CURVATURE ESTIMATION OF ORIENTED PATTERNS

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ABSTRACT

In digital pictures curvature which characterizes the local geometry of features cannot be calculated exactly and can only be estimated. This paper presents two methods to estimate curvature of oriented patterns at different scales which can be used even if the structures are very close. The first approach assumes that a structure is locally defined by an implicit isointensity contour. Curvature is obtained by a direct computation stemming from the differerential geometry. The second approach is based on an explicit representation of a structure defined by a set of points initially extracted. We assume this set represents an osculating circle and fit it with a circular arc. Both methods are applied to curvature estimation of seismic images for geological interpretation.

1 Introduction

Curvature estimation algorithms play an important role in the interpretation of digital pictures. Indeed curvature which characterizes the local geometry of features is a key notion in object recognition, biomedical data analysis [1], topographic feature extraction [2], ... The literature on the differential geometry [3] provides equivalent definitions of the curvature. Figure (1) illustrates one of them which is based on the local touching circle: the curvature of a curve is defined as the inverse of the radius of the osculating circle.

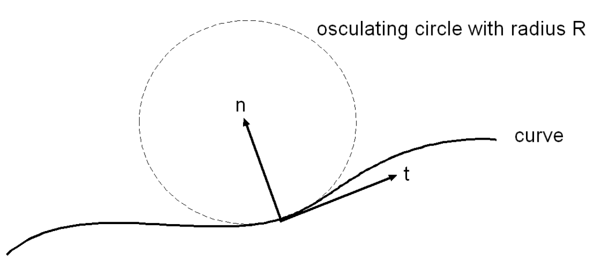

Figure 1: Definition of curvature

In digital pictures the curvature cannot be calculated exactly and hence must be estimated. Moreover curvature features have different levels of details and the estimation can be computed at different scales. This paper presents two methods to estimate curvature of structures at different scales which can be used even if the structures are very close. The first approach consists in a direct computation stemming from the differential geometry whereas the second one is based on the estimation of the osculating circle by assuming that a structure is defined by a discrete set of points. These methods are applied to curvature estimation of seismic images for geological interpretation.

2 2-D Curvature Estimation based on Differential Geometry

The first approach is based on the implicit representation of a structure. Assuming that the structure is locally defined by an isointensity contour, we compute the curvature from the first and second partial derivatives of the gray level fonction. Let $I(x, y)$ be an infinitely differentiable image and (C) an isointensity contour. We denote by $M(x, y)$ a point of (C) and by g the gradient of I at point M which is normal to (C). In a counter-clockwise fashion, the unit vector t tangent to (C) at point M is defined by:

$$t = \frac{1}{\sqrt{I_x^2 + I_y^2}} \begin{bmatrix} I_y \\ -I_x \end{bmatrix} \tag{1}$$

where I_x and I_y are the first partial derivatives of I. Then the curvature k can be computed [4] as:

$$k = -\frac{t^T H t}{||g||} \tag{2}$$

where $H = \begin{bmatrix} I_{xx} & I_{xy} \\ I_{yx} & I_{yy} \end{bmatrix}$ is the Hessian matrix of I and $I_{xx}, I_{yy}, I_{xy} = I_{yx}$ are the second partial derivatives of I.

Finally the curvature k is obtained from five partial derivatives:

$$k = \frac{2 I_x I_y I_{xy} - I_x^2 I_{yy} - I_y^2 I_{xx}}{\left(I_x^2 + I_y^2\right)^{3/2}} \tag{3}$$

We propose another expression by introducing the unit vector n normal to (C) at point M:

$$n = \frac{g}{\|g\|} \tag{4}$$

Then the curvature k can be expressed [5] as:

$$k = -div\,(n) \tag{5}$$

where $div\,(.)$ represents the divergence operator.

This formulation only requires four partial derivatives and a lower computation cost:

$$k = -\left(\frac{\partial n_1}{\partial x} + \frac{\partial n_2}{\partial y}\right) \tag{6}$$

with

$$n = \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \frac{1}{\sqrt{I_x^2 + I_y^2}} \begin{bmatrix} I_x \\ I_y \end{bmatrix} \tag{7}$$

Expression (3) is usually implemented with smoothed partial derivatives in order to obtain a continuous gradient vector field [4][6]. Unfortunately in the case of seismic images elementary structures are very close, which forbides the use of smoothed derivative operators. It leads to biased values of curvature.

In order to regularize the gradient vector field we use a principle component analysis (PCA) of the correlation matrix:

$$A = \begin{bmatrix} \overline{I_x^2} & \overline{I_x I_y} \\ \overline{I_x I_y} & \overline{I_y^2} \end{bmatrix} \tag{8}$$

where $\overline{(.)}$ denotes the mean operator computed in a rectangular window.

