SPATIAL AND TEMPORAL PROCESSING OF CYCLOSTATIONARY SIGNALS IN ARRAY ANTENNAS BASED ON LINEAR PREDICTION MODEL

Ami Kanazawa †, Hiroyuki Tsuj †, Jingmin Xin † and Blagovest Shishkov ‡

† Communications Research Laboratory, M.P.T. of Japan
3-4 Hikari-no-oka, Yokosuka 239-0847, Japan
‡ Institute of Applied Mathematics & Informatics, Technical University of Sofia
1756 Sofia, Bulgari

ABSTRACT

A novel direction-finding (DF) algorithm based on the linear prediction (LP) technique for exploiting cyclostationary statistical information (spatial and temporal) is explored. Its implementation is simple in comparison with that of existing cyclic methods. The effectiveness of the presented algorithm is demonstrated and compared with the existing conventional cyclic algorithms through numerical examples.

1. INTRODUCTION

In recent years, to satisfy the ever growing demands for a large number of mobiles on communications channels, the application of array processing has been expected for mobile communication systems to increase channel capacity and spectrum efficiency and to extend range coverage. However for this application, the knowledge of a reference signal, a training signal or the directions of arrival (DOA) of the signal of interest (SOI) is usually required to improve the system performance. Several methods have been proposed for the DOA estimation problem. The most attractive ones among them are the subspace algorithms, such as MUSIC (multiple signal classification) [1] and ESPRIT (estimation of signal parameters via rotational invariance techniques) [2], because of their high-resolution. However, like the conventional array processing methods, the subspace algorithms basically rely on the spatial properties (*i.e.* spatial delay) of the signals impinging on an array of sensors and exploit the eigenspace property of the array covariance matrix. One shortcoming of these approaches is that they ignore the temporal properties of the desired signals. Nevertheless, in general it is very difficult to efficiently combine the temporal and spatial information of the signal in determining the source DOA.

Many modulated signals arising in communications are cyclostationary due to the underlying periodicity arising from carrier frequencies or baud rates [3], [4]. By exploiting this special temporal property of the signals, some cyclic direction-finding (DF) algorithms for improving the signal detection capability have received much attention [4]-[10]. The cyclostationarity concept was first introduced into array signal processing in [5] and [6], where the correlation matrix estimate used in the general subspace algorithm is replaced by the cyclic array covariance matrix estimate. Since the cyclic correlation function is dependent on the lag parameter, the determining of the optimal lag parameter where the correlation function achieves its maximum is very important; unfortunately, in reality this lag parameter is rarely available. Three possible approaches used to over come this difficulty are proposed. One is to use the cyclic spectrum in place of the cyclic correlation [5], another is to combine all the cyclic array correlations of different lags [7], and the last one is to stack the cyclic subspaces corresponding to the different lags [8]. However, due to the additional computational cost of using the cyclic spectrum in the first approach and the burdensome evaluation of the cross correlations in the others, all these methods are not computationally efficient [7].

In this paper, a new approach is proposed for the estimating DOA of cyclostationary signal that utilizes a linear prediction (LP) technique. For resolving the problem of the choice of the lag parameter, we present a new alternative approach that exploits the cyclic statistical effectively in a forward-backward way, enabling robust high-resolution performance to be achieved. Furthermore, the presented scheme is simpler and more convenient to implement than those using the conventional cyclic algorithms, such as cyclic MUSIC and cyclic ESPRIT [5]-[10], where the computation of the cyclic array covariance matrix is burdensome. In addition, the important information contained in the cross correlation between the different sensor outputs is not considered in the SC-SSF (spectral correlation-signal subspace fitting) algorithm [7]. The effectiveness of the presented algorithm is demonstrated and compared with the conventional algorithms through numerical examples.

