

# ELECTROSTATIC FORMULATION FOR ADAPTATIVE DILATION

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## ABSTRACT

We introduce a new concept of caricature in order to exacerbate the main morphological characteristics of objects. Here, the term caricature implies that we are looking for a method leading to a simplification or an exaggeration of the particularities of objects.

Our technique is based on the shape of the object considered as an insulator charged with unipolar electricity. The electric forces exerted on each pixel belonging to the contour create the desired transformation. The contextual information is also taken into account through the definition of attractive forces between objects.

We consider only the case of the dilatation of objects in a binary image. The theory is illustrated through a problem concerning connection of objects in a binary image and through an example of contour closing.

## 1 ANISOTROPIC DILATION BY ENDOGENOUS ACTION

### 1.1 Dilation of a convex object

Generally, the morphological transformation of objects is based on well known techniques such as dilation and erosion. These techniques are isotropic and do not give good results in several applications where dilation and a connection of small objects is desired. The isotropic techniques are computed using the same pre-defined kernels [4]. The obtained dilation corresponds to an uniform growth in every direction. Our aim is, inversely, to dilate objects in preferential directions depending on the shape of objects. Figure 1 shows the expected result for a long object. In that case, the directional dilation might stretch the object along its principal axis without thickening.

There are some techniques proposed to obtain a directional dilation of objects taking into account their morphological properties [5]. We propose, here, to consider the object as an insulator imbedding electric charges.

An elementary electric charge is associated to each pixel, so a repulsive force is exerted by each pixel on the other pixels of the object. The repulsive force exerted by a pixel  $N$  on pixel  $M$  is defined as :

$$\vec{F}(M/N) = -\vec{F}(N/M) = \Psi(d(M, N)) \cdot \vec{u}_{NM} \quad (1)$$

where  $\Psi(d)$  is a positive function which tends to 0 when  $d(M, N)$  tends to infinity and  $\vec{u}_{NM}$  is the unit vector between  $N$  and  $M$ .

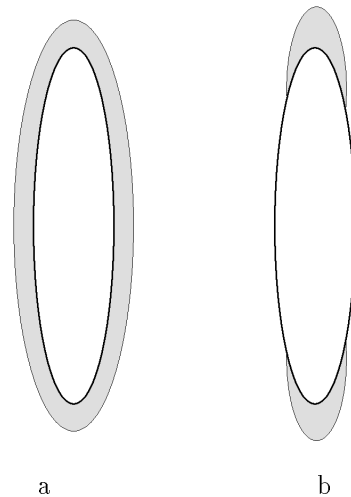


Fig.1 : Dilation of a long object. (a) isotropic dilation, (b) expected result.

In case of a strict analogy with Coulomb's forces,

$$\Psi(d(M, N)) = \frac{\lambda}{d^2(M, N)} \quad (2)$$

where  $\lambda$  is a parameter depending of the dielectric permittivity of the environment.

We can imagine other forms of distance function such as a gaussian (3) or, for instance a function inspired by Rayleigh law (4). The use of the function  $\Psi$  allows us to control the dependence on the distance between considered pixels in order to obtain different dilation effects.

$$\Psi(d) = \lambda e^{-\frac{d^2}{\sigma^2}} \quad (3)$$

$$\Psi(d) = \lambda d^\alpha e^{-\frac{1}{2}d^2} \quad (4)$$

The resulting force exerted on a pixel  $M$  by other pixels of a convex object  $X$  is then denoted by :

$$\begin{aligned}\vec{F}(M/X) &= \sum_{N \in X \setminus \{M\}} \vec{F}(M/N) \\ &= \sum_{N \in X \setminus \{M\}} \Psi(d(M, N)) \cdot \vec{u}_{NM} \quad (5)\end{aligned}$$

Due to the decreasing influence of pixels when  $d$  increases, the force defined in (5) does not take the overall shape of object into account.

In order to obtain a preferential dilation of the extremities, we modulate the force by a coefficient depending on the circular distribution  $p(\theta)$  of the elementary forces exerted on a pixel. Thereafter, we penalize the modulus of the resulting force by a function  $\xi(p)$  defined, for example, as :

$$\xi(p) = \left( \frac{|\sum \Psi \cdot \vec{u}|}{\sum \Psi} \right)^\delta \quad (6)$$

where  $\delta$  is a tuning parameter.

## 1.2 Extension to the dilation of a concave object

We propose to extend the previous theory to any type of object. Figure 2 shows a concave object and the expected result of a dilation. In this particular case, a dilation is supposed to close the ring while the forces defined above lead to a repulsion between extremities.

The generalization goes through the definition of the force exerted by a pixel  $N$  on a pixel  $M$  when the direct path between them is not a geodesic path (we can note : " $N$  is not seen by  $M$ ").

Considering that the permittivity of the object is negligible in comparison with the permittivity of the environment, we propose to consider that the force exerted by  $N$  on  $M$  is not depending on the euclidian distance but more generally on the geodesic distance between  $N$  and  $M$ . The direction of the force is then defined by the geodesic path as Figure 3 shows. In this paper, by simplifying, only the pixels seen by  $M$  (in the above sense) are used in the calculation of the resulting force. This simplification is justified if the function  $\Psi(d)$  is rapidly decreasing with  $d$ .

