LADDER SCHEME FOR PERFECT RECONSTRUCTION MODULATED FILTER BANKS

M. Gharbi, M. Colin, M. Gazalet and F.X. Coudoux
Dept. OAE/IEMN/ UMR-CNRS 9929, ENSIMEV University of Valenciennes,
B.P 311, Le Mont Houy, 59304 Valenciennex Cedex , FRANCE
Tel: +33 3 27141368; fax: +33 3 27141189
e-mail: gharbi@univ-valenciennes.fr

ABSTRACT

A product form of the polyphase filter matrix with adjustable overall delay which allows perfect reconstruction, is given for orthogonal and biorthogonal modulated filter banks. The lifting scheme has been used, yielding to simpler implementation and allowing for in-place computations, i.e. the transform can be calculated without allocating auxilary memory. The modulated filter bank system is decomposed into small subsystems in a ladder module configuration, which is profitable for many applications and easies the hardware implementation.

1 INTRODUCTION

For several years, the block transform and filter bank methods have been studied with a great interest and have found a wide variety of signal processing applications. They became today powerful tools for many applications, particularly for the signal or image coding for transmission or storage. References [1–3] cover the most significant aspects of the filter bank methods.

A typical M-channel maximally decimated filter bank is represented on Fig. 1. The analysis stage consists of M filters $h_k(n)$, $0 \leq k \leq M-1$ followed by M decimators. The analysis stage splits the input $x(n)$ into M subband signals $y_k(n)$. In coding applications, the signal resulting from each channel is quantified, coded and then transmitted. At the receiver end, the subband signals are interpolated, filtered by the synthesis filters $f_k(n)$ and combined. The overall system is generally designed so that, in absence of quantification and transmission errors, the output is a Perfect Reconstruction (PR) of the input signal is obtained $\widehat{x}(n) = x(n-r)$ where r is the global input-output delay.

Filter banks as well as transform methods can be represented by Fig. 1. The difference between these two families lies in the fact that the second uses filters with the same length L which is equal to the decimation factor M. So, adjacent blocks of the input signal are processed independently. Filter bank methods use filters with generally larger length. Typically their length L is equal to a multiple of the decimation factor M. These filters extend on several blocks of the input signal, and thus take account of the correlation between adjacent blocks and thus reduce the block artifacts inherent in the transform methods. The filter bank methods can thus be seen as a generalization of transform ones.

To reduce implementation complexity and filter design, Modulated Filter Banks (MFB) have been proposed where the analysis and synthesis filters are obtained by modulation respectively, of two low-pass prototype filters $h(n)$ and $f(n)$.

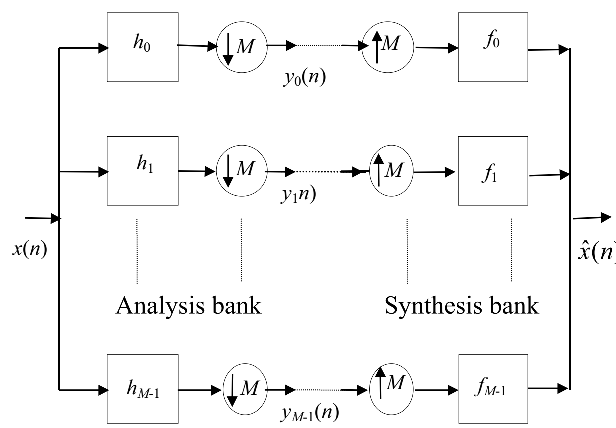

Figure 1: Maximally decimated filter bank

The PR-MFB were studied in the orthogonal case, initially for $L = 2M$, then generalized for arbitrary value of L. In references [4] and [5], the factorized form of the polyphase matrix for $L = 2mM$ was proposed independently. The orthogonal MFB is characterized by the fact that the analysis and synthesis filters are obtained by modulation of a same linear phase lowpass prototype and the overall input-output r is equal to $L-1$.

In [7] the conditions for perfect reconstruction for the biorthogonal case are given, where the overall input output delay r can be much lower than $L-1$. A low delay filter bank is often desirable particularly for real time applications. The two prototype filters can also be made equal and thus the filter bank design is facilitated. In [8,9] similar condition was obtained for modulations other than conventional cosine one.

