Optimised Wold-Like Decomposition of 2D Random Fields

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ABSTRACT

In this paper we address the problem of image texture modeling. In particular we adopt here a 2D Wold decomposition that separates a texture into regular and chaotic parts, allowing for simple texture parameterization, and for explicit extraction of periodic structures. The identification scheme based on the proposed decomposition improves the accuracy of the estimation of the model parameters as well as the visual resemblance of the model with respect to samples, owing to compliance of the chaotic component with the Julesz's conjecture.

1 INTRODUCTION

In the literature, different approaches have been proposed for the modeling of bidimensional random fields. In fact, to cite a few, they range from morphological models, and models using fractals or Markov Random Fields to autoregressive (AR) models or moving average (MA) models.

The choice of a suitable model is highly dependent on the application. For instance, AR (see [1]) or MA (see [2]) systems driven by independent identically distributed (i.i.d.) excitation have been proposed for classification or synthetic reproduction of natural textures. The use of such a kind of models simplifies the identification stage which is performed using deconvolution techniques. Higher order statistical analysis can be used in order to estimate the parameters characterizing the texture as pointed out in [3], [4]). Then the synthetic replicas can be obtained using the taken on model and the estimated parameters.

However the quality of the synthetic replicas depends on the capability of the used model to fit the given prototype. In order to obtain a more faithful synthesis, the model has to be complicated and more texture's characteristics have to be taken into account. In [5], it has been pointed out that, it is possible to take into consideration perceptive texture's characteristics such as *randomness*, *directionality*, *periodicity*, through the decomposition of the texture into its *unpredictable* and *deterministic* components according to the generalization to the 2D case of the Wold decomposition theorem [6], [7]. Referring to discrete time processes, the 1D Wold decomposition represents a process as a sum of two orthogonal components, the *deterministic* and the *unpredictable* component. The "deterministic" (perfectly predictable) component is constituted by a sum of harmonics. The "unpredictable" component is a causal AR process, *i.e.* a process generated by a white process acting as excitation, called innovation, driving a stable all pole filter. Extension to 2D processes of such a model is not straightforward, since it requires a definition of a causal ordering of the field elements. Using the classical lexicographic order, a half plane causal prediction mechanism can be set. With this convention, it is possible to define the unpredictable component as an AR causal series, while the deterministic component consists of a countable sum of bidimensional harmonics. In addition, a third component must be accounted for in the bidimensional case. This component, referred to as *evanescent field*, is the sum of a countable set of 1D fields (processes constant along one direction) and can be seen as a combination of plane waves. Thus, the identification of an extended Wold model of a 2D field requires not only the detection and estimation of harmonic components, and the identification of the AR model, but also the extraction and the identification of the evanescent field. In a recent paper, a method for 2D Wold decomposition has been proposed for texture analysis and synthesis [6]. The scheme aims to separate the texture into regular and chaotic parts, roughly associated to the deterministic and unpredictable components respectively. Even with some intrinsic limits coming from the purely additive modeling and from the Gaussian nature of the unpredictable part, the Wold-like decomposition allows for simple texture parameterization, and for explicit extraction of periodic structures. On the other hand, recently the authors,[8], [9], devised a quite different method for texture analysis and synthesis, directly inspired to visual criteria, and in particular to the well known Julesz's conjecture [10]. This conjecture admits that textures having the same first order and second order spatial distributions are distinguished with difficulty from the visual system, at least in a preattentive stage of vision. Actually, the method approximates a field with prescribed first and second order distributions by means of a system, driven by a realization of a white Gaussian random field, constituted by the cascade of a linear system, a zero-memory non-linearity, and finally another linear system. This model allows to perform an unsupervised synthesis of textures circumventing the problem of adding a human supervised component to the excitation, in order to copy the underlying structure of many textures encountered in practice (see [2]).

The aim of the present work is to merge the two approaches into a unique framework, and to optimize the overall identification process.

In essence, a modeling technique consisting of the following stages is proposed:

1. identification of the harmonic component, using a Fourier analysis of the prototype,
2. identification of the evanescent field, using the Radon transform, after having subtracted the harmonic component from the prototype,
3. identification of the unpredictable component, after having subtracted the two components obtained in the two previous steps, through the non–linear method described in [8].

The above modeling scheme can be used for analysis and synthesis purposes. For instance, it is well suited for automatic quality control of industrial products characterized by regular textured surfaces. In this case, the examination of the unpredictable part allows to detect even small deviations from regularity. In synthesis problems, it allows to produce textures with a great deal of naturalness, ranging from "clean" ideal periodic to rough versions of a given prototype.

In this paper, after a theoretical survey about the optimality of the procedures, algorithms are described in detail, then results of texture synthesis are shown.