It allows us to compute a local dominant orientation which is given by the eigenvector corresponding to the largest eigenvalue. Thus an estimation of the regularized unit vector n^* normal to (C) is available and the expression (6) directly provides the curvature, using n^* instead of n.

One should be aware of a major limitation in using a PCA of the gradient vector field for the estimation of the unit vector n^*. This leads to an orientation field instead of a vector field which has $\pm\pi$ ambiguity in phase angle. Hence curvature estimation which requires a vector field as input fail to yield the desired results. In the case of seismic images we can reasonably assume that elementary structures are never vertical. Then the choice of a phase angle of the unit vector n^* belonging to $[0, \pi]$ allows us to obtain a continuous unit vector field and a correct curvature estimation.

The rectangular window used in the calculation of the correlation matrix A provides discontinuities in the curvature estimation. The substitution of the convolution by a gaussian kernel for the mean operator enables us to get a more regular curvature. The new expression of the correlation matrix is:

$$A_\sigma = \begin{bmatrix} I_x^2 & I_{xy} \\ I_{xy} & I_y^2 \end{bmatrix} * G_\sigma \tag{9}$$

where G_σ represents a gaussian kernel with standard deviation σ.

Either the size of the rectangular window used for the computation of the PCA of the gradient vector field or the parameter σ can be considered as a scale factor, which is linked to the desired level of details of curvature features.

3 Curvature Estimation from Digital Data

The second approach is based on an explicit representation of a structure which is defined by a set S of points initially extracted:

$$S = \{(x_i, y_i) \mid i \in [1..N]\} \tag{10}$$

This set is composed of points located on both sides of the pixel where we estimate the curvature. Then the curvature at point M is computed by assuming that the set S is a discrete approximation of the osculating circle [7] to the structure. Therefore N the number of points which compose the set S characterizes the scale of curvature we aim at estimating.

3.1 Extraction of points

A way to extract points belonging to a structure is to use the estimation of the local dominant orientation obtained from the PCA of the gradient vector field. From the unit normal vector n estimated in each point we first define an unit tangent vector t. Then the local form of a structure can be reconstructed by integration of vector t which also provides a set of consecutive points.

3.2 Curvature Estimation

Our approach is based on the assumption that the set S represents the osculating circle. We fit a circular arc with radius R and center $C\,(x_c, y_x)$ on the local contour. Then the curvature of the structure is defined by:

$$k = \begin{cases} \frac{1}{R} & \text{locally convex structure} \\ \frac{-1}{R} & \text{locally concave structure} \end{cases} \tag{11}$$

A classical method [8] consists on the minimization of the error function:

$$J = \sum_{i=1}^{N} \left(R^2 - \left[(x_i - x_c)^2 + (y_i - y_c)^2\right]\right)^2 \tag{12}$$

Minimization of J leads to a linear matrix system with a direct solution. The major problem of this method arises when points are almost aligned: the estimated curvature is large whereas it should be nearly equal to zero.

Therefore we propose another criterion which combines (figure 2) the length of the chord defined by the extremal points of the set S and the area A of the surface delimited by the set S considered as a close curve:

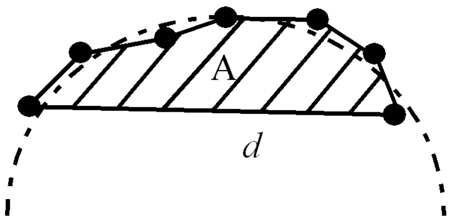

Figure 2: Length d of a chord and area A.

The radius R is then solution of the following equation:

$$A - R^2 \left[\arcsin\left(\frac{d}{2R}\right) - \frac{d}{2R}\sqrt{1 - \left(\frac{d}{2R}\right)^2} \right] = 0 \quad (13)$$

This function defined for $R \in \left[\frac{d}{2}, \infty\right[$ is strictly increasing and can be solved by a dichotomic algorithm. In practice less than ten iterations are required to get accurate results. Moreover in the case of large radius $d \ll R$, a direct estimation is given by:

$$R = \frac{d^3}{12A} \quad (14)$$

4 Results

The methods we propose have been applied to seimic images in order to detect points of high curvature. A 2-D seismic image (figure 3) is composed of seismic traces, sinusoid-like waveforms, which are a record of reflected waves arising from impedance contrasts between strata. The local extrema of seismic traces define spatially consistent lines, called seismic horizons. A major purpose in seismic data analysis is the detection and the characterization of seismic horizons separating homogeneous layers of rocks, sediments, ... Curvature estimation of these horizons is useful to help geological interpretation.