2. PROBLEM SATEMENT

2.1 Data Model

As shown in Fig. 1, we consider a uniform linear array (ULA) at a base station consisting of M identical isotropic sensors. The signals received from P sources $s_1(n),\cdots,s_P(n)$ arrive at the array from the directions $\theta_1,\theta_2,\cdots,\theta_P$ which are measured clockwise from the normal of the array. In the narrow band model, we assume that the carrier frequency is fairly large compared to the bandwidth of the modulating signal, then the discrete time narrow band signal model for the DF problem is given by

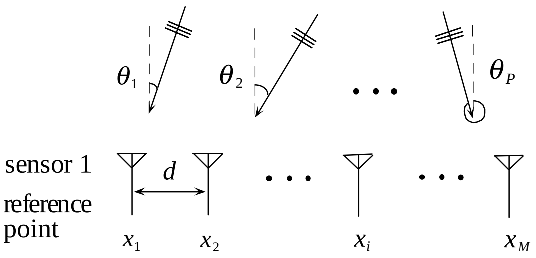

Fig. 1 The structure of a uniform linear array.

$$x_i(n) = \sum_{k=1}^{P} s_k(n) \exp\left(-j2\pi f_c (i-1)(d/c)\sin\theta_k\right) + v_i(n) \quad (1)$$

where

$x_i(n)\,[v_i(n)]$: received signal [additive noise] at the i th sensor,

$P\,[M]$: number of source signals [sensors],

and f_c, d, c are the carrier frequency, the intersensor spacing and the speed of propagation, respectively.

In this paper, the source signal $s_k(n)$ is assumed to be independent of $s_l(n)$ with $\theta_k \neq \theta_l$ for $k \neq l$. The additive noise $v_i(n)$ is assumed to be Gaussian distributed with zero-mean and uncorrelated with the source signals and the other noises. The received signal vector can be expressed as

$$\begin{aligned} \boldsymbol{x}(n) &= \boldsymbol{A}(\theta)\boldsymbol{s}(n) + \boldsymbol{v}(n) \\ &= \sum_{k=1}^{P} \boldsymbol{a}(\theta_k)\, s_k(n) + \boldsymbol{v}(n), \quad n = 0,1,\cdots,N-1 \end{aligned} \quad (2)$$

where

$\boldsymbol{x}(n) = [x_1(n),\cdots,x_M(n)]^T$: observed data vector,

$\boldsymbol{A}(\theta) = [\boldsymbol{a}(\theta_1),\cdots,\boldsymbol{a}(\theta_P)]$: array steering matrix,

$\boldsymbol{s}(n) = [s_1(n),\cdots,s_P(n)]^T$: source signal vector,

$\boldsymbol{v}(n) = [v_1(n),\cdots,v_M(n)]^T$: additive noise vector,

$\theta = [\theta_1,\theta_2,\cdots,\theta_P]^T$: angle vector,

and the steering vector is parameterized as $\boldsymbol{a}(\theta_k) = [1, \exp(-j2\pi f_c (d/c)\sin\theta_k),\cdots,\exp(-j2\pi f_c (M-1)(d/c)\sin\theta_k)]^T$.

2.2 Cyclostationarity

In communications, many different types of modulated signals can have highly distinct cyclic correlation functions with a known cycle frequency, which corresponds to the underlying periodicity arising from carrier frequencies or baud rates, while stationary noise and interference exhibit no cyclostationarity with the same one [3], [4]. By evaluating the cyclic correlations of the received data at certain cycle frequencies, we can extract only signals with the same cycle frequency and null out the stationary additive noise and all other co-channel interfering signals with different cycle frequencies, then the signal detection capability can be improved.