2 THE 2D WOLD–LIKE DECOMPOSITION

Let $\mathbf{T} = \{t[n_1, n_2], [n_1, n_2] \in \mathcal{Z}^2\}$ be a 2D regular and homogeneous random field. It can be shown, [6], that these kind of processes can be uniquely represented by the orthogonal decomposition

$$t[n_1, n_2] = y[n_1, n_2] + q[n_1, n_2] \tag{1}$$

where $\mathbf{Y} = \{y[n_1, n_2], [n_1, n_2] \in \mathcal{Z}^2\}$ is the so called *purely indeterministic field* and $\mathbf{Q} = \{q[n_1, n_2], [n_1, n_2] \in \mathcal{Z}^2\}$ is a *deterministic random field* which can be perfectly predicted from the field past samples. However in the bidimensional case, unlikely the monodimensional case, there is no natural order definition for "past". Let us recall, [6], that a family of non symmetrical half-plane (NSHP), whose border line is of rational slope $\gamma = \alpha/\beta$, can be defined. We will refer to the NSHP total–ordering definition, whose boundary line is of rational slope, as *Rational* NSHP (RNSHP). Let $\mathcal{O}$ be the set of the RNSHPs which is possible to define on the 2D support, then each realization $q[n_1, n_2]$ of the stochastic process $\mathbf{Q}$ can be expressed in the following way, [6]:

$$q[n_1, n_2] = p[n_1, n_2] + \sum_{(\alpha,\beta)\in\mathcal{O}} e_{(\alpha,\beta)}[n_1, n_2] \tag{2}$$

where the random field $\mathbf{P} = \{p[n_1, n_2], [n_1, n_2] \in \mathcal{Z}^2\}$ is *half–plane deterministic*, that is, it has no column–to–column innovations with respect to the RNSHP support, and the random field $\mathbf{E}_{(\alpha,\beta)} = \{e_{(\alpha,\beta)}[n_1, n_2], [n_1, n_2] \in \mathcal{Z}^2, (\alpha, \beta) \in \mathcal{O}\}$ is the *evanescent component* which takes into account the column–to–column innovations of the deterministic random field.

Therefore, if the process $\mathbf{T}$ is a 2D regular and homogeneous random field, using eqs.(1),(2) the following decomposition for $t[n_1, n_2]$ holds

$$t[n_1, n_2] = y[n_1, n_2] + p[n_1, n_2] + \sum_{(\alpha,\beta)\in\mathcal{O}} e_{(\alpha,\beta)}[n_1, n_2] \tag{3}$$

Using the arguments presented in [7], the *evanescent field* has the spectral density function which can be expressed as the sum of a finite number of 1D δ functions concentrated along lines of rational slope in the 2D Fourier domain.

In natural textures often encountered in practice, the *half plane deterministic* component can assume the form of an harmonic random field as expressed by the following formula

$$p[n_1, n_2] = \sum_{k=1}^{K} [C_k \cdot \cos 2\pi(n\omega_k + m\nu_k) + D_k \cdot \sin 2\pi(n\omega_k + m\nu_k)]$$

where C_k's and D_k's are mutually orthogonal random variables and (ω_k, ν_k) is the spatial frequency of the $k-th$ harmonic.

Finally, here we model the *purely indeterministic* component accordingly to the approach proposed by the authors in [8], [9], which is briefly summarized in the next section.

3 THE INDETERMINISTIC COMPONENT SYNTHESIS MODEL

The model, [8], employed to represent the *purely indeterministic* component of the texture is depicted in fig.1. A realization $u[n_1, n_2]$ of a Gaussian white random field $\mathbf{U} = \{u[n_1, n_2], n_1 \in N_1, n_2 \in N_2\}$ feeds the cascade of a linear system $c[n_1, n_2]$, a zero memory non–linearity (hard–limiter), a linear system $h[n_1, n_2]$, and a final histogram equalizer $\eta_\theta(\cdot)$. The output, $v[n_1, n_2]$, of the hard–limiter, driving the linear system $h[n_1, n_2]$, is

