BLIND IDENTIFICATION OF IIR MODEL BASED ON OUTPUT OVER-SAMPLING

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ABSTRACT

This paper deals with a blind problem for an IIR model in a time domain. Based on an output over-sampling scheme, the proposed algorithm can estimate parameters of an alternative multi-output model description first, then the parameters of original model can be obtained later. It can be clarified that an IIR model can be identified by using over-sampling scheme.

1 INTRODUCTION

A challengable problem which deals with unavailable source signal attracts much research and application interests in a wide range of signal processing areas. It is desired to identify a model with unknown input just through the output signal and some properties about the input signal, *i.e.*, to solve a blind identification problem.

In the last two decades many studies for blind problem were developed, which include the high-order statistics based algorithms (HOS) [1, 2], the maximum likelihood (ML) approach [3], and recently the attractive approaches based on the second-order cyclostationary statistics (SOCS) [4, 5, 6]. However, the existing HOS approaches exhibit slow convergence and have heavy computational load, while ML approaches require the statistics of the unknown signal. The approaches based on SOCS have considerably modest computation, however they have almost restricted availability to an FIR model. Though a frequency domain approach based on SOCS has been proposed for an IIR model [6], the variance of estimates in frequency domain may be larger than those in time domain in some cases. The effectiveness of over-sampling scheme has also been investigated for IIR model identification problem in time domain [7, 8, 9], while [7] discussed only noise free case, and [7, 8] require that the over-sampling rate is larger than the order of numerator polynomial. However, this requirement can be deleted in [9].

The purpose of this paper is to propose a novel blind identification algorithm for an IIR channel model using a subspace approach. We demonstrate that the over-sampled IIR model can be represented by a new multiple-output model description with a common denominator polynomial in its transfer function, and by making use of it we identify the IIR model from the only accessible over-sampled output.

2 PROBLEM STATEMENT

A linear discrete-time IIR model is considered in this paper. Let $\{s_m\}$ be the source sequence, $\{w(\cdot)\}$ is a white observation noise with zero mean, finite variance, and independent of $\{s_m\}$. $\{z(\cdot)\}$ and $\{y(\cdot)\}$ are noise-free and noise-corrupted output signal respectively. Though it can also be modeled by an FIR model, it may need long length of the impulse response to guarantee modeling accuracy, hence it is expected to obtain the IIR model identification in some application cases. Now the problem to be dealt with is how to identify the IIR model parameters just from the noise corrupted output $\{y(\cdot)\}$.

An input sequence considered here is given by

$$u(t) = \begin{cases} s_m : & t = mT \\ 0 : & t \neq mT \end{cases} \tag{1}$$

where T is the symbol period. In many cases, the sequence $\{s_m\}$ can be considered as an i.i.d sequence with zero mean and known variance σ_s^2. The proposed algorithm can also be extended to the case in which the input signal has duration time T with a zero-order holder.

The system output is over-sampled and it becomes to be a cyclostationary signal [4, 6]. The sampling interval Δ satisfies $\Delta = T/p$, where the integer p is referred to as an over-sampling rate. Under the sampling interval Δ, the IIR model called Δ-model can be characterized by

$$y(k) = \frac{B(q^{-1})}{A(q^{-1})} u(k) + w(k) \tag{2}$$

where

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{b_1 q^{-1} + \cdots + b_n q^{-n}}{1 + a_1 q^{-1} + \cdots + a_n q^{-n}}$$

Here n is the model order and $q^{-1} \equiv e^{-\Delta s}$ is denoted as a backward shift operator. Meanwhile the system input, output samples are denoted as $u(k)$ and $y(k)$ respectively, and $w(k)$ is a white additive observation noise.

The model in (2) can describe the linear channel efficiently, however, we will use an alternative model description for the blind identification.

3 MODEL DESCRIPTION FOR IDENTIFICATION

Let the following new variables corresponding to the over-sampling rate p be defined as

$$\begin{aligned} u_j(m) &= u(mp+j), \quad z_j(m) = z(mp+j) \\ w_j(m) &= w(mp+j), \quad y_j(m) = y(mp+j) \end{aligned} \tag{3}$$

where $j = 1, \cdots, p$. From the property of the transmitted sequence described in (1), $u_j(m)$ satisfies that

$$\begin{cases} u_1(m) = \cdots = u_{p-1}(m) = 0 \\ u_p(m) = s_{m+1} \end{cases} \tag{4}$$

By using the definitions and the property of the input given above, we have the following theorem.

