

DIRECTION-OF-ARRIVAL ESTIMATION OF CYCLOSTATIONARY COHERENT SIGNALS IN ARRAY PROCESSING

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ABSTRACT

A new approach is proposed for estimating the direction-of-arrival (DOA) of the cyclostationary coherent signals impinging on a uniform linear array (ULA) by utilizing the spatial smoothing (SS) technique. In order to improve the robustness of the DOA estimation by exploiting the cyclic statistical information sufficiently and handling the coherence effectively, we give a cyclic algorithm with multiple lag parameters and the optimal subarray size. The performance is verified and compared with the conventional methods through numerical examples.

1. INTRODUCTION

To overcome the limitations of the available frequency bands for future mobile communication systems, the applications of array antennas have received much attention, where a crux is the estimation of the number of the signals of interest (SOI) and their direction-of-arrival (DOA). The subspace algorithms such as multiple signal classification (MUSIC) have found considerable prominence in direction finding (DF) for their high resolution. However, the performance and applicability of these methods are limited in some communication applications, in which there are more signals than sensors in the array, and/or the spatial characteristics of noise and interference are unknown. As most different types of modulated signals can have highly distinct cyclic correlation functions with a known cycle frequency, while stationary noise and interference exhibit no cyclostationarity with the same one [1], more recently, many cyclostationarity based methods have been proposed to improve signal detection capability [2]-[6]. Unfortunately, the multipath propagation is often encountered in a variety of communication systems due to various reflections, with the result that the signals are coherent and cyclic matrix becomes singular, so that these cyclic techniques perform poorly as the ordinary MUSIC method. For overcoming the coherent sources problem, a preprocessing scheme such as the maximum likelihood (ML) method or the spatial smoothing (SS) approach [8] can be used. Although a cyclic least-squares method was proposed in [7], it can be interpreted as a cyclic ML DF method, where the ML method involves the multidimensional search and results in the more intensive computational burden.

Therefore the purpose of this paper is to investigate the DOA estimation of the coherent signals in communications. Firstly, a new cyclic approach is proposed to discriminate in favor of the desired signals against noise and interference impinging on a uniform linear array (ULA) by utilizing the SS and improved spatial smoothing (ISS) techniques [8], [10]. Secondly, the choice problems of the lag parameter and subarray dimension are considered. For improving the robustness of the DOA estimation, we give an algorithm with multiple lag parameters and the optimal subarray size to exploit the cyclic statistical information sufficiently and to handle the coherence effectively. The performance of the

proposed method is demonstrated and compared with the conventional methods through numerical examples.

2. PROBLEM STATEMENTS

2.1 Data Model

Here we consider that p narrowband source signals impinging on a ULA, which consists of M identical isotropic sensors with separating distance D , then the data $x_i(n)$ received by the i th sensor is given by

$$x_i(n) = \sum_{k=1}^p s_k(n) e^{j\omega_0(n-1)\tau_k} + w_i(n), \quad \text{for } i = 1, 2, \dots, M \quad (1)$$

where $\omega_0 = 2\pi f_c$, $\tau_k = (D/c)\sin\theta_k$. Here θ_k is the direction of the signal $s_k(n)$ measured relative to the normal of the array with $\theta_k \neq \theta_l$ for $k \neq l$, and f_c and c are respectively the carrier frequency and the speed of propagation. The additive noises $\{w_i(n)\}$ are assumed to be cyclically uncorrelated with themselves and with other source signals at the same cycle frequency α , which can be determined from the carrier frequency and baud rate [1].

We also assume that only p_a sources are self-cyclically correlated with the cycle frequency α and that the first q signals are coherent signals expressed by

$$s_k(n) = \beta_k s_1(n), \quad \text{for } k = 1, 2, \dots, q \quad (2)$$

where $1 \leq q \leq p_a \leq p$, and β_k is the multipath coefficient which represents the complex attenuation of the k th signal with respect to the first signal $s_1(n)$ with $\beta_1 = 0$ and $\beta_1 = 1$. Without loss of generality, we assume $q = p_a$ for simplicity of analysis, and which is known or estimated *a priori* [12].