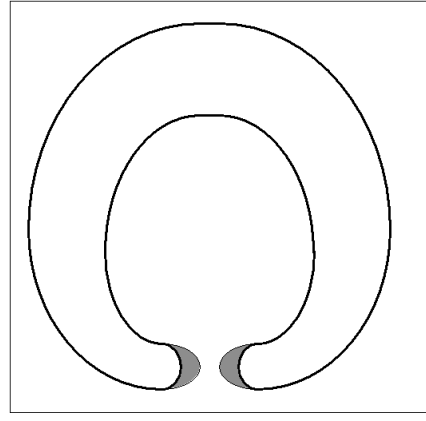


Figure 2 : Concave object and expected dilation

Let  $X_s$  be the set of pixels seen by  $M$ , the resulting force exerted on  $M$  is finally defined as :

$$\begin{aligned}\vec{F}_r(M/X) &= \sum_{N \in X_s \setminus \{M\}} \Psi(d(M, N)) \cdot \vec{u}_{NM} \\ &\times \left[ \frac{\left| \sum_{N \in X_s \setminus \{M\}} \Psi(d(M, N)) \cdot \vec{u} \right|}{\sum_{N \in X_s \setminus \{M\}} \Psi(d(M, N))} \right]^\delta \quad (7)\end{aligned}$$

The characterization of  $X_s$  is based on the definition of the discrete convexity given by Kim and Rosenfeld [3].

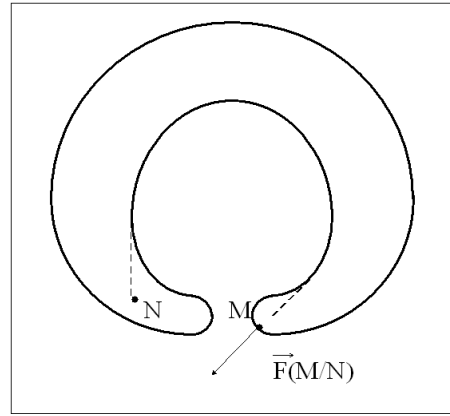


Figure 3 : Geodesic force

## 2 DILATION BY EXOGENOUS ACTION

Gleason and Tobin [2] proposed a directional approach to merge disconnected objects in a given image in order to obtain, for example, a curvilinear object. The principle of this approach based on gravitational forces is to dilate each object along a direction corresponding to the nearest and biggest objects. Finally a dilation kernel is obtained and applied to the whole object. We propose, still using the previous electric analogy, to define an attraction force between pixels which do not belong to the

same object. The aim is to obtain a preferential dilation of the extremity in order to connect objects without obscuring the general characteristics of the scene.

Like in the previous section, the objects are considered as insulators charged with unipolar electricity. The polarities of two distinct objects are supposed to be different to produce attractive forces between them. Note that we have to generalize the notion of polarity in order to introduce as many polarities as there are objects in the image.

The attraction force between two pixels with different polarities is defined as :

$$\vec{F}_a(M/N) = -\Psi(d(M, N)) \cdot \vec{w}_{NM} \quad (8)$$

Finally, the resultant of the forces exerted on a pixel  $M$  belonging to the contour of a given object  $X$  is expressed as :

$$\begin{aligned} \vec{F} &= \vec{F}_r + \vec{F}_a \\ &= \left[ \left( \lambda_r \sum_{N \in X_s \setminus M} \Psi \cdot \vec{w} \right) \times \xi(p) \right] \\ &\quad + \left[ \left( -\lambda_a \sum_{N \in X_v} \Psi \cdot \vec{w} \right) \times \xi(p') \right] \end{aligned} \quad (9)$$

where  $\lambda_a$  and  $\lambda_r$  are parameters which control the relative effect of the attraction and repulsion forces,  $p'$  is the circular distribution of the attraction forces and  $X_v$  is the set of pixels belonging to other objects.

### 3 APPLICATION TO CONTOUR CLOSING

An important problem in image analysis is edge detection. Since, in practice, the edges extracted by the use of classic detectors present some lacks due to noise or occultations, the edge detection is typically followed by a linking procedure. In this section, we introduce a method to connect the broken edges considering each piece of edge as a binary object.

The dilation technique described above is processed step by step. At each step, the resultant force is computed for all edge pixels. A set of candidate pixels is selected to be included in edges in order to merge them.

Thereafter, the dilation has to be decided taking into account the confidence that the candidate pixel is an edge pixel. We introduce an Utility Function (as defined in the Multiattribut Utility Theory [1]) based on the gradient. This function is used as a ponderation of the resultant and penalizes the dilation if the candidate pixel gradient is too small. The Utility Function may be different according to the severity of the gradient constraint. Figure 4 shows two examples of Utility Functions : one of them (b) is more restricting than the other.