In this paper, a product form of the polyphase matrix with adjustable overall delay obtained from the lifting scheme is given. It leads to a ladder implementation for orthogonal and biorthogonal MFB. The lifting scheme allows in-place computations and the implementation is facilitated.

2 THE PERFECT RECONSTRUCTION CONDITIONS

The PR reconstruction conditions for cosine MFB can be written in matrix form

$$\det(\mathbf{G}_l(z)) = -z^{-s} \qquad 0 \le l \le \frac{M}{2} - 1 \tag{1}$$

where

$$\mathbf{G}_l(z) = \sqrt{2M} \begin{pmatrix} -G_l(z) & G_{M-1-l}(z) \\ G_{M+l}(z) & G_{2M-1-l}(z) \end{pmatrix} \tag{2}$$

and $G_k(z) = \sum_{n=0}^{m-1} h(k+2Mn)z^{-n}$, $0 \le k \le 2M-1$, are the $2M$-polyphase components of the prototype filter $h(n)$. The synthesis prototype filter is given by $F(z) = (-1)^s H(z)$. The exponent s, $0 \le s \le 2(m-1)$, controls the overall input output delay r, which is equal to $2M(s+1)-1$ [7].

The MFB design is reduced to the prototype filter design satisfying (1). Notice that this PR conditions implies that $G_l(z)$ and $G_{M+l}(z)$ are relatively prime or have a monomial common divisor because any polynomial which divide them divides also the determinant. This property is used in the next paragraph to obtain the product form for the matrix $\mathbf{G}_l(z)$.

One can easily show that the polyphase matrix $\mathbf{E}(z)$ of the analysis bank can be written in terms the type-IV DCT modulation matrix and the matrix $\mathbf{G}_l(z)$, which play a central role in this paper [5].

Without loss of generality, we deals here only with the case of $L = 2mM$ and therefore all polyphase components of $h(n)$ have the same order $m-1$.

3 LIFTING FACTORIZATION

In the following, for considerations of causality, all polynomials are polynomials of z^{-1}. From the Euclidean algorithm, for two arbitrary polynomials, $a(z)$ and $b(z)$, one can write $\begin{pmatrix} a(z) \\ b(z) \end{pmatrix} = \begin{pmatrix} q(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} b(z) \\ r(z) \end{pmatrix}$ with the order of $r(z)$ less than the one of $b(z)$ and where $q(z) = 0$ if the order of $b(z)$ is greater than that of $a(z)$. To take into considerations hardware implementations, $q(z)$ is imposed monomial and therefore the order of the remainder $r(z)$ can only constrained to be less than that of $a(z)$.

The division can be performed also in ascending powers. In this case, with a monomial quotient also, the "remainder" $r(z)$ will not contain the constant term and therefore one can write $\begin{pmatrix} a(z) \\ b(z) \end{pmatrix} = \begin{pmatrix} q(z) & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & z^{-1} \end{pmatrix} \begin{pmatrix} b(z) \\ r(z) \end{pmatrix}$ with the order $r(z)$ less than that of $a(z)$.

The division can be iterated with $b(z)$ and $r(z)$ with either increasing or decreasing powers, and so on, until the last remainder polynomial equal to zero is obtained. This division process is assured to finish with a null remainder polynomial within a finite number of steps because at each step the order of $r(z)$ is diminished. If $a(z)$ and $b(z)$ are coprime or have a monomial factor, the last obtained divisor polynomial is a constant $K \ne 0$. Therefore, one obtains

$$\begin{pmatrix} a(z) \\ b(z) \end{pmatrix} = \prod_{i=0}^{N-1} \mathbf{Q}_i(z) \begin{pmatrix} K \\ 0 \end{pmatrix} \tag{3}$$

For arbitrary $c(z)$ and $d(z)$, if the $a(z)$ and $b(z)$ define the first column of the polynomial matrix $\mathbf{A}(z) = \begin{pmatrix} a(z) & c(z) \\ b(z) & d(z) \end{pmatrix}$ with $\det(\mathbf{A}(z)) = (-1)^N z^{-s}$, for some appropriate value of s, one can easily find (see [6]) a polynomial $q_N(z)$ such that

$$\mathbf{A}(z) = \prod_{i=0}^{N} \mathbf{Q}_i(z) \begin{pmatrix} 0 & 1/K \\ K & 0 \end{pmatrix} \tag{4}$$

where $\mathbf{Q}_N(z) = \begin{pmatrix} q_N(z) & 1 \\ 1 & 0 \end{pmatrix}$. The value s is the time where the division was taken in ascending powers.

