DFT based Optimal Blind Channel Identification

Claudio Becchetti, Giovanni Jacovitti, Gaetano Scarano

Dip. INFOCOM, Università di Roma "La Sapienza", Via Eudossiana 18, I-00184 Roma, Italy

Tel.+39.6.4458.5500, Fax. +39.6.4873.300, Emails: `cbecc, gjacov, gaetano@infocom.ing.uniroma1.it`

ABSTRACT

Optimal solutions for channel identification from oversampled Pulse Amplitude Modulated signals are presented. While (large sample) optimal identification derived in the time domain involves solution of matrix weighted linear systems of equations solved in a Least Squares sense, using Discrete Fourier Transforms of the received data, it is shown that optimal solutions are rather obtained through scalar weighted Least Squares. Since the matrix of weights is not *a priori* known, estimation is generally performed in two steps. Here, we describe a less computational demanding single step estimation procedure. In the small samples case, the single step estimation is slightly less accurate than two step procedures.

1 Introduction

In this contribution we address the problem of blind identification of FIR communication channels in applicative contexts characterized by rapidly time–variant channels and/or when small data samples are available. These conditions are particularly true in mobile communications.

In this applicative context, estimators based on the cyclostationary nature of the fractionally–sampled transmitted signal seem promising because good accuracy is obtained from short data length. Cyclostationarity is exploited to derive relations on the impulse response of the channel, referred in literature to as Cross Relations (CR) [1].

In presence of additive noise, the estimation of the channel may be not sufficiently accurate. Other approaches have been proposed in order to improve accuracy by organizing the relations in order to reduce the influence of the noise. In [2], the geometric properties of a suitable matrix of data are exploited by mimicking the MUSIC technique employed for DOA estimation. In such approach, the effect of the noise is properly taken into account by decomposing the space spanned from data into two orthogonal subspaces (namely the noise and the signal subspace). Estimation is carried out by imposing that the solution is orthogonal to the noise subspace.

From CR, asymptotically efficient solutions can be also derived. In fact, (10) can be organized in a linear system of equations where the unknown variables are the coefficients of the impulse response of the channel. An asymptotically efficient solution is obtained by optimally weighing the linear system of equations. The weighing matrix is dependent on the unknown channel response; therefore, optimal solutions can be obtained in a two step procedure. In the first step, a coarse channel estimate is obtained by solving unweighed CR relations; this coarse estimate is employed to build up the weighing matrix. In the second step, the CR relations are optimally weighed in order to obtain the final estimate. This approach has been employed in [3]. The optimal weighing has also been extensively investigated in [4]. In this work, closed form expressions of the weighing matrix are derived to minimize the asymptotic estimation variance.

Cyclostationary properties can be also employed in the frequency domain. In [5] it is exploited the fact that the bi-argumental autocorrelation function of a cyclostationary process is periodic in one of the indexes. The Fourier series of the autocorrelation over the periodic index, called cyclic spectrum, satisfies relations similar to the CR; channel estimation can be performed by exploiting the structure of such "frequency domain" relations.

In [6], the CR are drawn from the Discrete Time Fourier Transform of the data. In the same work, an asymptotically two step efficient estimator based on cyclic spectra is also proposed.

In this contribution, we propose a DFT based approach for the blind estimation of the channel response which is applicable under some conditions commonly encountered in data communications channels. We show that this approach is equivalent to the CR based approaches in the noiseless case while, for noisy data, some benefits are obtained w.r.t. time domain approaches. In particular, we show that the optimal weighing is performed at reduced computational complexity. Moreover, we derive a one step asymptotically efficient procedure w.r.t to the two step procedures proposed in literature [3, 6, 7].