From (1), we have a vector form to express the obtained sensor data as

$$\mathbf{x}(n) = \mathbf{A}(\theta)\mathbf{s}(n) + \mathbf{w}(n) \quad (3)$$

where $\mathbf{x}(n) = [x_1(n), x_2(n), \dots, x_M(n)]^T$, $\mathbf{A}(\theta) = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \dots, \mathbf{a}(\theta_p)]$, $\mathbf{s}(n) = [s_1(n), s_2(n), \dots, s_p(n)]^T$, $\mathbf{w}(n) = [w_1(n), w_2(n), \dots, w_M(n)]^T$, and $\mathbf{a}(\theta_k) = [1, e^{j\omega_0\tau_k}, \dots, e^{j\omega_0(M-1)\tau_k}]^T$.

2.2 Degradation of Cyclic MUSIC

From the definition of the cyclic correlation function [1], and under the assumptions for the source signals and noise, we have the cyclic array covariance matrix (CACM) as

$$\begin{aligned} R_x^a(\tau) &= \langle \mathbf{x}(n + \tau/2) \mathbf{x}^H(n - \tau/2) e^{-j2\pi\alpha n} \rangle \\ &= \mathbf{A}(\theta) R_s^a(\tau) \mathbf{A}^H(\theta) \end{aligned} \quad (4)$$

where $\langle \cdot \rangle = \lim_{N \rightarrow \infty} (1/N) \sum_{n=1}^N (\cdot)$, H denotes Hermitian (complex conjugation) transpose, and $R_s^a(\tau)$ is the cyclic source covariance matrix (CSCM) which remains as nondiagonal so long as the source signals are at most partially correlated. In that case, the rank of CSCM $R_s^a(\tau)$ still is p_a , if the eigenvalue decomposition (EVD) of $R_s^a(\tau)$ is given by

$$R_s^a(\tau) = \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{U}_s^H \quad (5)$$

where $U_a = [u_{a1}, \dots, u_{aM}]$, $\Lambda_a = \text{diag}(\lambda_{a1}, \dots, \lambda_{aM})$, $\lambda_{a1} \geq \dots \geq \lambda_{ap_a} \geq \lambda_{ap_a+1} = \dots = \lambda_{aM} = 0$, $\{\lambda_{ai}\}$ and $\{u_{ai}\}$ are respectively the eigenvalues and the corresponding eigenvectors, and the noise subspace is spanned by the eigenvectors $\{u_{ap_a+1}, \dots, u_{aM}\}$. From the properties of EVD, then we have

$$a''(\theta_k)u_{oi} = 0, \quad \text{for } i = p_a + 1, \dots, M \quad (6)$$

Hence the cyclic MUSIC method from (6) will give high-resolution DOA estimation of the desired signals, since the contributions of the additive noise and interfering signals vanish in (4) by selecting the cycle frequency α appropriately. However, when the source signals are coherent as in (2), then the CSCM $R^a(\tau)$ in (4) becomes singular, i.e. $\text{rank}(R^a(\tau)) = 1$, so that it will be impossible to estimate any true arrival angle θ_k from the relation in (6) by using the cyclic MUSIC. In contrast to the cyclic least-squares method in [7], in this paper, we propose a new cyclic method for the DOA estimation of the cyclostationary coherent signals.