Let us resume the matrix given in (2) verifying (1). We will see how it can be written as 4. For each l, $0 \le l \le \frac{M}{2} - 1$, denoting $a(z) = -\sqrt{2M}G_l(z)$ and $b(z) = \sqrt{2M}G_{M+l}(z)$, both of order $m-1$, one can use the above described procedure of Euclidean division either in increasing or decreasing powers of z^{-1}. Notice that only half of coefficients of the prototype $h(n)$ are used. One obtain the product (3) where the total number of division steps N is equal to $2m-1$ and the division in ascending powers was performed s times. So, it only remains to determine the last quotient $q_N(z)$ to write $\mathbf{G}_l(z)$ as the product form (4). This quotient is linearly related to the prototype coefficients and therefore is easily derived by identification (equaling (4) to $\mathbf{G}_l(z)$). So $det(\mathbf{G}_l(z)) = -z^{-s}$. Notice that, as the four polynomial entries of $\mathbf{G}_l(z)$ have the same order $m-1$, then $q_N(z)$ is only a constant polynomial.

Alternatively, as in our simulations, one can determine globally for all values of l, these last quotients by minimization of the stopband error of the lowpass prototype filter $\int_{\omega_s}^{\infty} \left|H(e^{j\omega})\right|^2 d\omega$. As this stopband error is quadratic with the prototype filter coefficients, its minimization leads to a linear equations easily resolved. To refine the results (correct rounding errors, ...), one can slightly disturb $G_l(z)$ and $G_{M+l}(z)$, $0 \le l \le \frac{M}{2} - 1$ and recompute $G_{M+l}(z)$ and $G_{2M-1-l}(z)$. An optimization algorithm could be used for that purpose.

Finally, for each l, $\mathbf{G}_l(z)$ has been written as the product (4) and therefore is factorized.

Inverting $\mathbf{G}_l(z)$ is trivial and thus the synthesis stage can be derived without any difficulties.

4 IMPLEMENTATION COMPLEXITY AND EXAMPLES

Only the implementation of the matrix $\mathbf{G}_l(z)$ is considered here since the modulation part is often the same as in conventional cosine MFB. The analysis filter bank is implemented as $M/2$ two-channel ladder filter bank where each channel depends only on $\mathbf{G}_l(z)$. An example of implementation is given in Fig. 4 were the following trick was used $\begin{pmatrix} p & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} q & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & p \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ q & 1 \end{pmatrix}$. Notice that, in (4), each quotient matrix $\mathbf{Q}_i(z)$ requires one addition and one multiplication per output sample. Therefore the implementation of $\mathbf{G}_l(z)$, requires $2m$ adders and $2m+2$ multipliers. Its complexity is independent of the overall delay defined by s. On can show that the ladder implementation of the MFB use the minimal number of delays equal to $max(m-1, s)M + 1$. So, the implementation is minimal for any value of s.

Some examples are given to illustrate the factorization scheme. Fig. 2 and 3 shows impulse response and magnitude response of prototype filter for $M = 8$ and $L = 96$.

In our simulations, the procedure described above starts from a lowpass prototype filter obtained from the QCLS method [7] and the Matlab optimization package was used. In all cases, we have found the ladder implementation designated by the QCLS method.

5 Conclusion

In this paper, a product form of the polyphase matrix that leads to ladder implementation of biorthogonal MFB with variable overall system in given. The analysis stage is implemented as $M/2$ two-channel ladder structure followed by the modulation matrix transform. The complexity is nearly the same as in the conventional lattice-based one in the paraunitary case. The overall system delay is controllable and can be imposed much lesser than in the orthogonal case that can be useful in real time applications.

