications such as subband coding. In this case, the periodicity of $s_m(\mathbf{n})$ can not be canceled each other. As a result, $y(\mathbf{n})$ is not constant and the checkerboard effect is generated.

3.3 Conditions for MD Multirate Filter Banks without Checkerboard Effect

As shown in the previous subsection, the checkerboard effect is caused by the synthesizer.

In the following, we derive the conditions for MD maximally decimated PR filter banks.

Theorem 4. *A necessary and sufficient condition for MD maximally decimated PR filter banks to avoid the checkerboard effect is given as*

$$R_{0,0}(\mathbf{1}) = \cdots = R_{M-1,0}(\mathbf{1}), \quad (20)$$

where $R_{l,0}(\mathbf{z})$ denotes the DC gain of a polyphase filter of the lowpass filter $F_0(\mathbf{z})$ in the synthesizer.

In addition, the condition Eq.(20) is equal to

$$H_m(\mathbf{1}) = \sum_{l=0}^{M-1} E_{m,l}(\mathbf{1}) = 0, \quad m = 1, \cdots, M-1, \quad (21)$$

where $H_m(\mathbf{1})$ denotes the DC gain of the analysis filters except for the lowpass channel.

Proof. First, let us show that Eq.(20) is a necessary and sufficient condition.

In general, the DC component of $x(\mathbf{n})$ is not zero. Thus, f or the lowpass channel($m = 0$), Eq.(20) is a necessary and sufficient condition, because Eq.(20) is the condition given by Theorem 1. Therefore, it is clear that Eq.(20) is a necessary condition.

Next, let us show that Eq.(20) is a sufficient condition. Substituting $\mathbf{z} = e^{j\boldsymbol{w}}|_{\omega_0=0,\cdots,\omega_{D-1}=0}$ into Eq.(17) yields

$$\mathbf{R}(\mathbf{1})\mathbf{E}(\mathbf{1}) = \mathbf{E}(\mathbf{1})\mathbf{R}(\mathbf{1}) = \mathbf{I}_M. \quad (22)$$

Form Eq.(22), when Eq.(20) is satisfied, we obtain

$$R_{0,0}(\mathbf{1}) \sum_{l=0}^{M-1} E_{m,l}(\mathbf{1}) = 0 \quad m = 1, \cdots, M-1. \quad (23)$$

Since $R_{0,0}(\mathbf{1}) \neq 0$ from Eq.(22) , Eq.(23) is reduced to

$$\sum_{l=0}^{M-1} E_{m,l}(\mathbf{1}) = H_m(\mathbf{1}) = 0 \quad m = 1, \cdots M-1. \tag{24}$$

This means the subband signal $a_m(\mathbf{n}) = 0, m = 1, \cdots, M-1$. That is ,the DC component of $x_m(\mathbf{n})$ in Fig.3 is zero value. By substituting Eq.(24) into Eq.(19), $s_m(\mathbf{n})$ except for $m = 0$ becomes

$$s_m(\mathbf{n}) \quad = \quad 0, \qquad m = 1, \cdots, M-1. \tag{25}$$

Thus, $s_m(\mathbf{n})$ except for the lowpass channel has zero value. Therefore, Eq.(20) is a sufficient condition.

Next, we show that Eq.(21) is equal to Eq.(20). From the above consideration, it is clear that if Eq.(20) is satisfied , Eq.(21) is done. Therefore, let us show that if Eq.(21) is satisfied, Eq.(20) is done.

From Eq.(22), we have the following equation

$$\begin{bmatrix} E_{0,0}(\mathbf{1}) & \cdots & E_{0,M-1}(\mathbf{1}) \\ \vdots & \ddots & \vdots \\ E_{M-1,0}(\mathbf{1}) & \cdots & E_{M-1,M-1}(\mathbf{1}) \end{bmatrix} \begin{bmatrix} R_{0,0}(\mathbf{1}) \\ \vdots \\ R_{M-1,0}(\mathbf{1}) \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}.^T \tag{26}$$