3. DOA ESTIMATION OF COHERENT SIGNALS

3.1 SS-Based Cyclic MUSIC

Firstly we consider a new cyclic DOA estimation of the cyclostationary coherent signals by utilizing the SS technique [8], [10], which is a spatial averaging preprocessing that groups the total array into overlapping subarrays and then forms the average of the subarray covariance matrix to decorrelate the coherent signals. By dividing the total array into L overlapping subarrays with m sensors ($m \geq p_a + 1$), where $L = M - m + 1$, then the vector $x_l(n)$ of received data in the l th forward subarray can be expressed by

$$x_l(n) = A_m(\theta)D^{l-1}s(n) + w_l(n), \quad \text{for } l = 1, 2, \dots, L \quad (7)$$

where $x_l(n) = [x_{l1}(n), x_{l2}(n), \dots, x_{lm}(n)]^T$, $A_m(\theta) = [a_m(\theta_1), a_m(\theta_2), \dots, a_m(\theta_p)]$, $D = \text{diag}(e^{j\omega_1\tau_1}, \dots, e^{j\omega_p\tau_p})$, $w_l(n) = [w_{l1}(n), w_{l2}(n), \dots, w_{lm}(n)]^T$, and $a_m(\theta_k) = [1, e^{j\omega_k\tau_k}, \dots, e^{j\omega_k(m-1)\tau_k}]^T$.

From (2) and (7), we can obtain the CACM for the l th subarray as

$$\begin{aligned} R_{l,l}^a(\tau) &= \langle x_l(n + \tau/2)x_l^H(n - \tau/2)e^{-j2\pi\alpha n} \rangle \\ &= A_m(\theta)D^{l-1}R^a(\tau)(D^{l-1})^H A_m^H(\theta) \end{aligned} \quad (8)$$

where $R^a(\tau) = \beta R_{11}^a(\tau)\beta^H$, $R^a(\tau)$ is the cyclic auto-correlation function (CACF) of the signal $s_1(n)$, $\beta = [\beta_1, \beta_2, \dots, \beta_p]^T$ and $\beta_k = 0$ for $k = p_a + 1, \dots, p$. Hence the spatial smoothed cyclic covariance matrix (SSCCM) $\bar{R}_l^a(\tau)$ of the subarrays is given by

$$\bar{R}_l^a(\tau) = \frac{1}{L} \sum_{l=1}^L R_{l,l}^a(\tau) = A_m(\theta)\bar{R}^a(\tau)A_m^H(\theta) \quad (9)$$

where $\bar{R}^a(\tau) = R_{11}^a(\tau)CC^H/L$, $C = B\Psi$, $B = \text{diag}(\beta_1, \dots, \beta_p)$, $\Psi = [\psi_1, \dots, \psi_p]^T$, and $\psi_k = [1, e^{j\omega_k\tau_k}, \dots, e^{j\omega_k(L-1)\tau_k}]^T$.

Due to $\beta_k \neq 0$ for $k = 1, \dots, p_a$ whereas $\beta_k = 0$ for $k = p_a + 1, \dots, p$, the ranks of the diagonal matrix B and the Vandermonde matrix Ψ are given by $\text{rank}(B) = p_a$ and $\text{rank}(\Psi) = \min(p, L)$, respectively; hence, we have $\text{rank}(C) = \min(p_a, L)$ for the $p \times L$ matrix C . With the result that if $L \geq p_a$, we can prove that $\text{rank}(\bar{R}^a(\tau)) = \min(p_a, L) = p_a$; i.e. the smoothed CSCM $\bar{R}^a(\tau)$ has the same form as the CSCM $R^a(\tau)$ in (4) regardless of the coherence of the source signals. Therefore the eigenvectors $\{u_i\}$ of the matrix $\bar{R}_l^a(\tau)$ in (9) are orthogonal to the actual array response vectors $\{a(\theta_k)\}$. That is

$$a''(\theta_k)u_i = 0, \quad \text{for } i = p_a + 1, \dots, m \quad (10)$$

Thus from the positions of peaks of the spectrum given by

$$P(\theta) = \frac{1}{\sum_{i=p_a+1}^m |a''(\theta)u_i|^2} \quad (11)$$

where $a(\theta) = [1, e^{j\omega_1\tau}, \dots, e^{j\omega_p(m-1)\tau}]^T$ and $\tau = (D/c)\sin\theta$, the DOA $\{\theta_k\}$ of the desired signals can be estimated.