The results of the differential approach given in figure 4 have been obtained by computing the divergence of the unit normal vector. The curvature estimation is sensitive to noise when the unit normal vector is obtained by a PCA computed from low size windows. Therefore we use a PCA computed from 21×21-sized windows (figure 4) which allows us to estimate only large scale curvatures. Moreover the use of a gaussian kernel with standard deviation $\sigma = 0.5$ provides a more regular curvature.

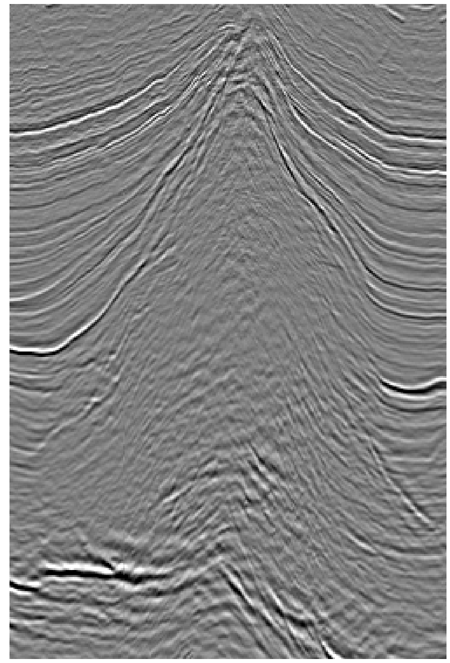

Figure 3: A seismic image

One can observe on this image that the loci of the detected points describes the axis of the principal seismic dome. Moreover the results point out the axis of two differents domes almost visually imperceptible on the original image. This was confirmed by geologists.

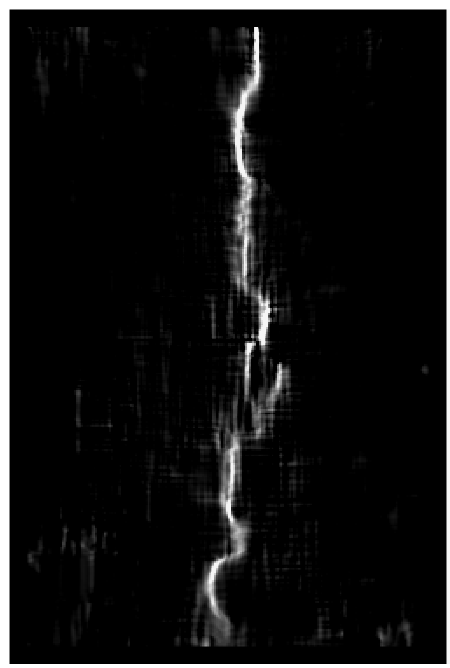

Figure 4: Convex curvature estimation based on the differential geometry (mean operator of size 21×21).

The seismic image was also processed with the second approach: the radius of the osculating circle is computed from a set of 15 points. The high curvature loci is quite regular as shown in figure 5.

Figure 5: Convex curvature estimation based on the osculating circle (15 points, PCA of size 11×11*).*

We compare the noise robustness of both approaches using a synthetized image (figure 6) made up of circular arcs. Different amounts of gaussian noise (standard deviation $\sigma_n = 1, 5, 10, 15, 20, 25$) have been added. The curvature estimation based on the differential geometry is processed with a PCA of size 19×19. The second method is computed from sets of 7, 9 and 11 points provided by integrating of unit tangent vector obtained from a PCA of size 13, 11 and 9 respectively. We can notice that the neighbourhood used in both cases is similar. The results (figure 7) point out that the curvature based on digital data is more robust to noise. Unfortunately this approach can only be used with sets of few points extracted by our method: the assumption that the set of points initially extracted is a discrete approximation of the osculating circle must be verified.

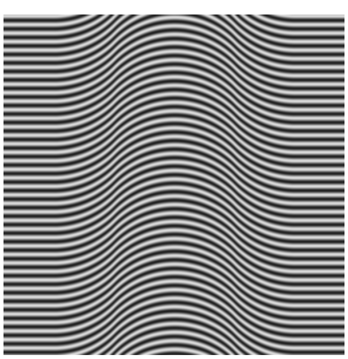

Figure 6: Synthetized image of arcs.

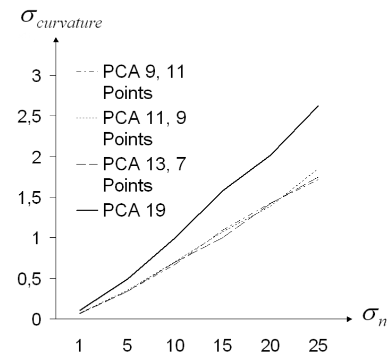

Figure 7: Standard deviation of curvature estimation.

5 Conclusion

The 2-D curvature estimation has been extended to the 3-D case successfully. From a computationnal point of view the differential geometry method is directly adapted whereas the discrete arc approximation method is implemented as a multi 2-D approach.

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