2 DFT *Based Blind Channel Identification*

With reference to fig.1, let us consider the discrete–time equivalent model of the received signal $r[n]$ in a Pulse Amplitude Modulation (PAM) signaling scheme:

$$r[n] = \sum_m s_m \cdot h[n - mP] + v[n] \tag{1}$$

where s_m denotes the generic transmitted symbol drawn from a discrete constellation and $v[n]$ is additive noise, possibly colored by the receiving shaping filter. The samples $h[n]$ are the impulse response of the overall channel (including both the shaping filters in transmission and reception) obtained by fractional sampling at P times the symbol rate.

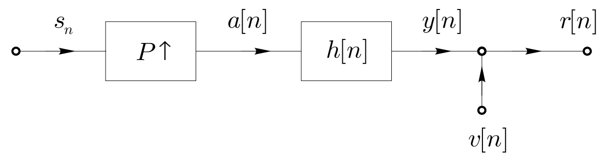

Figure 1: *Discrete–time model of digital transmission through communication channels.*

Denoting by $y[n] = \sum_m s_m \cdot h[n - Pm]$ the useful component in (1), and referring, for simplicity, to a fractional sampling with $P = 2$, we introduce the expanded sequence of symbols

$$a[n] = \sum_p s_p \cdot \delta[n - 2p] = \begin{cases} s_{n/2} & \text{for } n \text{ even} \\ 0 & \text{for } n \text{ odd} \end{cases} \tag{2}$$

so to write

$$y[n] = h[n] * a[n] \tag{3}$$

The N point DFT of the expanded sequence of symbols $a[n]$ has the property to be $N/2$ periodic. In fact, defining

$$A[k] \stackrel{\text{def}}{=} \mathrm{DFT}_N\{a[n]\} = \sum_{n=0}^{N-1} a[n] \cdot e^{j2\pi nk/N}$$

the zero interleaving in the expansion (2) induces the property:

$$A[k] = A[k + N/2] \quad ; \quad k = 0, \cdots, N/2 - 1 \tag{4}$$

This property will be exploited to obtain a relation between the channel response and the noiseless observations.

In fact, if we assume that the noiseless received sequence $y[n]$ is characterized by proper time guards (as for example in block transmission of cellular communications), discrete linear convolutions can be considered as circular convolutions. In this respect, using the DFT, (3) is rewritten as follows:

$$Y[k] = H[k] \cdot A[k] \tag{5}$$

where $Y[k]$ and $H[k]$ are the N point DFT's of $y[n]$ and $h[n]$ respectively.

Now, considering the pair of equations obtained from (5) for k and $k+N/2$, invoking the periodicity of the expanded sequence of symbols (4), it is possible to write:

$$\begin{aligned} Y[k]H[k + N/2] = Y[k + N/2]H[k] \\ k = 0, \cdots, N/2 - 1 \end{aligned} \tag{6}$$

In (6), there are $N/2$ equations in N the unknown parameters $H[k], k = 0, \cdots, N/2 - 1$. Assuming that the channel frequency response depends on $L \leq N/2$ parameters, these latter are recoverable from (6), under suitable conditions.

For FIR channels of order L, the k-th DFT sample of the channel $H[k]$ is expressed by:

$$H[k] = \mathbf{f}^{\mathrm{T}}(k)\mathbf{h} \tag{7}$$

where $\mathbf{h}$ is the vector containing the $L+1$ coefficients of the channel impulse response $\mathbf{h} \stackrel{\text{def}}{=} \left[h[0], ..., h[L]\right]^{\mathrm{T}}$ and $\mathbf{f}^{\mathrm{T}}(k)$ is the k-th vector of the DFT basis $\|\mathbf{f}^{\mathrm{T}}(k)\|_p = e^{j2\pi kp/N}$. In this case, the homogeneous equations (6) become linear

$$\begin{aligned} \left(Y[k] \cdot \mathbf{f}^{\mathrm{T}}(k + N/2) - Z[k + N/2] \cdot \mathbf{f}^{\mathrm{T}}(k)\right) \cdot \mathbf{h} = 0 \\ k = 0, \cdots, N/2 - 1 \end{aligned} \tag{8}$$