Theorem 1 *The input signal to a linear discrete-time IIR model is given in* (1)*, where the symbol period is* T*. The output is over-sampled at interval* $\Delta = T/p$*, where* p *is an over-sampling rate, then the* Δ*-model given by* (2) *can also be described by an SIMO model description with a common denominator polynomial* $G(z^{-1})$ *and* p *numerator polynomials as*

$$y_j(m) = \frac{H_j(z^{-1})}{G(z^{-1})} s_m + w_j(m) \tag{5}$$

where the denominator $G(z^{-1})$ *is given by* (6)

$$G(z^{-1}) = \det\left(\boldsymbol{I} - z^{-1}\boldsymbol{A}^p\right) = 1 + \sum_{i=1}^{n} g_i z^{-i} \tag{6}$$

and p *numerator polynomials* $H_j(z^{-1})$ *are given in* (7).

$$\begin{aligned} H_j(z^{-1}) &= \boldsymbol{c} \cdot adj\left(\boldsymbol{I} - z^{-1}\boldsymbol{A}^p\right) \cdot \boldsymbol{A}^{j-1}\boldsymbol{b} \\ &= \sum_{i=1}^{n} h_{j,i} z^{-i+1} \end{aligned} \tag{7}$$

where

$$\boldsymbol{A} ! \doteq \begin{bmatrix} -a_1 & 1 & 0 & 0 \\ -a_2 & 0 & \ddots & 0 \\ \vdots & \vdots & 0 & 1 \\ -a_n & 0 & \cdots & 0 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \tag{8}$$

$$\boldsymbol{c} = [1 \quad 0 \quad \cdots \quad 0]$$

And $z^{-1} \equiv e^{-\Delta T}$ *is denoted as a backward shift operator different from* q^{-1}.

Since the observation interval of $y_j(m)$ is T, the model in (5) can be considered as a

$$\begin{aligned} \sum_{i=0}^{n} \left(a_i y_{\beta(k-i)}(\alpha(k-1-i))\right) &= \sum_{i=1}^{n} \left(b_i s_{\alpha(k-i)}\right) \\ &+ \sum_{i=0}^{n} \left(a_i w_{\beta(k-i)}(\alpha(k-1-i))\right) \end{aligned} \tag{9}$$

where $\alpha(x)$ is an integer part of the quotient x/p, $\beta(x)$ is the remainder of devision x/p, $k = mp+1, \cdots, (m+1)p$. Substituting (5) leads to

$$\begin{aligned} &\sum_{i=0}^{n} \left(a_i H_{\beta(k-i)}(z^{-1}) z^{\alpha(k-1-i)-\alpha(k-1)}\right) \\ &= G(z^{-1}) \sum_{i=1}^{n} \left(b_i z^{\alpha(k-i)-\alpha(k-1)}\right) \end{aligned} \tag{10}$$

Then we can obtain a set of linear equations about a_i and b_i when $G(z^{-1})$ and $H_j(z^{-1})$ are known or estimated, thus the Δ-model parameters can be determined by solving an over-determined linear equation set.

Based on the results above, a blind identification can be established in the next section.

4 BLIND IDENTIFICATION ALGORHIM

Now let us show how to estimate the numerator and denominator polynomials. Here we consider the case for $p = 2$, and it can be extended into the case of $p > 2$ easily.

4.1 Estimation of Numerator Polynomials

From the SIMO representation given in (5), we have

$$\begin{cases} y_1(m) = \frac{H_1(z^{-1})}{G(z^{-1})} s_m + w_1(m) \\ y_2(m) = \frac{H_2(z^{-1})}{G(z^{-1})} s_m + w_2(m) \end{cases} \tag{11}$$