In cyclic algorithms, it is assumed that there are P_α SOI sharing the same cycle frequency α, where $P_\alpha \le P$ and α is either known or estimated from the carrier frequency and baud rate [3], [4]. In this paper, we suppose that it is known. From (1), the cyclic autocorrelation function (CACF) of the received signal $x_i(n)$, and cyclic cross-correlation function (CCCF) between the data $x_i(n)$ and $x_m(n)$ are respectively expressed as follows,

$$\begin{aligned} R^{\alpha}_{x_i,x_i}(\tau) &= \left\langle x_i(n) x_i^*(n-\tau) e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} \\ &= \sum_{k=1}^{P_\alpha} R^{\alpha}_{s_k,s_k}(\tau) \end{aligned} \quad (3)$$

$$\begin{aligned} R^{\alpha}_{x_i,x_m}(\tau) &= \left\langle x_i(n) x_m^*(n-\tau) e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} \\ &= \sum_{k=1}^{P_\alpha} R^{\alpha}_{s_k,s_k}(\tau) \exp(j2\pi f_c (m-i)(d/c)\sin\theta_k) \end{aligned} \quad (4)$$

where τ is the lag parameter, and the notation $<\cdot>_{N\to\infty}$ denotes the discrete time averaging as

$$\left\langle z(n) \right\rangle_{N\to\infty} = \lim_{N\to\infty} \frac{1}{N} \sum_n \{z(n)\} \quad (5)$$

3. LP BASED DOA ESTIMATION

The linear predictive method estimates the received data of one sensor using linear combinations of the remaining sensor minimizes the mean square prediction error [11]-[13]. Here we assume the output of m th sensor can be predicted from the remaining $M-1$ sensor outputs at any instant as

$$\hat{x}_m(n) = -\sum_{i\neq m} a_i x_i(n) \quad (6)$$

where $1 \le m \le M$, $i = 1,\cdots,M$ and $i \neq m$. The prediction error $\varepsilon_m(n)$ between the actual output and the estimate is given

$$\begin{aligned} \varepsilon_m(n) &= x_m(n) - \hat{x}_m(n) \\ &= x_m(n) + \sum_{i\neq m} a_i x_i(n) \end{aligned} \quad (7)$$

where $\varepsilon_m(n)$ is assumed to be spatially and temporally white Gaussian noise.

Under the assumptions, by multiplying equation (7) by $x_m^*(n-\tau)e^{-j2\pi\alpha(n-\tau/2)}$ and applying the time-average operator $<\cdot>_{N\to\infty}$ in (5), we have [12]

$$R^{\alpha}_{x_m,x_m}(\tau) + \sum_{i\neq m} a_i\, R^{\alpha}_{x_i,x_m}(\tau) = 0\,. \quad (8)$$

where the CACF $R^{\alpha}_{x_m,x_m}(\tau)$ and CCCF $R^{\alpha}_{x_i,x_m}(\tau)$ are given in (3) and (4). The choice of the position m of the array for predicting its output is flexible, and it affects the resolution capability and the bias in the DOA estimation corresponding the SNR and the directional separation of sources [11]. Here for simplicity, we selcete $m = M-1$, and choose $\tau = 0, \pm 1, \cdots, \pm(Q-1)$, then from (10), we have

$$\begin{bmatrix} R_{x_{M-1},x_M}(-Q+1) & \cdots & R_{x_1,x_M}(-Q+1) \\ R_{x_{M-1},x_M}(-Q+2) & \cdots & R_{x_1,x_M}(-Q+2) \\ \vdots & \ddots & \vdots \\ R_{x_{M-1},x_M}(0) & \cdots & R_{x_1,x_M}(0) \\ \vdots & \ddots & \vdots \\ R_{x_{M-1},x_M}(+Q-2) & \cdots & R_{x_1,x_M}(+Q-2) \\ R_{x_{M-1},x_M}(+Q-1) & \cdots & R_{x_1,x_M}(+Q-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_{M-1} \end{bmatrix} = - \begin{bmatrix} R_{x_M,x_M}(-Q+1) \\ R_{x_M,x_M}(-Q+2) \\ \vdots \\ R_{x_M,x_M}(+Q-1) \end{bmatrix} \quad (9)$$

where the notation by 'α' is omitted next. From (3) and (4), we can find that the contributions from the interference and noises to the CCCF's and CACF in (9) vanish, by selecting the cycle frequency α appropriately. The equation (9) can be rewritten as

$$\boldsymbol{R}\boldsymbol{a}=\boldsymbol{y} \tag{10}$$

As discussed below, solving equation (10) depends upon a number of factors including the relation size of Q and M and the rank of matrix $\boldsymbol{R}$. The following case will be considered:

1. *Square matrix:* $2Q-1=M-1$. If $\boldsymbol{R}$ is nonsingular, then the inverse matrix $\boldsymbol{R}^{-1}$ is uniquely defined by
$$\hat{\boldsymbol{a}}=\boldsymbol{R}^{-1}\boldsymbol{y} \tag{11}$$
2. *Rectangular matrix:* $2Q-1<M-1$. If the rank of $\boldsymbol{R}$ is $2Q-1$ (the rows of $\boldsymbol{R}$ are linearly independent), then the $(2Q-1)\times(2Q-1)$ matrix $\boldsymbol{R}\boldsymbol{R}^H$ is invertible and the minimum norm (MN) solution is
$$\hat{\boldsymbol{a}}_{\mathrm{MN}}=\boldsymbol{R}^H(\boldsymbol{R}^H\boldsymbol{R})^{-1}\boldsymbol{y} \tag{12}$$
The matrix $\boldsymbol{R}^{+}=\boldsymbol{R}^H(\boldsymbol{R}^H\boldsymbol{R})^{-1}$ is known as the pseudo inverse of the matrix $\boldsymbol{R}$ for the undetermined problem.
3. *Rectangular matrix:* $2Q-1>M-1$. If the colums of $\boldsymbol{R}$ are linearly independent ($\boldsymbol{R}$ has full rank), then the matrix $\boldsymbol{R}\boldsymbol{R}^H$ is invertible and the least square (LS) solution is
$$\hat{\boldsymbol{a}}_{\mathrm{LS}}=(\boldsymbol{R}^H\boldsymbol{R})^{-1}\boldsymbol{R}^H\boldsymbol{y} \tag{13}$$
The matrix $\boldsymbol{R}^{+}=(\boldsymbol{R}^H\boldsymbol{R})^{-1}\boldsymbol{R}^H$ is known as the pseudo inverse of the matrix $\boldsymbol{R}$ for the overdetermined problem.

With the singular value decomposition (SVD) of $\boldsymbol{R}$, we obtain

$$\boldsymbol{R}=\boldsymbol{U}\boldsymbol{\Lambda}\,\boldsymbol{V}^H\,, \tag{14}$$

where $\boldsymbol{U}=[u_1,\cdots,u_{M-1}]$, $\boldsymbol{V}=[v_1,\cdots,v_{M-1}]$ and $\boldsymbol{\Lambda}=\mathrm{diag}(\lambda_1,\cdots,\lambda_{M-1})$. It is then clear that the rank of $\boldsymbol{R}$ should be equal to the number P_α of the SOI with the cycle frequency α. By using the SVD and the number of P_α, the estimate of the coefficients $\boldsymbol{a}$ is obtained by

$$\hat{\boldsymbol{a}}=\sum_{i=1}^{P_\alpha}\frac{\boldsymbol{v}_i\,\boldsymbol{u}_i^H}{\lambda_i}\boldsymbol{y} \tag{15}$$

Turn to the array response vector in (2), let's introduce λ as the associated carrier wavelength and D as the normalized distance between the reference element and the second sensor $D=d/(\lambda/2)$. By substituting $\omega_k=\pi\sin\theta_k$, then the array response vector will be modified as follows

$$\boldsymbol{a}(\omega_k)=[1,\exp(-jD\omega_k),\cdots,\exp(-j(M-1)D\omega_k)]^T \tag{16}$$

The power spectral density $S_{x_M}(\omega)$ of the LP model is given by

$$S_{x_m}(\omega)=\frac{1}{\left|1+a_1z^{-1}+\cdots+a_{M-1}z^{-(M-1)}\right|^2} \tag{17}$$

where $z=e^{j\omega}$. Therefore after the parameters $\{a_i\}$ have been estimated, then the desired DOA can be found by searching the positions of peaks of the spectrum in (17).