3.2 ISS-Based Cyclic MUSIC

For improving the performance of the SS-based subspace algorithms, the forward-backward SS technique has usually been used [9], but it is easy to see that any information in the cross correlations of the subarrays is still ignored. Here, by using the cross subarray correlations between the l th and l' th subarrays in (7), we present a new cyclic MUSIC based on the improved SS (ISS) scheme [10], where the improved spatial smoothed cyclic covariance matrix (ISSCCM) $\tilde{R}_l^a(\tau)$ is given by

$$\tilde{R}_l^a(\tau) = \frac{1}{L} \sum_{l=1}^L \sum_{l'=1}^L R_{l,l'}^a(\tau) R_{l',l}^a(\tau) = A_m(\theta)\tilde{R}^a(\tau)A_m^H(\theta) \quad (12)$$

where $\tilde{R}_l^a(\tau) = (1/L) \sum_{l=1}^L D^{l-1} S^a(\tau) (D^{l-1})^H$, $S^a(\tau) = \sum_{l=1}^L R_{l,l}^a(\tau) (D^{l-1})^H A_m^H(\theta) A_m(\theta) D^{l-1} R_{l,l}^a(\tau) = R_{11}^a(\tau) \beta \beta^H$, and the $Lm \times 1$ vector c is given by $c = [\beta^H A_m^H(\theta), \beta^H D^H A_m^H(\theta), \dots, \beta^H (D^{L-1})^H A_m^H(\theta)]^H$.

Applying the analysis for SS-based cyclic MUSIC algorithm, it is obvious that $\text{rank}(S^a(\tau)) = p_a$, so we can prove easily that the ranks of the smoothed matrices $\bar{R}_l^a(\tau)$ and $\tilde{R}_l^a(\tau)$ in (12) equal p_a if $L \geq p_a$. It is clear that the procedure in (10) and (11) can be applied to estimate the actual DOA of the desired signals.

4. IMPROVED CYCLIC ESTIMATION

4.1 Parameter Determination

In the above section, the SS and ISS-based cyclic DF methods are proposed; however, the choices of the statistically significant lag parameter τ and the optimal subarray size m are very crucial.

As the cyclic correlation function is dependent on lag τ [1], if the cyclic correlation of one source is zero or insignificant for a given τ , then this signal will be not resolved. The choice of the optimal lag parameter is very important, specially, if one wants to detect different desired signals just by varying the parameter of cycle frequency. No value of lag parameter τ is specified for use in the cyclic MUSIC and cyclic ESPRIT [2]. Here we propose the use of multiple lag parameters as a sensible choice to exploit the cyclic statistical information sufficiently. As the cyclic correlation function is usually symmetrical about $\tau = 0$, now we choose τ as $\tau = -T, \dots, -1, 0, 1, \dots, T$, then the spatial smoothed cyclic matrices in (9) and (12) can be modified respectively as

$$\bar{R}_l^a = \frac{1}{2T+1} \sum_{\tau=-T}^T \bar{R}_l^a(\tau) \quad \text{and} \quad \tilde{R}_l^a = \frac{1}{2T+1} \sum_{\tau=-T}^T \tilde{R}_l^a(\tau) \quad (13)$$

where T is a positive integer, which is chosen so that $R_{11}^a(\tau) = 0$ for $|\tau| > T$.

In addition, improving the decorrelation of the SS technique is obtained at the cost of a reduction in array aperture, which usually causes the degradation in DOA estimation. A trade-off between goodness of decorrelation and potential resolution of DOA estimation is required. Although the performance of the SS scheme was widely analyzed, it is difficult to derive a general analytical expression of the optimal subarray size, which provides the best effectiveness of the spatial smoothing, because it

generally depends on the source coordinates and the relative source phases. As the goal of the SS technique is to increase the distance between the signal and noise subspaces eigenvalues, here we determine the optimal subarray size by maximizing the eigenvalue distance $D_{sn}(m)$ [11],

$$m_{opt} = \max_m \{D_{sn}(m)\}, \quad \text{for } m = p_a + 1, \dots, M-1, \quad (14)$$

where $D_{sn}(m) = \lambda_{p_a} - \lambda_{p_a+1}$, and $\{\lambda_i\}$ are the eigenvalues of the $m \times m$ modified matrix \hat{R}_t^a or \hat{R}_t^a in (13).