Define a matrix $\boldsymbol{Y}_{1,2}(L)$ as

$$\boldsymbol{Y}_{1,2}(L) = [\boldsymbol{Y}_1(L) \quad -\boldsymbol{Y}_2(L)] \tag{12}$$

where

$$\boldsymbol{Y}_j(L) = \begin{bmatrix} y_j(L) & \cdots & y_j(1) \\ \vdots & \cdots & \vdots \\ y_j(N+L-1) & \cdots & y_j(N) \end{bmatrix}_{N \times L}$$

where $L \geq n$. From the definitions in (3), it can also be expressed by

$$\boldsymbol{Y}_{1,2}(L) = \boldsymbol{Z}_{1,2}(L) + \boldsymbol{W}_{1,2}(L) \tag{13}$$

where $\boldsymbol{Z}_{1,2}(L)$, $\boldsymbol{W}_{1,2}(L)$ have the same structure as $\boldsymbol{Y}_{1,2}(L)$. Then matrix $\boldsymbol{R}_{1,2}^{Z}(L)$ has following property.

Lemma *If* $H_1(z^{-1})$ *and* $H_2(z^{-1})$ *are coprime, and the source sequence* $\{s_m\}$ *satisfies the PE condition, then*

$$Rank\left(\boldsymbol{R}_{1,2}^{Z}(L)\right) = L + n - 1 \tag{14}$$

where $\boldsymbol{R}_{1,2}^{Z}(L) = \left(\boldsymbol{Z}_{1,2}^{T}(L)\boldsymbol{Z}_{1,2}(L)\right)/N$.

However, the observations are corrupted by observation noise, we can only obtain $\boldsymbol{R}_{1,2}^{Y}(L)$. Since the observation noise $w(\cdot)$ is a white stationary noise and independent of the input signal, then we have that

$$\lim_{N\to\infty} \boldsymbol{R}_{1,2}^{Y}(L) = \lim_{N\to\infty} \boldsymbol{R}_{1,2}^{Z}(L) + \sigma_w^2 \boldsymbol{I} \tag{15}$$

Let the EVD of $\boldsymbol{R}_{1,2}^{Y}(L)$ can be written as

$$\boldsymbol{R}_{1,2}^{Y}(L) = \boldsymbol{U}\boldsymbol{\Gamma}\boldsymbol{U}^{T} \tag{16}$$

Then from Lemma, the eigenvectors can be divided into two subspaces, one is the signal subspace $\boldsymbol{U}_s$ determined by the columns of $\boldsymbol{H}_{1,2}(L)$, where

$$\boldsymbol{H}_{1,2}(L) = \begin{bmatrix} \boldsymbol{H}_i \\ -\boldsymbol{H}_j \end{bmatrix}$$

$$\boldsymbol{H}_j = \begin{bmatrix} h_{j,1} & \cdots & h_{j,n} & & \\ & \ddots & & \ddots & \\ & & h_{j,1} & \cdots & h_{j,n} \end{bmatrix}_{L \times (L+n-1)}$$

And the noise subspace $\boldsymbol{U}_\perp$, *i.e.*,

$$\boldsymbol{R}_{1,2}^{Y}(L) = [\boldsymbol{U}_s \quad \boldsymbol{U}_\perp] \begin{bmatrix} \boldsymbol{\Gamma}_s & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\Gamma}_\perp \end{bmatrix} \begin{bmatrix} \boldsymbol{U}_s^T \\ \boldsymbol{U}_\perp^T \end{bmatrix} \tag{17}$$

where the noise subspace $\boldsymbol{U}_\perp$ is orthogonal to the signal subspace $\boldsymbol{U}_s$, *i.e.*, $\boldsymbol{U}_\perp \perp \boldsymbol{H}_{1,2}(L)$, therefore, we have

$$\boldsymbol{U}_\perp^T \boldsymbol{H}_{1,2}(L) = \mathbf{0} \tag{18}$$

where the noise subspace has dimension of $2L \times (L-n+1)$. Based on the orthogonal property given in (18), the numerator polynomials $H_1(z^{-1})$, $H_2(z^{-1})$ and any eigenvector of the noise subspace $\boldsymbol{U}_\perp(:,l)$ has following relationship as

$$\boldsymbol{U}_{\perp,12}^{(l)} \boldsymbol{h}_{1,2} = \mathbf{0} \tag{19}$$

where

$$\boldsymbol{U}_{\perp,12}^{(l)} = \begin{bmatrix} \boldsymbol{U}_{\perp,1}^{(l)} & \boldsymbol{U}_{\perp,2}^{(l)} \end{bmatrix}$$