In the presented algorithm, we can choose the cyclic correlation functions as $\boldsymbol{R}^{\alpha}_{x_i,x_M}(\tau)$ or its conjugate counterpart, where $i=1,2,\cdots,M$. Really we use cyclic information from all the sensors but in a shorter way in comparison with the cyclic subspace methods, such as the cyclic MUSIC and ESPRIT algorithms, where the cyclic covariance matrix of all sensors is required [5]-[8]. In this paper, we exploit all the temporal information contained in the cyclic correlation functions avoiding some of the drawbacks of existing cyclic algorithms, and the proposed algorithm is simpler and more convenient than other cyclic subspace methods known until now [5]-[8].

4. SIMULATION RESULTS

We finally present the simulation results to show the effectiveness of the proposed method. We considered a ULA having eight elements with half-wavelength spacing. The signal to noise ratio (SNR) for each source is defined as the ratio of the power of each source to that of the background noise.

There are two signals impinging on the array, one is the BPSK signal with 0.5 roll-off factor and 0.25 baud rate ($\alpha=0.25$) which arrives from 50º, and the other is the AM signal arrives from –40º ($\alpha=0.4$). The length of the sample was 1024, and the lag parameter was $Q=7$. Fifty trials are performed for searching DOA of the BPSK and AM signals with $\alpha=0.25$ and $\alpha=0.4$ in the cases where the SNR of BPSK is 10dB while SNR of AM varies from 0dB to 10dB.

Table 1 Comparison of the DOA estimation of the BPSK signal with $\alpha=0.25$.

SNR of AM	Proposed Method		Cyclic MUSIC	
(dB)	mean	RMSE	mean	RMSE
0	51.705	0.241	48.186	0.257
2	51.587	0.224	48.189	0.256
4	50.802	0.113	48.177	0.258
6	51.083	0.153	48.174	0.258
8	51.378	0.195	48.163	0.260
10	49.091	0.129	48.174	0.258

Table 2 Comparison of the DOA estimation of the AM signal with $\alpha=0.4$.

SNR of AM	Proposed Method		Cyclic MUSIC	
(dB)	mean	RMSE	mean	RMSE
0	-40.061	0.009	-39.045	0.135
2	-40.037	0.005	-39.045	0.135
4	-40.037	0.005	-39.053	0.134
6	-40.019	0.003	-39.044	0.135
8	-40.014	0.002	-39.041	0.136
10	-40.008	0.001	-39.033	0.137

The simulation results of DOA estimation are shown in Table 1 and 2, where the mean and root mean squared-error (RMSE) are compared with the cyclic MUSIC and the proposed approach. Fig.1 and 2 plot the spatial spectra for the BPSK and AM signals with SNR of BPSK is 10dB and that of AM is 0dB where only ten trials are shown in each figure respectively. As shown in the tables and figures, we can find the proposed method has the advantage of the cyclic MUSIC.

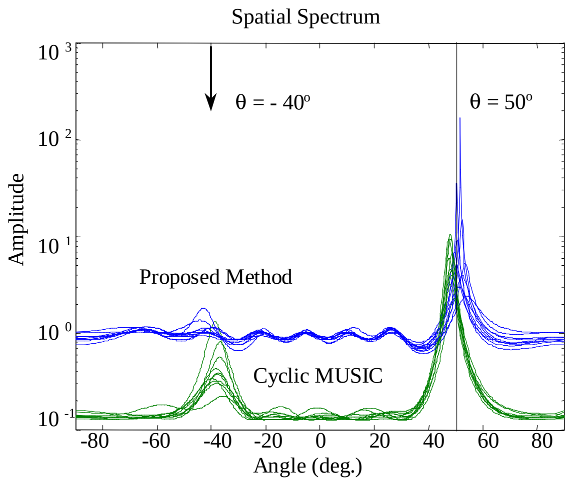

Figure 1 The spatial spectra of the DOA estimation of the BPSK signal by using the cyclic MUSIC and the proposed method with $\alpha = 0.25$.