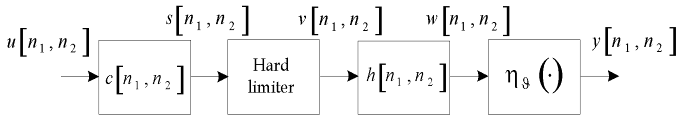

Figure 1: Indeterministic component's model

a binary random field obtained by minimizing the difference between the first and the second order statistics of the model's output and of the image sample. The filter $h[n_1, n_2]$ accomplishes the task of reconstructing the texture from the binary image, while a further refinement is obtained performing a histogram equalization, matching the synthetic texture with the original one. The rational of this choice stems from the Julesz's conjecture, that requires the coincidence of first and second order statistics to generate a texture visually close to the given prototype, at least in a preattentive stage of vision. In essence, the i.i.d. excitation used in classical approaches, (see *e.g.* [1],[2]), to feed a linear system is replaced by a more structured field tailored to retain the morphological behavior of the prototype. The choice of a binary excitation presents the advantage that, under weak symmetry conditions, the whole of its second order distributions is set by its autocorrelation function (a.c.f.). This in turn implies that it is possible to generate a binary excitation, $v[n_1, n_2]$, with wanted second order statistics by hard–limiting a realization of a Gaussian random field $s[n_1, n_2]$ with a suitable a.c.f.. In fact, as stated by the *arcsin law* [11], the a.c.f. $R_{ss}[n_1, n_2]$ of a realization of a random field $s[n_1, n_2]$ and the a.c.f. $R_{vv}[n_1, n_2]$ of its binarized version $v[n_1, n_2]$ are related through the following formula

$$\frac{R_{vv}(k,l)}{\sigma^2_{v_r}} = \frac{2}{\pi} \cdot \arcsin\left(\frac{R_{ss}(k,l)}{\sigma^2_s}\right) \tag{4}$$

The identification of the binary excitation and of the filter $h[n_1, n_2]$ is performed using a Bussgang blind deconvolution algorithm.

4 THE IDENTIFICATION AND SYNTHESIS PROCEDURES

The proposed identification and synthesis technique can be summarized in the following steps:

4.1 Identification of the harmonic component

Unlike in [6], it is accomplished starting from the estimation of the bidimensional a.c.f. of the texture from a selected sample. First, a coarse estimate of the Periodic Spatial Grid (PSG) is determined looking at the a.c.f. maxima nearest to the origin. Using this coarse estimate, values of the harmonic components in the Fourier plane are picked up, and finely adjusted to maximize the captured spectral energy. Let us indicate with $z[n_1, n_2]$ the impulse response of the filter retaining the information on the harmonic components of the given texture.

4.2 Identification of the evanescent field

Once identified, the additive periodic component of the texture is subtracted from the sample. Then, the presence of evanescent components is detected by thresholding the corresponding maximum likelihood ratio. As pointed out in [12], an efficient implementation of the Bayesian joint detection and estimation procedure can be done in the Radon transform (RT) domain. Given an image $i(x, y)$, let $g_i(s, \theta)$ be its RT usually defined as the line integral along a line inclined at an angle θ from the y-axis and at a distance s from the origin, that is:

$$g_i(s,\theta) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} i(x,y)\delta(x\cos\theta + y\sin\theta - s)dxdy$$

and let $\hat{g}_i(s, \theta)$ be the *filtered projections* obtained by filtering each *projection* $g_i(s, \theta)$ along s by a one-dimensional filter $\mathcal{H}_s$ whose frequency response is $|\xi|$. More specifically, introducing the operator $\widehat{\mathcal{R}}_s = \mathcal{H}_s\mathcal{R}_s$, and denoting with $G_i(\xi, \theta) = \mathcal{F}_1\{g_i(s, \theta)\}$ the 1D Fourier transform of $g_i(s, \theta)$ along s, the filtered projections are:

$$\hat{g}_i(s,\theta) = \widehat{\mathcal{R}}_{s,\theta}\{i(x,y)\} = \mathcal{H}_s\{g_i(s,\theta)\} = \int_{-\infty}^{+\infty} |\xi| G_i(\xi,\theta) e^{j2\pi\xi s} d\xi$$

Then, for Gaussian stationary observation noise, the *filtered projections* are sufficient statistics for both detection and estimation problems, and the canonical configuration of the Likelihood processor is constituted by the operator $\widehat{R}_{s,\theta}$ cascaded with a *whitening filter* and a Wiener filter adapted to the power density spectrum of the 1D cross-section of each planar wave [12]. Thus, whenever the energy of the output of the Wiener filter exceeds a given threshold, the said output is retained as an evanescent component of the texture at the corresponding direction.

4.3 Identification of the unpredictable component

The identification of the unpredictable component can be performed after having subtracted the identified evanescent field from the sample. Let us indicate with $f[n_1, n_2]$ the estimate of the inverse of the filter $h[n_1, n_2]$, with $\mathbf{f}$ the filter operator associated to $f[n_1, n_2]$, and with $\mathbf{y}$, $\mathbf{w}$, $\mathbf{v}$, the vectors obtained rearranging $\{y[n_1, n_2]\}$, $\{w[n_1, n_2]\}$, $\{v[n_1, n_2]\}$ columns in lexicographic order. In Table 1, the identification procedure, without its detailed derivation (which can be found in [8]), is summarized. The unpredictable component is identified using the non-linear method described in Table 1. Once estimated the a.c.f. $\widehat{\mathbf{R}}_{vv}[k, l]$ of the binarized prototype, we apply the inverse arcsin law to find the a.c.f. of $\widehat{\mathbf{R}}_{ss}[k, l]$ whose spectral factorization leads to the design of the filter $c[n_1, n_2]$.