6 References

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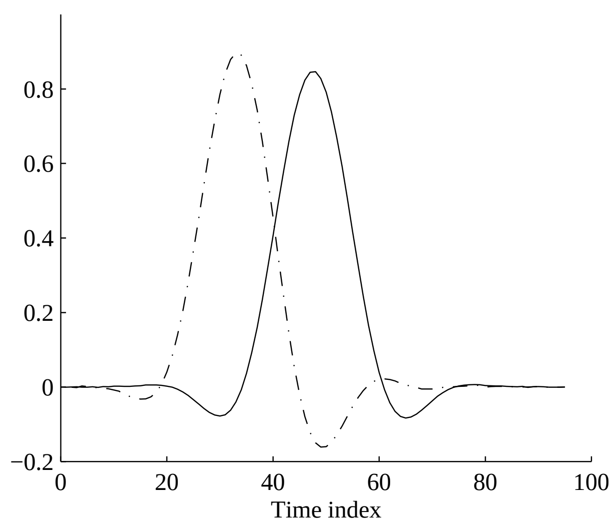

Figure 2: Prototype impulse responses for M=8, L=96 and two values of s ($s = 3$: '_ .' and $s = 5$: '__').

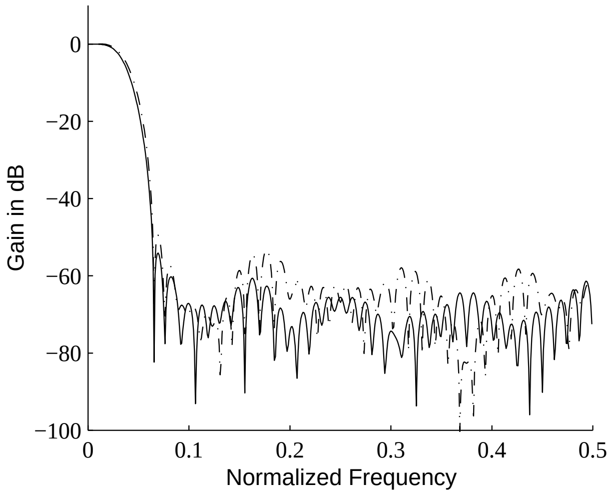

Figure 3: Magnitude frequency responses of filters in Fig. 2.

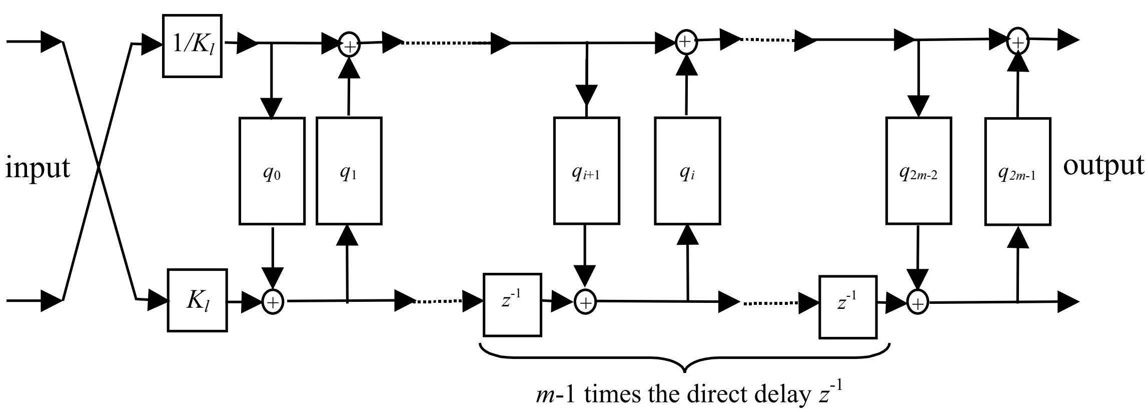

Figure 4: Ladder implementation of the $\mathbf{G}_l(z)$ block for $s = m - 1$.