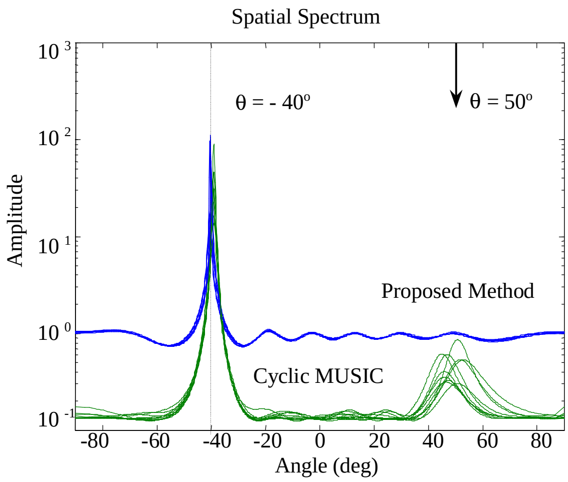

Figure 2 The spatial spectra of the DOA estimation of the AM signal by using the cyclic MUSIC and the proposed method with $\alpha = 0.4$.

5. CONCLUSIONS

For the DOA estimation of the cyclostationary signals in communications, a new signal selective approach based on LP model was proposed. In the presented method, the problem of the choice of the optimal lag parameter is resolved by exploiting the cyclic statistical effectively in a forward-backward way, and the robust high-resolution performance can be achieved. Furthermore, the presented scheme is simpler and more convenient to implement than those using the conventional cyclic algorithms. The effectiveness of the presented algorithm is demonstrated and compared with the conventional algorithm through numerical example.

REFERENCES

[1] Schmidt, R.O., A signal subspace approach to multiple emitter location and spectral estimation. Ph.D. dissertation, Stanford, CA., 1981.

[2] Roy, R.H., ESPRIT – estimation of signal parameters via rotational invariance techniques. Ph.D. dissertation Stanford, CA., 1987.

[3] Gardner, W.A., Exploitation of spectral redundancy in cyclostationary signals. *IEEE Signal Processing Magazine*, pp. 14-16, April 1991.

[4] Gardner, W.A., *Cyclostationarity in Communications and Signal Processing*. IEEE Press, New York, 1994.

[5] Gardner, W.A., Simplification of MUSIC and ESPRIT by exploitation of cyclostationarity. *Proc. IEEE*, vol. 76, pp. 845-847, 1988.

[6] Schell, S.V., R.A. Calabretta, W.A. Gardner, and B.G. Agee, Cyclic MUSIC algorithms for signal - selective direction finding. *Proc. IEEE ICASSP'89*, vol. 4, pp. 2278-2281, 1989.

[7] Xu, G. and T. Kailath, Direction-of-arrival estimation via exploitation of cyclostationarity – a combination of temporal and spatial processing. *IEEE Trans. SP*, vol. 40, pp. 1775-1786, 1992.

[8] P. Burns and Z. Ding, Robustification of cyclostationary array processing techniques. *Proc. IEEE ICASSP'95*, pp. 1589-1592, 1995.

[9] Izzo, L., L. Paura and G. Poggi, An interference-tolerant algorithm for localization of cyclostationary-signal sources. *IEEE Trans. Signal Processing*, vol. 40, pp. 1682-1686, 1991.

[10] Gelli, G., L. Izzo and L. Paura, A cyclic method for signal-selective DOA estimate, *Proc. EUSIPCO'94*, pp. 752-755, 1994.

[11] Johnson, D.H., The application of spectral estimation methods to bearing estimation problems. Proc. IEEE, vol. 70, pp.1018-1028, 1982.

[12] S. Haykin, *Adaptive Filter Theory*. Prentice-Hall, New Jersey, 1996.

[13] Pillai, S.U., *Array Signal Processing*. Springer-Verlag, 1989.