4.2 DOA Estimation Algorithm

The matrix $R_{x,l}^a(\tau)$ in (12) is just a submatrix of the CACM $R_t^a(\tau)$ in (4), and it can be formed simply and directly as the $m \times m$ block matrix along the $2L-1$ diagonal of $R_t^a(\tau)$. On the analysis basis as mentioned above, we can propose an algorithm for DOA estimation of coherent signals based on SS or ISS technique as follows.

Step 1: Calculate the estimates of the CACM $R_t^a(\tau)$ in (4) for $\tau = -T, \dots, -1, 0, 1, \dots, T$ from the finite N samples $x(n)$ as

$$\hat{R}_t^a(\tau) = \frac{e^{-j2\pi\alpha\tau}}{N} \sum_{n=0}^{N-1} x(n+\tau)x^H(n)e^{-j2\pi\alpha n}, \quad \text{for } \tau \geq 0 \quad (15)$$

$$\hat{R}_t^a(\tau) = \frac{e^{-j2\pi\alpha\tau}}{N} \sum_{n=0}^{N-1} x(n)x^H(n-\tau)e^{-j2\pi\alpha n}, \quad \text{for } \tau < 0 \quad (16)$$

Step 2: Set $m = p_a + 1, \dots, M-1$ and form the $m \times m$ estimated submatrix $\hat{R}_{x,l}^a(\tau)$ from the estimated CACM $\hat{R}_t^a(\tau)$ in (15) and (16) for $l, l' = 1, 2, \dots, L$.

Step 3: Calculate the estimate of the $m \times m$ modified spatial smoothed cyclic matrix \hat{R}_t^a or \hat{R}_t^a in (13) as

$$\hat{R}_t^a = \frac{1}{2T+1} \sum_{\tau=-T}^T \hat{R}_t^a(\tau), \quad \hat{R}_t^a = \frac{1}{2T+1} \sum_{\tau=-T}^T \hat{R}_t^a(\tau) \quad (17)$$

where the estimates of the SSCCM $\hat{R}_t^a(\tau)$ and ISSCCM $\hat{R}_t^a(\tau)$ are obtained from (9) and (12).

Step 4: Calculate the eigenvalue distance for the $m \times m$ estimated matrix in (17) as

$$\hat{D}_{sn}(m) = \hat{\lambda}_{p_a} - \hat{\lambda}_{p_a+1}$$

where $\{\hat{\lambda}_i\}$ are the eigenvalues of the matrix \hat{R}_t^a or \hat{R}_t^a .

Step 5: Determine the optimal subarray size m_{opt} as the value of $\{p_a + 1, \dots, M-1\}$ for which $\hat{D}_{sn}(m)$ is maximized.

Step 6: Estimate the DOA of the desired signals from the positions of peaks of the spectrum given by

$$S(\theta) = \frac{1}{\sum_{i=p_a+1}^{m_{opt}} |\hat{a}^H(\theta)\hat{u}_i|^2} \quad (18)$$

where $\{\hat{u}_i\}$ are the eigenvectors of the $m_{opt} \times m_{opt}$ matrix \hat{R}_t^a or \hat{R}_t^a .

5. NUMERICAL EXAMPLES

The ULA has $M = 8$ sensors with half-wavelength spacing. The sensor outputs are collected at the rate $f_s = 4$ MHz, the speed of propagation is $c = 3 \times 10^8$ m/s, the length of samples is $N = 1024$, and the lag parameter T is chosen as $T = 10$. The additive noise is assumed to be complex white noise with the same variance σ_n^2 .