$$\boldsymbol{U}_{\perp,j}^{(l)} = \begin{bmatrix} \boldsymbol{U}_\perp((2-j)L+1,l) & & \\ \vdots & \ddots & \\ \boldsymbol{U}_\perp((2-j)L+L,l) & \vdots & \boldsymbol{U}_\perp((2-j)L+1,l) \\ & \ddots & \\ & & \boldsymbol{U}_\perp((2-j)L+L,l) \end{bmatrix}$$

$$\boldsymbol{h}_{1,2} = \begin{bmatrix} h_{1,1} & \cdots & h_{1,n} & h_{2,1} & \cdots & h_{2,n} \end{bmatrix}^T$$

Then parameters of $H_j(z^{-1})$ can be estimated from any eigenvector of the noise subspace, which is illustrated in the following theorem.

Theorem 2 *Assume that $H_1(z^{-1})$ and $H_2(z^{-1})$ are coprime, and the source sequence $\{s_m\}$ satisfies the PE condition, the subspaces of $\boldsymbol{R}_{1,2}^Y(L)$ are given by (17). Then $\boldsymbol{U}_{\perp,12}^{(l)}(:,2:2n)$ has column full rank $2n-1$. Further, the estimate of $\boldsymbol{h}_{1,2}$ with a scalar ambiguity η which will be estimated later can be given by*

$$\hat{\boldsymbol{h}}_{1,2} = \begin{bmatrix} \eta \\ \eta \left(\boldsymbol{\Psi}_l^T \boldsymbol{\Psi}_l\right)^{-1} \boldsymbol{\Psi}_l^T \boldsymbol{\Psi}_l \end{bmatrix} \tag{20}$$

where

$$\boldsymbol{\Psi}_l = \boldsymbol{U}_{\perp,12}^{(l)}(:,2:2n), \quad \boldsymbol{\psi}_l = \boldsymbol{U}_{\perp,12}^{(l)}(:,1) \tag{21}$$

It is reasonable that the estimate accuracy of $\boldsymbol{h}_{1,2}$ increases if all the noise subspace eigenvectors are used, *i.e.*, $l = 1, \cdots, L-n+1$.

4.2 The Denominator Polynomial Estimation

Define the auto-covariance $R_j(\tau)$ as

$$R_j(\tau) = E\{y_j(m) y_j(m-\tau)\}$$

then from (5)

$$\begin{aligned} & E\left\{\left(y_j(m) + \sum_{i=1}^{n} g_i y_j(m-i)\right) y_j(m-\tau)\right\} \\ & = E\left\{\left(\sum_{i=1}^{n} h_{j,i} s(m-i) + \sum_{i=0}^{n} g_i w_j(m-i)\right) \cdot y_j(m-\tau)\right\} \end{aligned} \tag{22}$$

Denote the impulse response sequence of $1/G(z^{-1})$ as

$$\frac{1}{G(z^{-1})} = \sum_{i=0}^{\infty} \gamma_i z^{-1} \tag{23}$$

Then $y_j(m)$ can also be written as

$$y_j(m) = \sum_{i=1}^{\infty} \lambda_{j,i} s(m-j) + w_j(m) \tag{24}$$

where

$$\begin{bmatrix} \lambda_{j,1} \\ \lambda_{j,2} \\ \lambda_{j,3} \\ \lambda_{j,4} \\ \vdots \end{bmatrix} = \begin{bmatrix} h_{j,1} & & & \\ \vdots & \ddots & & \\ h_{j,n} & \cdots & h_{j,1} & \\ & h_{j,n} & \cdots & h_{j,1} \\ & & \ddots & \ddots & \ddots \end{bmatrix} \begin{bmatrix} \gamma_0 \\ \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \vdots \end{bmatrix}$$