A similar system is derived in the time domain [1], making use of the odd and the even sample sequence of $y[n]$:

$$\begin{aligned} y_o[n] \stackrel{\text{def}}{=} y[2n + 1] = s_n * h_o[n] \\ y_e[n] \stackrel{\text{def}}{=} y[2n] = s_n * h_e[n] \end{aligned} \tag{9}$$

where $h_o[n] \stackrel{\text{def}}{=} h[2n+1]$ and $h_e[n] \stackrel{\text{def}}{=} h[2n]$ are the odd and the even samples of the impulse response of the channel, respectively. From (9), eliminating the symbols s_n, the following relation, similar to that obtained in the DFT domain, are obtained:

$$y_o[n] * h_e[n] = y_e[n] * h_o[n] \tag{10}$$

2.1 *Channel Estimation*

Let us now consider the noise corrupted DFT observations $R[k] \stackrel{\text{def}}{=} \mathrm{DFT}_N\{r[n]\}$. Denoting by $V[k]$ the DFT of the noise samples $v[n]$, we have $R[k] = V[k] + Y[k]$. Using $R[k]$, (6) is rewritten as follows:

$$\begin{aligned} \left(R[k] \cdot \mathbf{f}^{\mathrm{T}}(k + N/2) - R[k + N/2] \cdot \mathbf{f}^{\mathrm{T}}(k)\right)\mathbf{h} = U[k] \\ k = 0, \cdots, N/2 - 1 \end{aligned} \tag{11}$$

where $U[k]$ is a residual given by:

$$U[k] = V[k]H[k + N/2] - V[k + N/2]H[k] \tag{12}$$

The residuals $U[k]$ are not known as they are dependent on the noise $V[k]$. Nevertheless, a channel estimate can be obtained solving (11) in a Least Squares (LS) sense:

$$\begin{aligned} \hat{\mathbf{h}} = \arg\min_{\mathbf{h}} \Bigg\{ \sum_{k=0}^{N/2-1} \Big| \big(R[k] \cdot \mathbf{f}^{\mathrm{T}}(k + N/2) \\ - R[k + N/2] \cdot \mathbf{f}^{\mathrm{T}}(k)\big)\mathbf{h}\Big|^2 \Bigg\} \end{aligned} \tag{13}$$

Loosely speaking, the solution is obtained by minimizing the energy of the residuals $U[k]$, under some constraint on $\mathbf{h}$.

Similar LS solutions are obtained in the time domain [1].

The LS solution is heavily affected by the presence of additive noise. A more accurate solution can be accomplished by properly weighing the LS system so that the covariance of the (weighted) residuals is proportional to the identity matrix $\mathbf{I}$. If the noise samples belong to a zero mean, stationary process, the solution is Minimum Variance Unbiased (MVU) and if the process is also Gaussian, the solution is Maximum Likelihood (ML).

2.2 The Optimal Weighting

Assuming uncorrelated noise samples $v[k]$, the DFT samples $V[k]$ remain uncorrelated. Hence, from (12), it follows that the covariance matrix K_U of the DFT residuals $U[k]$ is diagonal, with non-zero entries given by:

$$\|K_U\|_k = \sigma_U^2(k) = \sigma_V^2 \left\{|H[k]|^2 + |H[k+N/2]|^2\right\} \quad (14)$$

where $\sigma_V^2 = \mathrm{E}\left\{V^2[k]\right\}$ is the variance of the samples of the noise in the DFT domain. Therefore, the MVU solution is obtained from a scalar weighted LS solution where the weights of each equation in (13) are simply $1/\sigma_U(k)$.

Unfortunately, the covariance of the residuals depends on the solution $\mathbf{h}$ and therefore the MVU solution cannot be obtained in a closed form.

Nevertheless, a two step procedure can be devised to obtain an asymptotical (large sample) MVU solution. In the DFT domain, as indicated in [7], this procedure consists of the following three steps.