By substituting Eq.(21) into Eq.(26) and using the Cramer's rule, $R_{i,0}(\mathbf{1})$ is given by

$$R_{i,0}(\mathbf{1}) \quad = \quad \frac{1}{|\mathbf{E}(\mathbf{1})|} \begin{vmatrix} E_{1,1}(\mathbf{1}) \cdots & E_{1,M-1}(\mathbf{1}) \\ \vdots \quad \cdots & \vdots \\ E_{1,1}(\mathbf{1}) \cdots & E_{M-1,M-1}(\mathbf{1}) \end{vmatrix}. \tag{27}$$

Since the right-hand side of Eq.(27) does not depend on the index i, all polyphase filters $R_{i,0}(\mathbf{z}), i = 0, \cdots, M-1$ for the lowpass channel have the same DC gain. Therefore, when Eq.(21) is satisfied, Eq.(20) is also satisfied. Thus, Eq.(20) is equal to Eq.(21). $\diamond$

4 Example

In order to verify the significance of the derived theorems, we show a example. As processing examples, we convert the resolution of the image in Fig.5(a) by the 2D multirate filter, where $\mathbf{M}_U$ and $\mathbf{M}_D$ are

$$\mathbf{M}_U = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \mathbf{M}_D = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}. \tag{28}$$

This means that the size of image is expanded by factor 3/2 in vertical and horizontal directions respectively. The used $F(z_0, z_1)$ have 5×5 taps and they are optimized by minimizing the stopband.

stopband:$2\pi/3 < \omega_0$ and $2\pi/3\omega_1$.

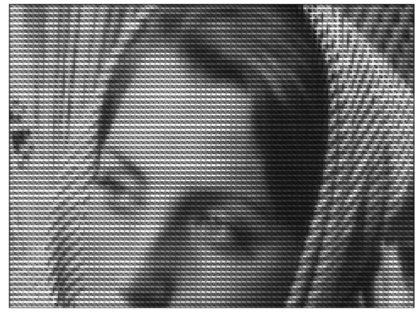
The processed images are shown in Fig.5(b),(c). Fig.5(b) is converted by the filter which does not satisfy Eq.(5) and Fig.5(c) is done by the filter which satisfies Eq.(5).

From Fig.5, we see that the checkerboard effect can be avoided by using the proposed condition.

5 Conclusion

In this work, we considered the checkerboard effect in MD multirate filters and filter banks. Some theorems about the conditions for the MD multirate filters and filter banks to avoid the checkerboard effect are derived. Simulation example show that the checkerboard effect can be avoided by using the proposed conditions.

(a)



(b)

(c)

Figure 5: processed image

References

[1] Strang G. and Nguyen T. "Wavelets and Filter Banks", Wellesley-Cambridge Press 1996.

[2] M. Kato, T. Yoshida, and A. Nishihara "Design of Multirate Filters with Time-Domain Constraints",IEICE Technical Report DSP96-24 pp. 23-28(Jun.1996)

[3] Y. Harada, S. Muramatsu, and H. Kiya "Two-Channel QMF Bank without Checkerboard Effect and Its Lattice Structure",IEICE Trans. Fundamentals, J80-A,11,(1997).

[4] Y. Harada, and H. Kiya "Multirate Filter without Checkerboard Effect and Its Zero Point Location",IEICE Technical Report CAS97-01 pp. 83-90(Jan.1997)

[5] M. Kato, T. Yoshida, and A. Nishihara "Necessary and Sufficient Condition for *M*-channel Perfect Reconstruction filter Banks to Avoid Checkerboard Effects",IEICE Technical Report CAS96-79 pp.7-12(Jan.1997)

[6] Vaidyanthan P. P."Multirate Systems and Filter Banks", Prentice Hall, Englewood Cliffs, 1993.

[7] Ramesh A. and C.Sidney: "On Upsampling, Downsampling, and Rational Sampling Rate Filter Banks",IEEE Trans. Signal Processing **42**, 4(1994)

[8] Y. Harada, S. Muramatsu, and H. Kiya "Multidimensional Multirate Filter and Filter Bank without Checkerboard Effect",IEICE Trans. Fundamentals, (subimitted)