From the independence property of sequence $\{s(m)\}$, (22) can be written as

$$\begin{aligned} & E\left\{\left(y_j(m) + \sum_{i=1}^{n} g_i y_j(m-i)\right) y_j(m-\tau)\right\} \\ & = \sum_{i=1}^{\max(0,n-\tau+1)} h_{j,i+\tau} \lambda_{j,i} \sigma_s^2 + g_\tau \sigma_w^2 \end{aligned} \tag{25}$$

where $\tau \geq 0$. For $\tau = 0, \cdots, n-1$, we have that

$$\left(\sigma_w^2 \boldsymbol{J} - \boldsymbol{R}_j\right) g + \boldsymbol{\Omega}_j \boldsymbol{\gamma} = \boldsymbol{r}_j - \sigma_w^2 \boldsymbol{e}_1 \tag{26}$$

where

$$\boldsymbol{R}_j = \begin{bmatrix} R_j(-1) & R_j(-2) & \cdots & R_j(-n) \\ R_j(0) & R_j(-1) & \cdots & R_j(-n+1) \\ \vdots & \ddots & \ddots & R_j(-n+2) \\ R_j(n-2) & \cdots & R_j(0) & R_j(-1) \end{bmatrix}$$

$$\boldsymbol{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{e}_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$$

$$\boldsymbol{\Omega}_j = \begin{bmatrix} h_{j,1} & \cdots & h_{j,n} \\ \vdots & \cdot^{\cdot^{\cdot}} & \\ h_{j,n} & & \end{bmatrix} \begin{bmatrix} h_{j,1} & & \\ \vdots & \ddots & \\ h_{j,n} & \cdots & h_{j,1} \end{bmatrix}$$

$$\boldsymbol{r}_j = \begin{bmatrix} R_j(0) & \cdots & R_j(-n+1) \end{bmatrix}^T$$

Let $\bar{\boldsymbol{R}}_j = \boldsymbol{R}_j - \sigma_w^2 \boldsymbol{J}$, $\bar{\boldsymbol{r}}_j = \boldsymbol{r}_j - \sigma_w^2 \boldsymbol{e}_1$, then for $j = 1, 2$

$$\begin{bmatrix} \bar{\boldsymbol{R}}_1 & \boldsymbol{\Omega}_1 \\ \bar{\boldsymbol{R}}_2 & \boldsymbol{\Omega}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{g} \\ \boldsymbol{\gamma} \end{bmatrix} = \begin{bmatrix} \bar{\boldsymbol{r}}_1 \\ \bar{\boldsymbol{r}}_2 \end{bmatrix} \tag{27}$$

Notice that

$$R_j(\tau) = \lim_{M\to\infty} \frac{1}{M} \sum_{m=\tau}^{M+\tau-1} y_j(m) y_j(m-\tau) \tag{28}$$

Thus by making use of the available observation data, g_i can be estimated from (27). η can be calculated by

$$\hat{\eta} = \sqrt{\gamma_0} \tag{29}$$

Moreover, the parameters of Δ-model can also be calculated by using the approach given in Section 3.

5 NUMBERICAL SIMULATIONS

The transmitted sequence $\{s_m\}$ is with emission period $T = 1.0$, variance $\sigma_s^2 = 1.0$, which is an i.i.d signal. We consider the discrete-time channel model with sampling interval $\Delta = 0.5$, *i.e.*, $p = 2$. The observation noise w is a white noise with zero mean and variance σ_w^2 and is independent of $\{s_m\}$, and the signal to noise ratio is set to SNR= 20dB. We consider two IIR models with different impulse response shape.

Case 1. The over-sampled Δ-model is given by

$$A(q^{-1}) = 0.0545q^{-1} - 0.0179q^{-2} - 0.0460q^{-3}$$

$$+0.2168q^{-4} + 0.1642q^{-5}$$
$$B(q^{-1}) = 1 - 1.2479q^{-1} + 0.8574q^{-2} - 0.2902q^{-3}$$

Its shape of impulse response may often be encountered with in communication area. The true parameters of T-model polynomials are given by

$$G(z^{-1}) = 1 + 0.1575z^{-1} + 0.0109z^{-2} - 0.0842z^{-3}$$
$$H_1(z^{-1}) = 0.0545z^{-1} - 0.0216z^{-2} + 0.3901z^{-3} + 0.2037z^{-4}$$
$$H_2(z^{-1}) = 0.0501z^{-1} + 0.1599z^{-2} + 0.3774z^{-3} + 0.0477z^{-4}$$