First, a coarse estimate of $\mathbf{h}$ by the LS solution in (13) is obtained.

Then, this coarse solution is used to compute an (estimated) covariance of the residuals $\hat{\sigma}_U^2(k)$.

Finally, the asymptotical MVU solution is computed by solving a new LS system where each equation is scaled using the covariance of the residuals estimated in the second step.

The procedure is asymptotical MVU as the first step yields a consistent estimate of $\mathbf{h}$.

In the time domain, a similar two step procedure has been described in [3], but a more involved weighing procedure is required since the covariance of the residuals turns out to be not (proportional to) a diagonal matrix; therefore, computation of the pseudoinverse of this covariance matrix is needed.

It is worth noting that, since the DFT domain based LS solution involves uncorrelated residuals, it results to be "more close" to the MVU estimate w.r.t. the correspondent time domain LS solution, since in the time domain the residuals are correlated. Moreover, the two step DFT based solution is less computational expensive w.r.t. the time domain two step solution, due to the simpler weighing procedure. In particular, the computation load is reduced from $O(N^3)$ to $O(N/2\log(N))$ operations [7]. The major drawback of the DFT based estimation is that proper time guards are required.

2.3 The Single Step Asymptotical MVU solution

The first step in two step procedures consists of estimating the weights. From (14), it is seen that the weighing depends on the channel through its energy density spectrum only. Therefore, assuming i.i.d. symbols, the weights may be directly estimated from the observations $R[k]$. In fact, it results

$$\mathrm{E}\left\{|R[k]|^2\right\} = \sigma_V^2 + \sigma_A^2 \cdot |H[k]|^2 \quad (15)$$

where $\sigma_A^2 = \mathrm{E}\left\{|A[k]|^2\right\}$ is the variance of the DFT of the symbols. Assuming σ_V^2 known, an unbiased estimate of $\sigma_U^2(k)$ is simply given by:

$$\hat{\sigma}_U^2 = \frac{\sigma_V^2}{\sigma_A^2}\left(\left|R[k]\right|^2 + \left|R[k+N/2]\right|^2 - 2\sigma_V^2\right) \quad (16)$$

Estimating the weights from (16), a single step algorithm is obtained, scaling the LS equations (13) by $\hat{\sigma}_U^2$. Note that the estimate (16) resembles the periodogram estimate and it turns out to be not consistent. Therefore, an asymptotical MVU estimation cannot be achieved using the weights $\hat{\sigma}_U^2$ estimated through eqrefeq:pesononconsistente.

A consistent estimate of σ_U^2 is obtained noting that, for channels of order L, the N point DFT $|H[k]|^2$ depends on $2L+1$ parameters which are the samples coefficients of the autocorrelation (acf.) $c_h[m] = h[m] * \overline{h}[-m]$. In fact,

$$|H[k]|^2 = \sum_{p=0}^{L}\sum_{q=0}^{L} h[p] \cdot h^*[q] \cdot e^{-j2\pi(p-q)k/N} \quad (17)$$

Substituting $m = p - q$, we have

$$|H[k]|^2 = \sum_{m=-L}^{L} c_h[m] \cdot e^{-j2\pi mk/N} \quad (18)$$

resulting also $c_h[m] = \overline{c}_h[-m]$. Considering (18), we observe that the weights σ_U^2 expressed in (14) depend only on the acf. coefficients as well. In fact, using the (18), we can write for even L:

$$\begin{aligned}\sigma_U^2(k) &= \sigma_V^2\left(2c_h[0] + \sum_{m=-L}^{L} c_h[m] \cdot e^{-j2\pi mk/N}(1+(-1)^m)\right) \\ &= 2\sigma_V^2\left(c_h[0] + 2\sum_{m=1}^{L/2} \Re\left\{c_h[2m] \cdot e^{-j2\pi 2mk/N}\right\}\right)\end{aligned}$$