Example 1: Heavy Loading Case

We consider a scenario that the total number of arriving signals is larger than that of sensors with SNR's are 10dB, where $p = 9$ and $p_a = 3$. The reference signal of the BPSK

1 source arrives from -40° with 1MHz baud rate ($\alpha = 0.25$), and it undergoes multiple reflection and results in two additional coherent arrivals along -15° and 50° with the multipath coefficients as $\beta_2 = -0.45 + j0.34$ and $\beta_3 = 0.29 + j0.2$, respectively. The six interfering signals, i.e. BPSK 2 and BPSK 3 impinge from 20° and -32.5° with 0.8 and 1.33MHz baud rates ($\alpha = 0.2$ and 0.33); AM 1 and AM 2 come from 10° and -53° with 0.3 and 0.18MHz carrier frequencies ($\alpha = 0.15$ and 0.09); FM 1 and FM 2 signals arrive from 28° and -4° with 0.04 and 0.1MHz carrier frequencies ($\alpha = 0.02$ and 0.05), respectively.

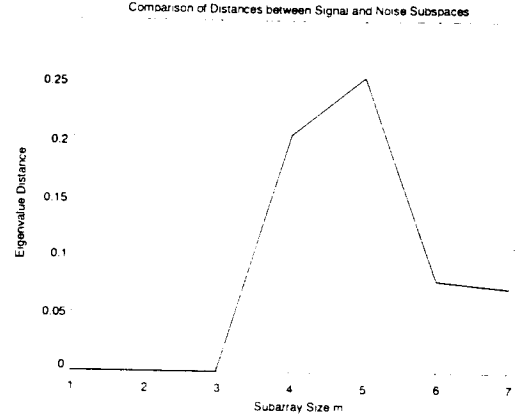


Fig. 1 The determination of the optimal size of the subarrays in the SS-based method in Example 1.

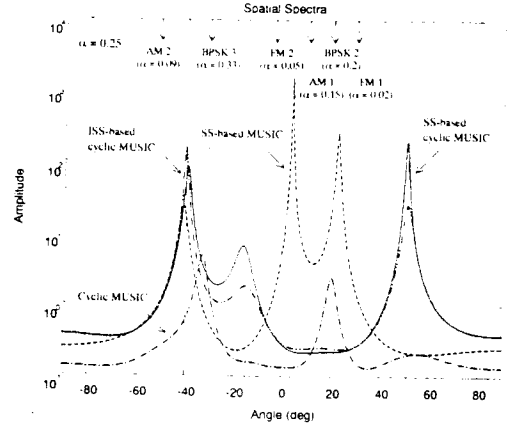


Fig. 2 The comparison of DOA estimation by using the proposed cyclic algorithms, the SS-based MUSIC and cyclic MUSIC methods in Example 1.

For employing the presented SS-based cyclic approaches, we have to choose the optimal subarray size, which gives the good effectiveness of decorrelation and accuracy in DOA estimation. From Fig. 1, it is shown that the optimal subarray dimension can be decided as $m_{opt} = 5$, which gives the maximum distance between the signal and noise subspaces. It is known that the conventional MUSIC is impossible to estimate the DOA as the signal number is greater than that of the sensors. For verifying the performance of the proposed SS and ISS-based methods, 50 trial runs are performed with $m_{opt} = 5$ and $\alpha = 0.25$, and the averaged spatial spectra are illustrated in Fig. 2, where these results are compared with the conventional SS-based MUSIC and cyclic MUSIC (with $\alpha = 0.25$ and multiple τ) algorithms. Due to the more impinging signals and strongly adverse interference environments where the signal to interference plus noise ratio (SINR) is less than -5.5 dB or the coherency of SOI, the resolution of the ordinary SS-

based MUSIC and cyclic MUSIC methods decreases and fails completely. However the proposed approach is superior and has no problem in resolving the coherent source, and the gain in detection and accuracy can be obtained by applying the presented SS and ISS-based cyclic algorithms.