By using the observations during $2000T$, the estimated parameters could be obtained as follows:

$$\hat{H}_1(z^{-1}) = 0.0527z^{-1} - 0.0233z^{-2} + 0.3905z^{-3} + 0.2023z^{-4}$$
$$\hat{H}_2(z^{-1}) = 0.0479z^{-1} + 0.1579z^{-2} + 0.3779z^{-3} + 0.0441z^{-4}$$
$$\hat{G}(z^{-1}) = 1 + 0.1472z^{-1} + 0.0005z^{-2} - 0.10182z^{-3}$$
$$\hat{A}(q^{-1}) = 1 - 1.2591q^{-1} + 0.8736q^{-2} - 0.3054q^{-3}$$
$$\hat{B}(q^{-1}) = 0.0526q^{-1} - 0.0184q^{-2} - 0.0453q^{-3} + 0.2157q^{-4} + 0.1632q^{-5}$$

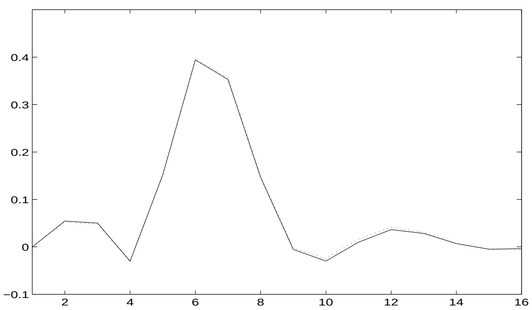

Figure 1: The Estimated impulse response. Solid line: True; Dotted line: Estimate

Case 2. The discrete-time model with a general impulse response shape is given by

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{2.0q^{-1} - 1.38q^{-2} - 0.756q^{-3}}{1 - 1.9101q^{-1} + 1.5183q^{-2} - 0.4607q^{-3}}$$

The SIMO model polynomials are given by

$$G(z^{-1}) = 1 - 0.6117z^{-1} + 0.5453z^{-2} - 0.2122z^{-3}$$
$$H_1(z^{-1}) = 2.0z^{-1} - 0.3553z^{-2} - 1.7836z^{-3}$$
$$H_2(z^{-1}) = 2.4401z^{-1} - 2.6179z^{-2} - 0.3483z^{-3}$$

The estimated parameters of the SIMO transfer function from the observations within $1200T$ are as follows:

$$\hat{G}(z^{-1}) = 1 - 0.6261z^{-1} + 0.5615z^{-2} - 0.2398z^{-3}$$
$$\hat{H}_1(z^{-1}) = 1.9915z^{-1} - 0.3861z^{-2} - 1.7570z^{-3}$$
$$\hat{H}_2(z^{-1}) = 2.4232z^{-1} - 2.6320z^{-2} - 0.3155z^{-3}$$

The original model parameters are calculated as follows

$$\frac{\hat{B}(q^{-1})}{\hat{A}(q^{-1})} = \frac{2.0105q^{-1} - 1.4914q^{-2} - 0.6529q^{-3}}{1 - 1.9401q^{-1} + 1.5690q^{-2} - 0.4897q^{-3}}$$

The estimated impulse response with sampling interval Δ is shown in Figure 2.

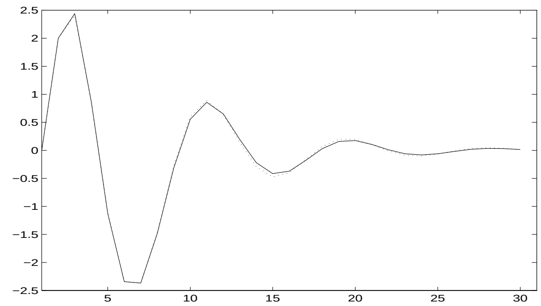

Figure 2: The Estimated impulse response. Solid line: True; Dotted line: Estimate

6 CONCLUSIONS

We have proposed a novel approach to solve the blind identification problem of an IIR model. By using the only accessible output observations sampled at higher sampling rate than the symbol rate, the SISO model can be described by an SIMO model with common denominator polynomial in the transfer function, then the parameters of the SIMO model and original SISO model can be estimated. It can be clarified that an IIR model can be identified just using the over-sampled output.

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