Introducing the vectors

$$\begin{aligned}\boldsymbol{\sigma}_U^2 &= \left[\sigma_U^2(0) \cdots \sigma_U^2(N/2-1)\right]^{\mathrm{T}} \\ \mathbf{c}_h &= \left[c_h[0], c_h[2], \ldots c_h[L]\right]^{\mathrm{T}}\end{aligned}$$

and the matrix $\mathbf{Q}$ whose components are

$$\|\mathbf{Q}\|_{(k,m)} \stackrel{\mathrm{def}}{=} 2\sigma_V^2(1+\delta[m]) \cdot e^{-j2\pi 2mk/N}$$

the previous equation can be written as:

$$\boldsymbol{\sigma}_U^2 = \mathbf{Q} \cdot \mathbf{c}_h \tag{19}$$

Denoting by $\mathbf{R}$ the $N/2$ vector of components ($k = 0, ..N/2-1$):

$$\|\mathbf{R}\|_k = \left(|R[k]|^2 + |R[k+N/2]|^2 - 2\sigma_V^2\right)/\sigma_A^2$$

a consistent unbiased estimation of the $L/2+1$ vector $\mathbf{c}_h$ can be obtained as follows:

$$\hat{\mathbf{c}}_h = \mathbf{Q}^+ \cdot \mathbf{R} \tag{20}$$

where $^+$ denotes pseudoinversion. The vector $\mathbf{c}_h$ is directly estimated by the power spectrum of the observations and therefore it remains well estimated even when channel order is overestimated. Therefore, the channel order may be estimated by analysing the amplitude of the coefficients in $\mathbf{c}_h$.

From (19) and (20), a direct estimation of the weights is obtained as:

$$\hat{\boldsymbol{\sigma}}_U^2 = \mathbf{Q}\mathbf{Q}^+ \cdot \mathbf{R} \tag{21}$$

The matrix $\mathbf{Q}\mathbf{Q}^+$ does not depend on the observed data and therefore it can be computed once. Since the estimate of $\boldsymbol{\sigma}_U^2$ is consistent, then the weighted LS solution given by scaling (13) with $\hat{\boldsymbol{\sigma}}_U^2$ is also asymptotically efficient. Note that since the weighing consists only in normalizing the equations by $\hat{\boldsymbol{\sigma}}_U^2$, then numerical problems due to small values may be easily monitored.

3 *Simulation* Results *and Conclusion*

Simulation results are provided to assess the validity of the proposed one step optimal solution (FCR), in comparison to the CR method [1], the two step procedure (TSML) and the Cramer Rao Bound (CRB) drawn in [3]. The experimental conditions are the same reported in [3]. In particular, the channel impulse response is $\mathbf{h}=[1, 1, -2\cos\theta, -2\cos(\theta+\delta), 1, 1]^{\mathrm{T}}$ for $\theta = \pi/10$. The parameter δ affects the identifiability of the channel: for $\delta = 0$ the channel is not identifiable. The simulations are performed *vs.* SNR, and *vs.* the channel parameter δ and refer to $N_r = 100$ Montecarlo trials. The SNR is defined as $\mathrm{SNR}_{dB} = 20\log_{10}\left(\frac{\|\mathbf{h}\|\sigma_a}{\sqrt{2}\sigma_u}\right)$. A sequence of $N = 30$, zero-mean, i.i.d., binary (± 1) symbols is considered. The mean square error of the channel parameters is $\mathrm{MSE}_{\mathrm{dB}} = 20\log_{10}\left(\frac{1}{\|\mathbf{h}\|}\sqrt{\frac{1}{N_r}\sum_{i=1}^{N_r}\|\tilde{\mathbf{h}}-\mathbf{h}\|^2}\right)$.

Comparable performances between TSML and FCR are observed as shown in figs. 2 and 3 where the mean square error of the channel estimate is plotted *vs.* SNR and a parameter δ . Although slightly less accurate w.r.t. the TMSL method, the proposed FCR method presents considerable computational advantages; in fact FCR method requires $O(N/2\log(N))$ operations *vs.* $O(N^3)$ operations required by TMSL and the FCR solution is directly obtained through a single step procedure.