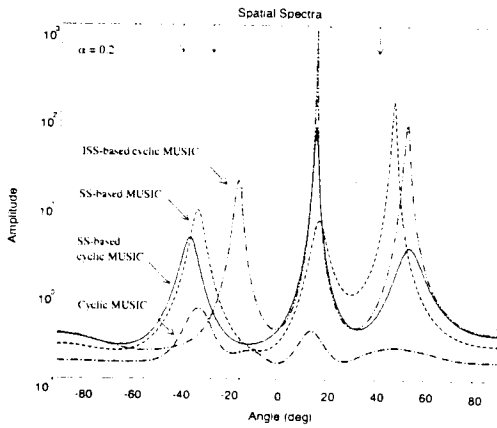


Fig. 3 The comparison of DOA estimation for the BPSK 2 coherent source in Example 2.

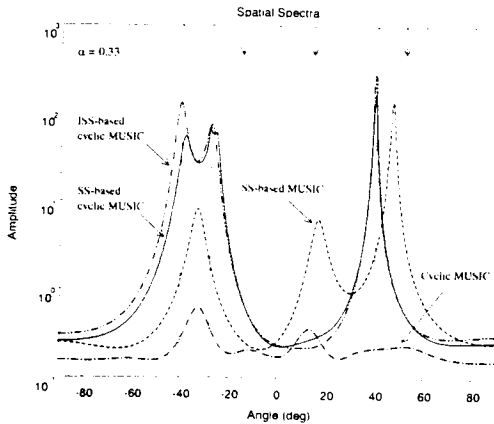


Fig. 4 The comparison of DOA estimation for the BPSK 3 coherent source in Example 2.

Example 2: Two Coherent Sources Case

Here we illustrate the behavior of the presented approach in the two coherent sources case, where the two reference signals from the BPSK 2 and BPSK 3 sources come from the directions 15° and -27.5° with 0.8MHz and 1.33 MHz baud rates ($\alpha = 0.2$ and 0.33), and SNR's are assumed to be 10dB. There are two additional coherent arrivals for each source. The pair of the BPSK 2 source has the angles of arrival of -14° and 53° with the multipath coefficients $-0.45 + j0.34$ and $0.3 - j0.7$, and the pair of the BPSK 3 source comes from the directions -40° and 40° with the multipath coefficients $0.4 + j0.8$ and $0.5 - j0.6$, respectively.

For detecting the two coherent pairs, 50 trials are run with $m_{opt} = 5$, and $\alpha = 0.2$ and 0.33 , respectively, the averaged spatial spectra of the proposed method are demonstrated in Fig. 3 and 4, and compared with the traditional SS-based MUSIC and cyclic MUSIC (with multiple τ , and $\alpha = 0.2$ and 0.33 , respectively) algorithms. In this case, the ordinary SS-based MUSIC method has a problem in resolving the two coherent sources. Due to the close impinging angles of two coherent sources, the peaks of the SS-based cyclic MUSIC spectra give some wrong arrival estimates in the region of -14° to -40° . However, since the information of the cyclic cross correlations of the subarrays is exploited

sufficiently, the ISS-based cyclic MUSIC algorithm is still superior and has a better performance. All three directions of each coherent source can be clearly identified and the improvement of the ISS-based algorithm in terms of resolvability is also clarified.

6. CONCLUSIONS

In this paper, a new approach for DOA estimation of cyclostationary coherent signals was proposed by applying the SS technique. For attaining the robust performance of DOA estimation, the SS and ISS-based algorithms were improved by using the multiple lag parameters and the optimal subarray size to exploit the cyclic statistics sufficiently and handle the coherence effectively. The effectiveness of the proposed algorithm was illustrated and compared with the ordinary SS-based MUSIC and cyclic MUSIC methods through numerical examples. It was clarified that the presented method can provide satisfactory performance, even in strongly adverse interference environments, and give good accuracy even when the number of impinging signals is larger than that of the array sensors.

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