Table 1: Indeterministic component iterative identification procedure.

(*initialization*)
$\eta^{-1}_{\theta(\mathbf{0})}(\mathbf{y}) = \mathbf{y}$
$\mathbf{f}^{(0)} = [0\ 0 \cdots 1 \cdots 0\ 0]^{\mathrm{T}}$
j =0
Repeat
$\mathbf{w}^{(j)} = \eta^{-1}_{\theta(j)}(\mathbf{y})$
$\mathbf{z}^{(j)} = \mathbf{D}_{\mathbf{w}^{(j)}} \mathbf{f}^{(j)}$
$\mathbf{v}^{(j)} = sign\left[\mathbf{z}^{(j)}\right]$
$\mathbf{f}^{(j)} = \widehat{\mathbf{R}}^{-1}_{\mathrm{w(j)}\mathrm{w(j)}} \cdot \widehat{\mathbf{R}}_{\mathrm{v(j-1)}\mathrm{w(j)}}$
$\theta^{(j)} = \arg\underset{\theta}{\text{histogram matching}} \left(\mathbf{G}^{-1}_{\mathbf{f}^{(j)}} \mathbf{v}^{(j)}, \mathbf{y}\right)$
$j = j + 1$
Until $\widehat{\mathbf{R}}_{\mathrm{v}^{(j+1)}\mathrm{w}^{(j)}} = \text{const} \cdot \widehat{\mathbf{R}}_{\mathrm{v}^{(j)}\mathrm{w}^{(j)}}$

4.4 Synthesis of the harmonic component

The synthetic texture's harmonic component is obtained passing a realization of a white Gaussian random field through the filter $z[n_1, n_2]$ estimated in the analysis stage.

4.5 Synthesis of the indeterministic component

The indeterministic component is synthesized employing the model depicted in fig.(1) driving the estimated filter $c[n_1, n_2]$ with a realization of a white Gaussian random field $u[n_1, n_2]$. The signal so obtained is then binarized, passed through the filter $h[n_1, n_2]$, and equalized.

Finally these two components are added to the original texture's evanescent component producing the synthetic replica of the given prototype.

5 EXPERIMENTAL RESULTS AND CONCLUSIONS

To give an example of how the proposed method works, in fig.2, a whole sequence of images taken at various processing stages, starting from the prototype to the final results is shown. It is worth notice that such a methodology not only separates regular and 1D components from random components, but also improves parameter estimation accuracy and visual resemblance of the model with respect to samples, owing to compliance of the indeterministic component with Julesz's conjecture.

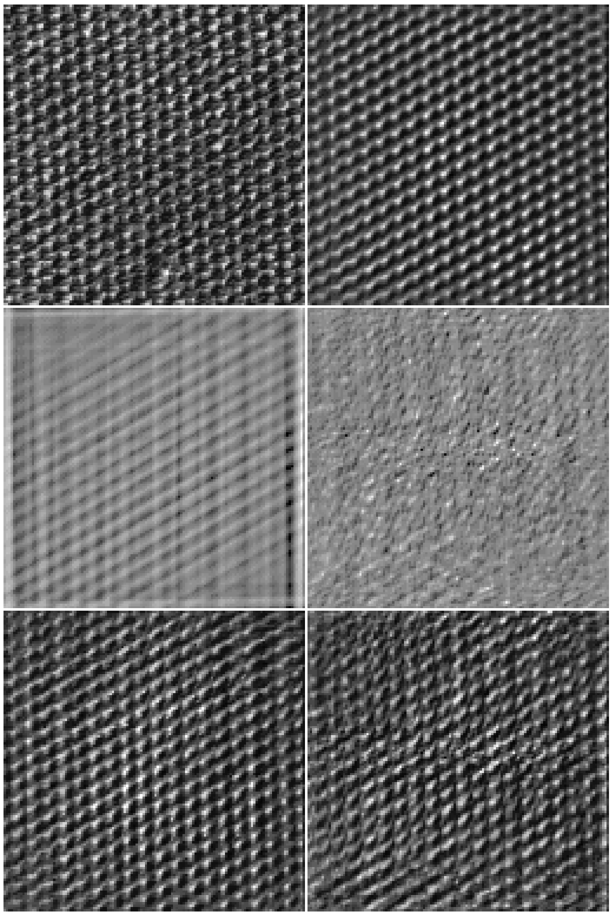

Figure 2: First row: left, D77(cotton canvas); right, synthetic harmonic component. Second row: left, evanescent field; right, synthetic unpredictable component. Third row: left, synthetic texture (synthetic harmonic plus evanescent components); right, synthetic texture (synthetic harmonic plus evanescent plus synthetic unpredictable components)

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