References

[1] G. Xu, H. Liu, L. Tong, T. Kailath, "A deterministic approach to blind channel identification", *IEEE Trans. Sig. Proc.*, Dec. 1995.

[2] E. Moulines, P. Duhamel, J.F. Cardoso, "Subspace methods for the blind identification of multichannel FIR filter", *IEEE Trans. Signal Processing*, Vol. 43, n. 2, pp. 516-525, Feb. 1995.

[3] Y .Hua, "Fast Maximum likelihood for blind identification of multiple FIR channel", *IEEE Trans. Sig. Proc.*, March 1996.

[4] K. Meraim, J.F. Cardoso, A.Y. Gorokhov, P. Loubaton, E. Moulines, "On subspace methods for the blind identification of single-input multiple-output FIR systems", *IEEE Trans. Signal Processing*, Vol. 45, n. 1, pp. 42-55, Jan. 1997.

[5] L. Tong, G. Xu, B. Habissi, T. Kailath, "Blind channel identification based on second-order statistics: a frequency-domain approach", *IEEE Trans. Inform. Theory*, Vol. 41, n. 1, pp. 329-334, Jan 1995.

[6] G.B. Giannakis, "Linear cyclic correlation approaches for blind identification of FIR channel", *Proc. IEEE 28th Asilomar Conf. on Sig., Syst. and Computers*, Pacific Grove, CA, Nov. 1994.

[7] C. Becchetti, G. Jacovitti, G. Scarano, "Fast blind identification of FIR communications channels", *Proc. of the IEEE Int. Conf. on Acoustics, Speech and Signal Processing*, Seattle (WA), USA, May 1998.

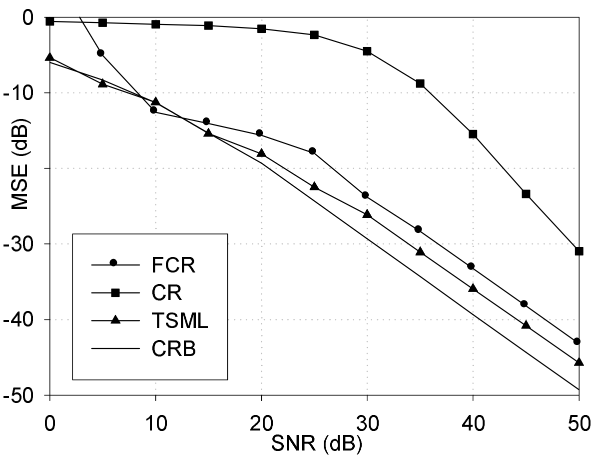

Figure 2: *Mean Square Error of channel parameter estimates versus SNR. ($\delta = \pi/10$).*

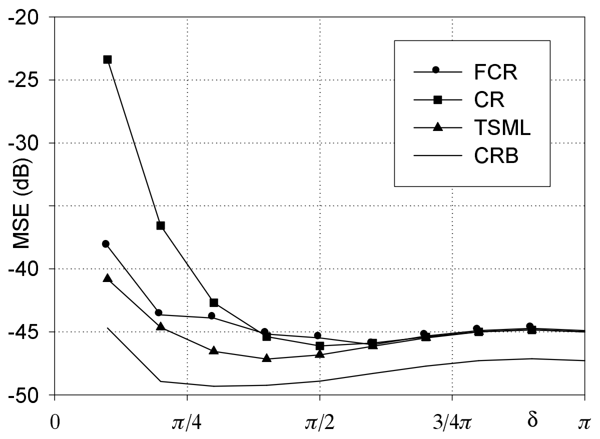

Figure 3: *Mean Square Error of channel parameter estimates versus the parameter δ related to identifiability of the channel. ($SNR = 45dB$).*