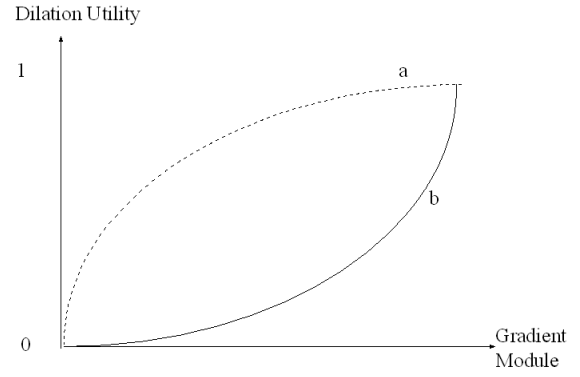


Figure 4 : Examples of Utility Function based on gradient module.

## 4 RESULTS

### 4.1 Connection of piece-wise objects

In order to illustrate the efficiency of our approach, we consider a binary image of defect distributions on a wafer presented in [1]. The image contains several disconnected scratches which are, in fact, parts of bigger defects. The aim is to merge the scratches without thickening in order to improve the defect analysis.

Figure 5(a) shows the original image. Figure 5(b) shows the result of an isotropic dilatation, Figure 5(c) is the result of the method proposed by Gleason and Tobin[2] and, finally, Figure 5(d) is the result of our dilation with a gaussian function  $\Psi$ .

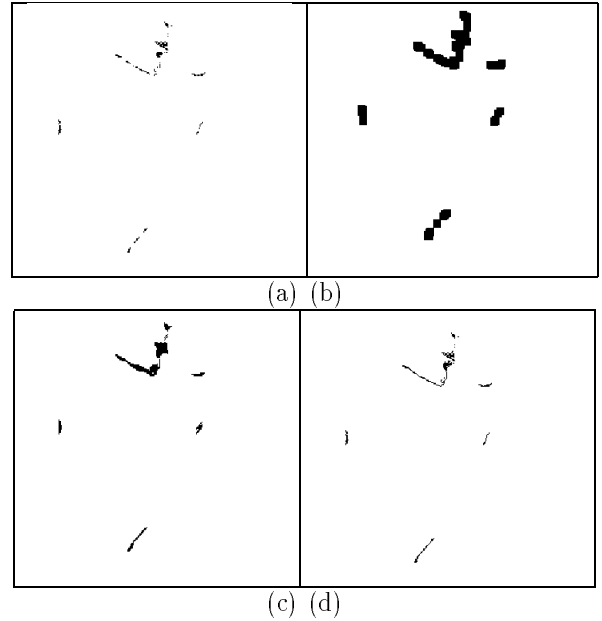


Figure 5 : (a) image of disconnected scratches on a semiconductor wafer. (b) Isotropic dilation result, (c) directional dilation based on gravitational attraction, (d) dilation based on electric analogy.

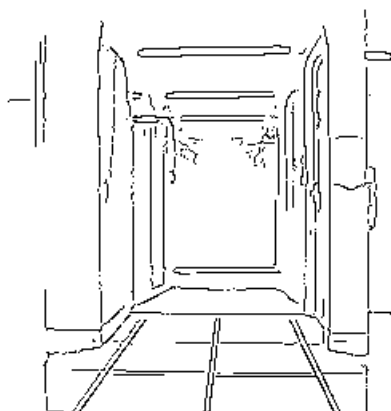
The result produced by the isotropic dilatation is not well-adapted to the obtaining of a good information on the scratches. The method based on gravitational attraction of objects improves significantly the feature extraction but our approach connects the objects without any thickening and the result obtained is a better representation of defect distribution. The original image contains 29 disconnected objects. The connections presented in both Figure 4 (c) and (d) allow for the reduction of this number to 9.

## 4.2 Contour closing

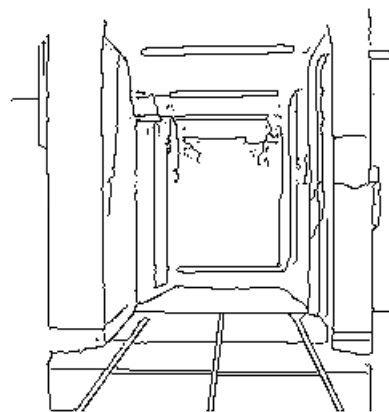
Figure 6b shows the contours obtained from figure 6a by a Deriche operator ( $\alpha=1$ ). In order to merge the disconnected contours, we used the electric forces as defined in this paper. Moreover, we introduced the utility function described in section 3. This function will make the dilation impossible when it leads to incorporate a pixel with a too small gradient into the contour. The result of the closing is shown Figure 6(c).



(a) Original image



(b) contours obtained by a Deriche operator ( $\alpha=1$ )



(c) contour closing on image of (b)

Figure 6 : contour closing based on electric analogy

## 5 CONCLUSION

The definition of electric forces exerted on each pixel permits us to obtain a directional dilation of objects. This approach gives good results in the case where a connexion between small binary objects is desired. In particular, we applied our technique to contour closing issue. Further research will be conducted to extend the procedure to dilate object in a grey-scale image.

## References

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